1. Prove that $n! < n^n$, where $n \geq 2$ is an integer.

   Remember to divide your proof up into these three steps:

   Basis step: Show for $n = 2$.

   Inductive hypothesis: Assume the statement for some n.

   Inductive step: Show that your assumption for n means the statement is true for $n+1$ as well. (To complete the inductive step, you'll need to use your inductive hypothesis at least once!)

2. Prove that for any positive integer n

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

3. Prove that for any positive integer n

$$\sum_{i=1}^{n} i \cdot i! = 1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$$

4. Prove that for any positive integer n

$$\sum_{i=1}^{n} i(i+1)(i+2) = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4$$

5. Prove that for any nonnegative integer n, 6 divides $n^3 - n$.

6. Prove that if n is an integer where $n \geq 3$, then $n^2 - 7n + 12$ is nonnegative.

7. Prove that for any nonnegative integer n where $n \neq 2$ and $n \neq 3$, the inequality $n^2 \leq n!$ is true.