Consensus and Reliable Broadcast

Broadcast

- If a process sends a message m, then every process eventually delivers m

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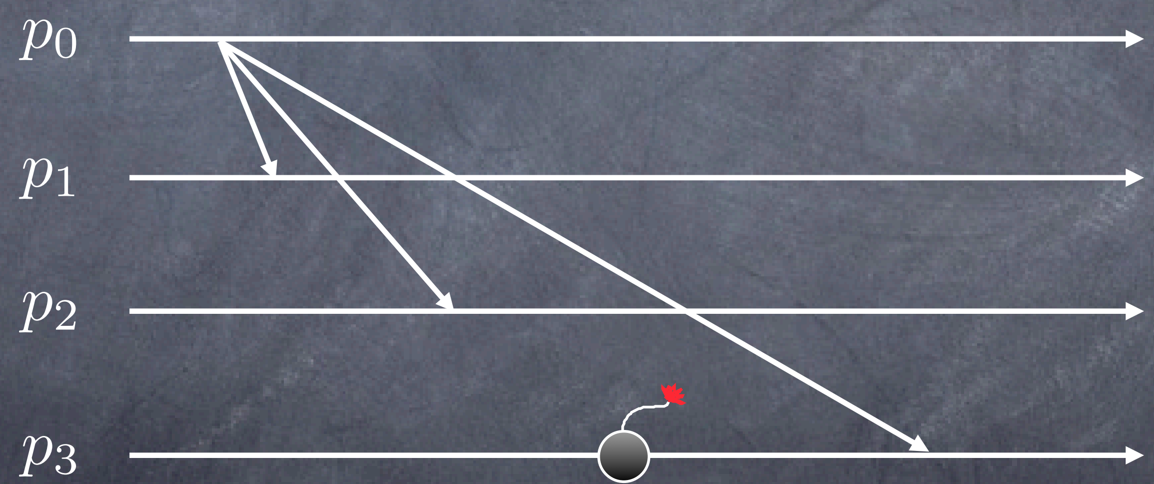

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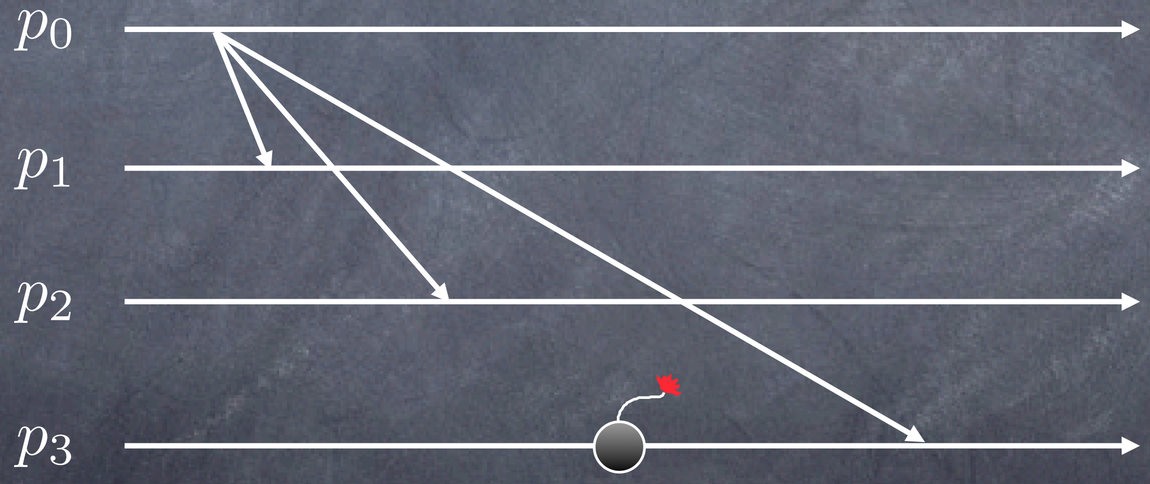
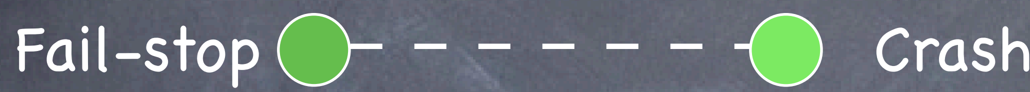


- How can we adapt the spec for an environment where processes can fail? And what does "fail" mean?

A hierarchy of failure models

- Crash

A hierarchy of failure models



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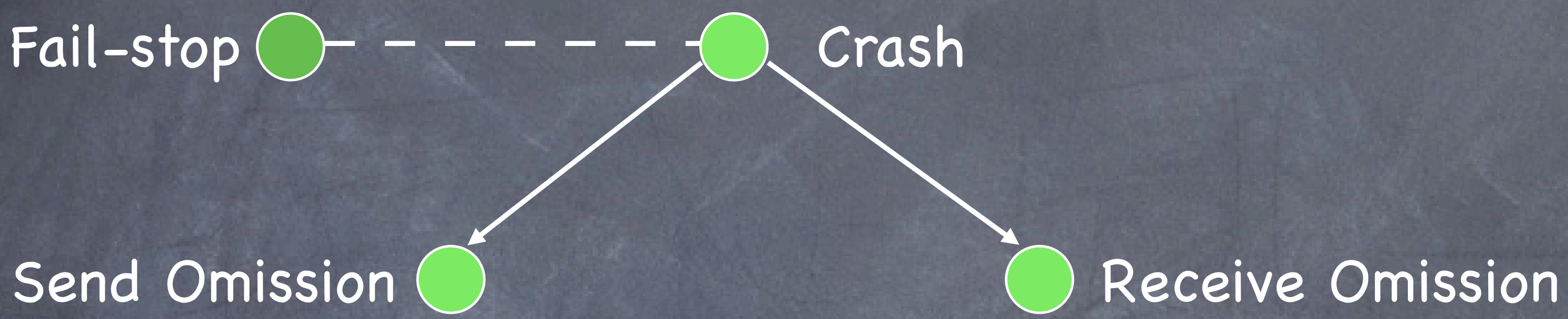

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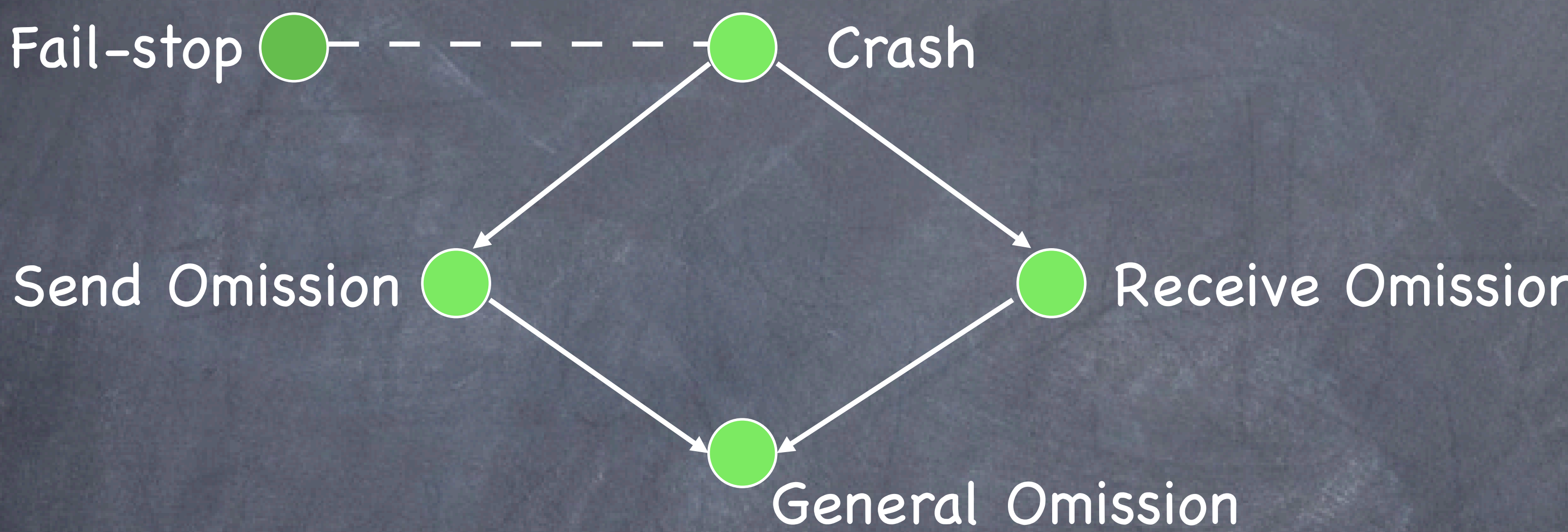

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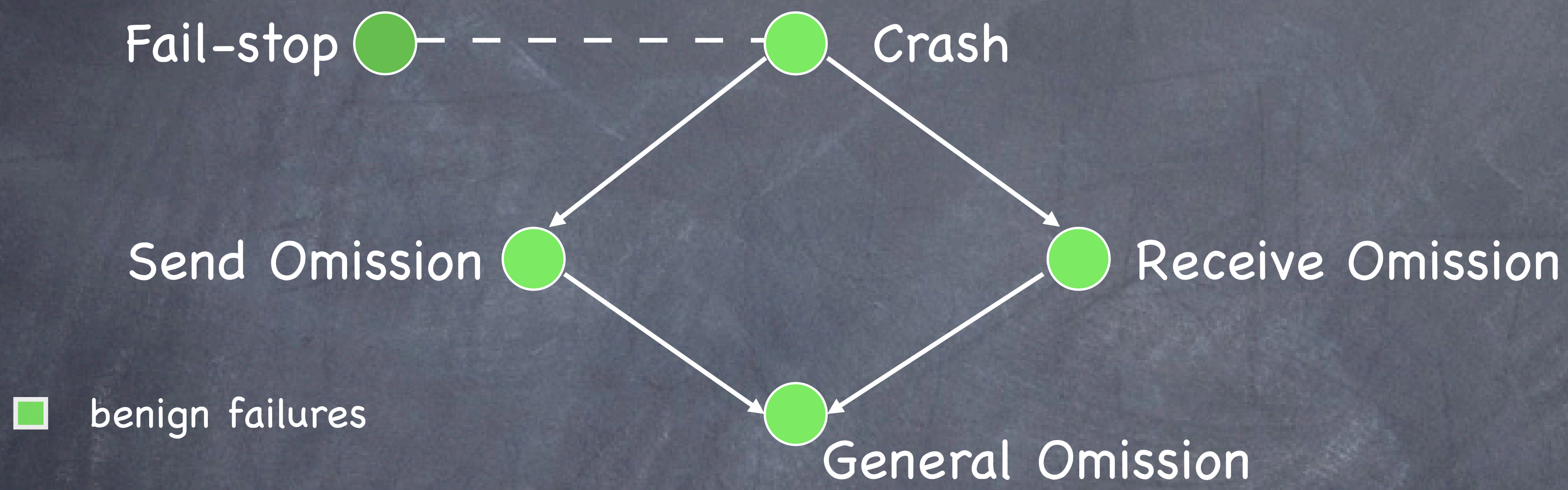

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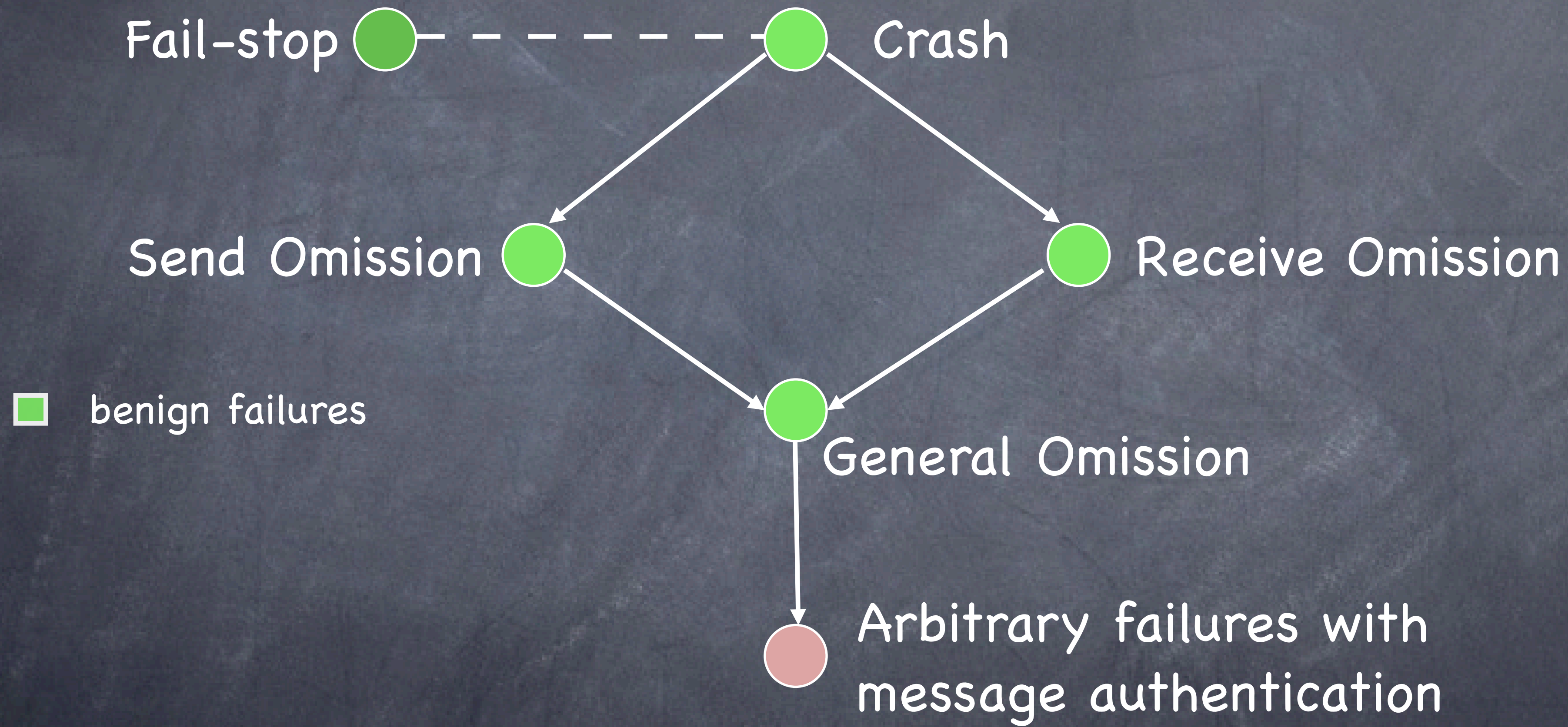

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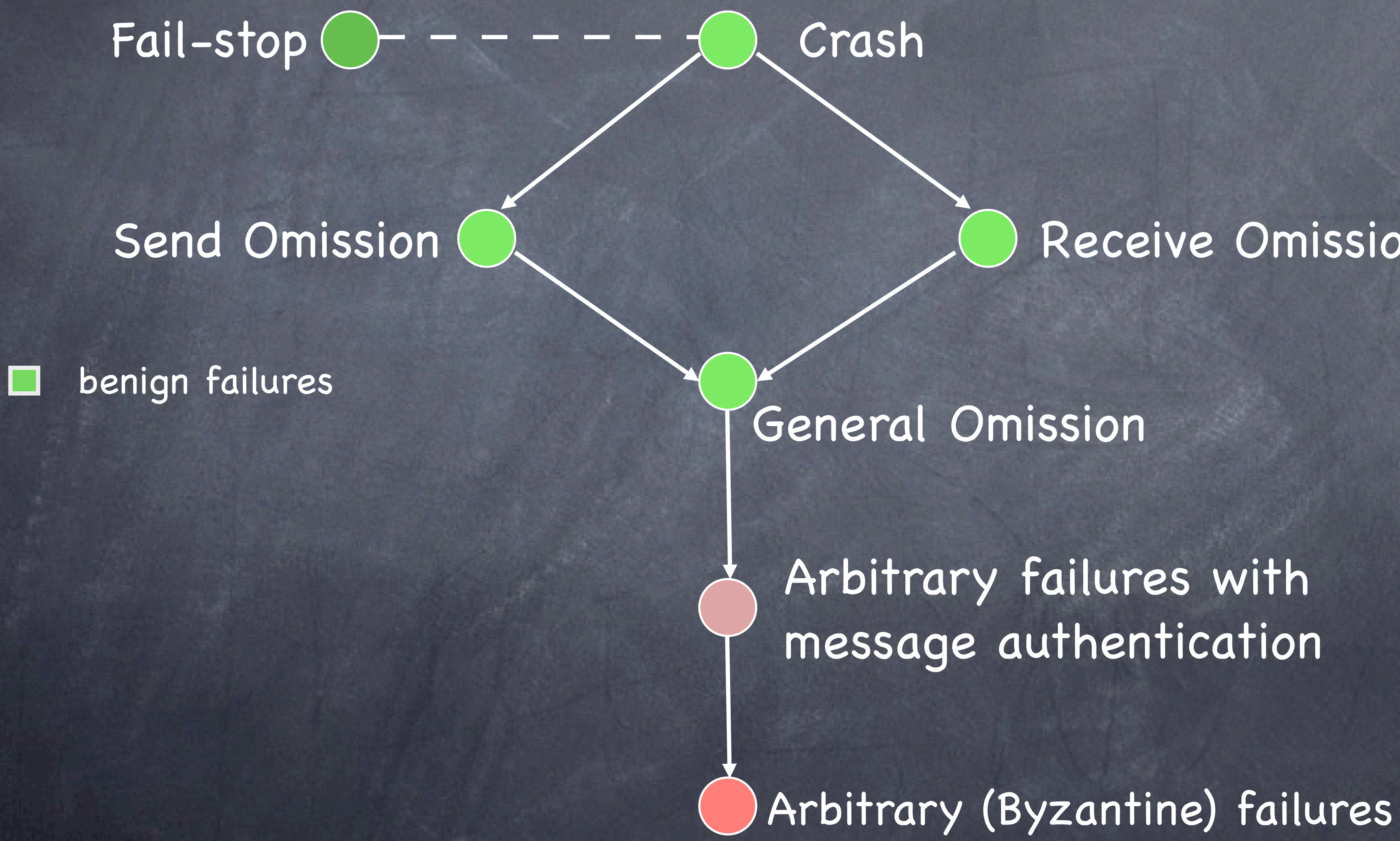

Reliable Broadcast

Validity	If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m
Agreement	If a correct process delivers a message m, then all correct processes eventually deliver m
Integrity	Every correct process delivers at most one message, and if it delivers m, then some process must have broadcast m

Terminating Reliable Broadcast

Validity	If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m
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Termination	Every correct process eventually delivers some message

Consensus

Validity	If all processes that propose a value propose v, then all correct processes eventually decide v
Agreement	If a correct process decides v, then all correct processes eventually decide v
Integrity	Every correct process decides at most one value, and if it decides v, then some process must have proposed v
Termination	Every correct process eventually decides some value

Properties of send(m) and receive(m)

Benign failures:

Validity If p sends m to q, and p , q, and the link between them are correct, then q eventually receives m

Uniform* Integrity For any message m, q receives m at most once from p, and only if p sent m to q

* A property is uniform if it applies to both correct and faulty processes

Properties of send(m) and receive(m)

Arbitrary failures:

Integrity For any message m , if p and q are correct then q receives m at most once from p, and only if p sent m to q

Questions, Questions...

- Are these problems solvable at all?
- Can they be solved independent of the failure model?
- Does solvability depend on the ratio between faulty and correct processes?
- Does solvability depend on assumptions about the reliability of the network?
- Are the problems solvable in both synchronous and asynchronous systems?
- If a solution exists, how expensive is it?

Plan

- Synchronous Systems
  - Consensus for synchronous systems with crash failures
  - Lower bound on the number of rounds
  - Reliable Broadcast for arbitrary failures with message authentication
  - Lower bound on the ratio of faulty processes for Consensus with arbitrary failures
  - Reliable Broadcast for arbitrary failures
- Asynchronous Systems
  - Impossibility of Consensus for crash failures
  - Failure detectors
  - PAXOS

Model

- Synchronous Message Passing
  - Execution is a sequence of rounds
  - In each round every process takes a step
    - sends messages to neighbors
    - receives messages sent in that round
    - changes its state
- Network is fully connected (an n-clique)
- No communication failures

A simple Consensus algorithm

Process p_i:

Initially $V = \{v_i\}$

To execute propose(v_i)

1: send $\{v_i\}$ to all

decide(x) occurs as follows:

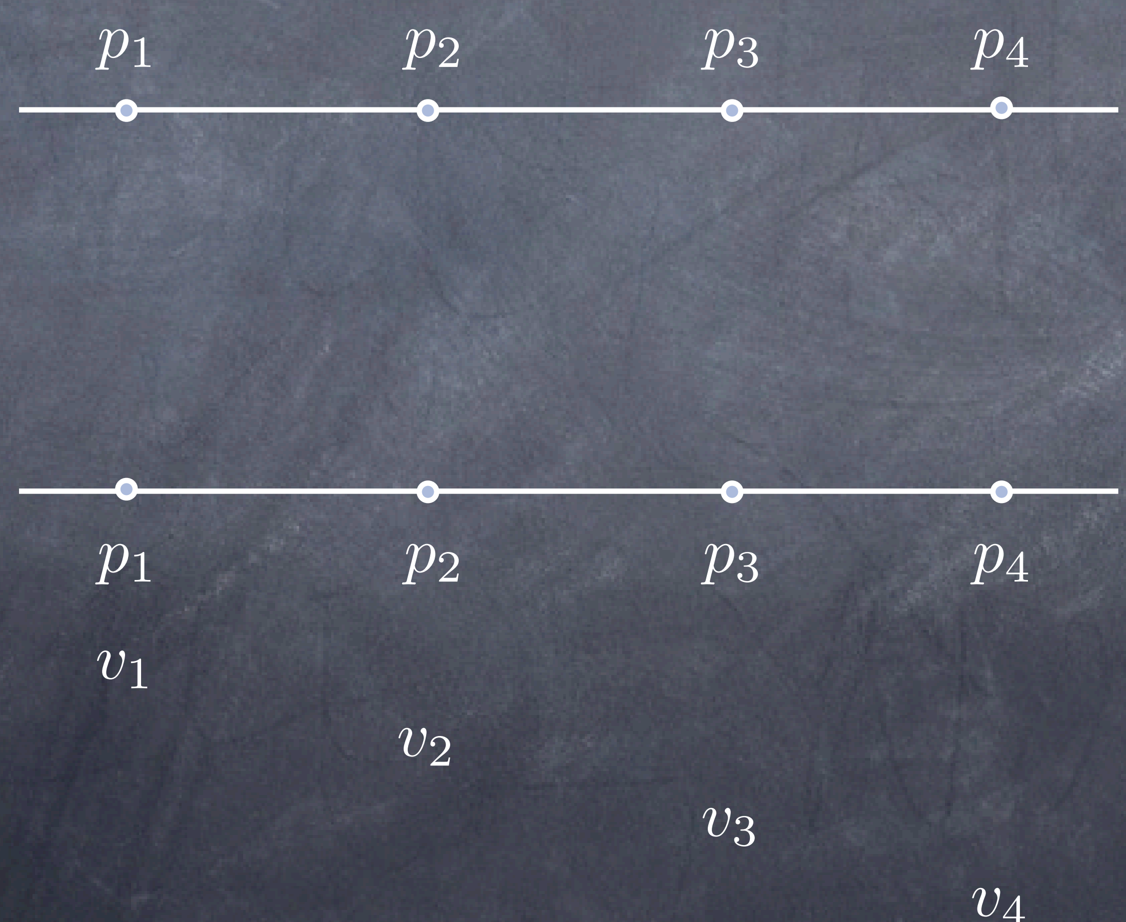
2: for all j, $0 \le j \le n-1$, $j \neq i$ do

3: receive S_j from p_j

4: $V := V \cup S_j$

5: decide min(V)

An execution



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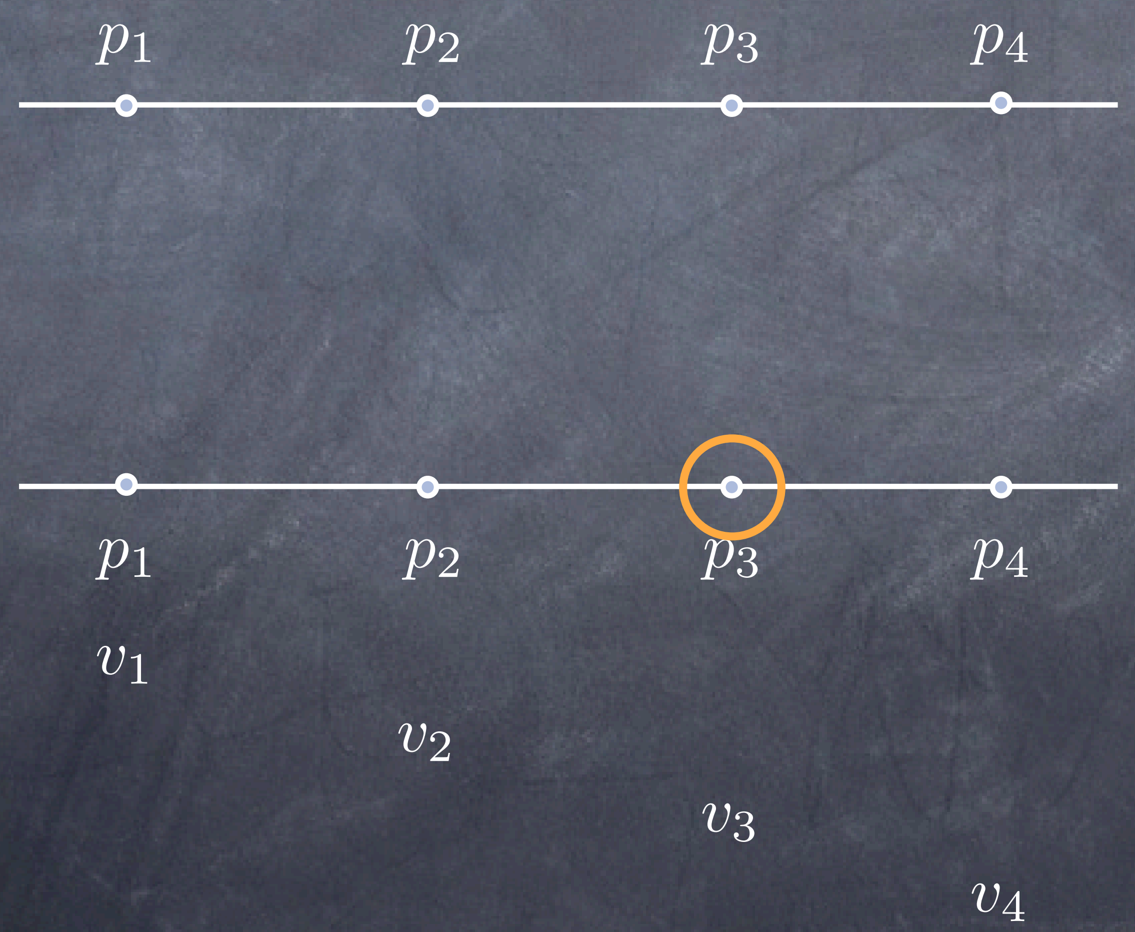

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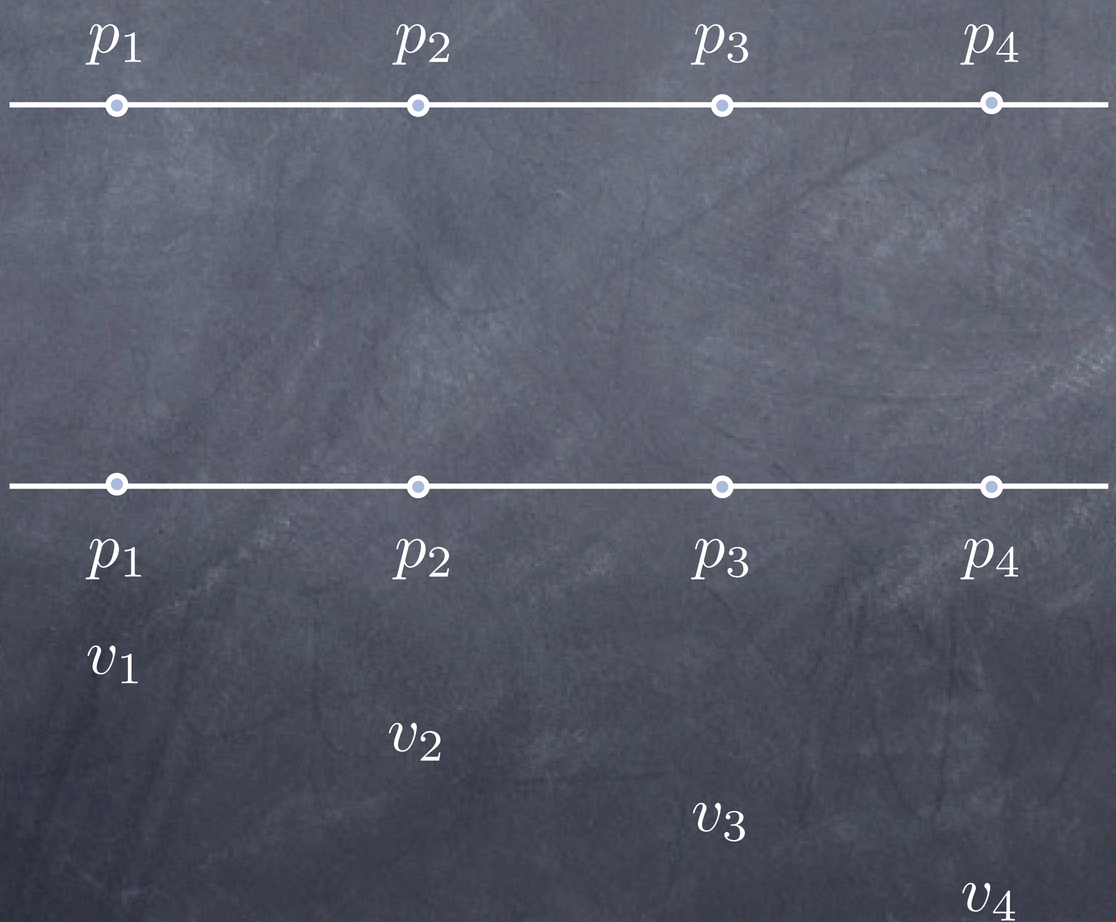
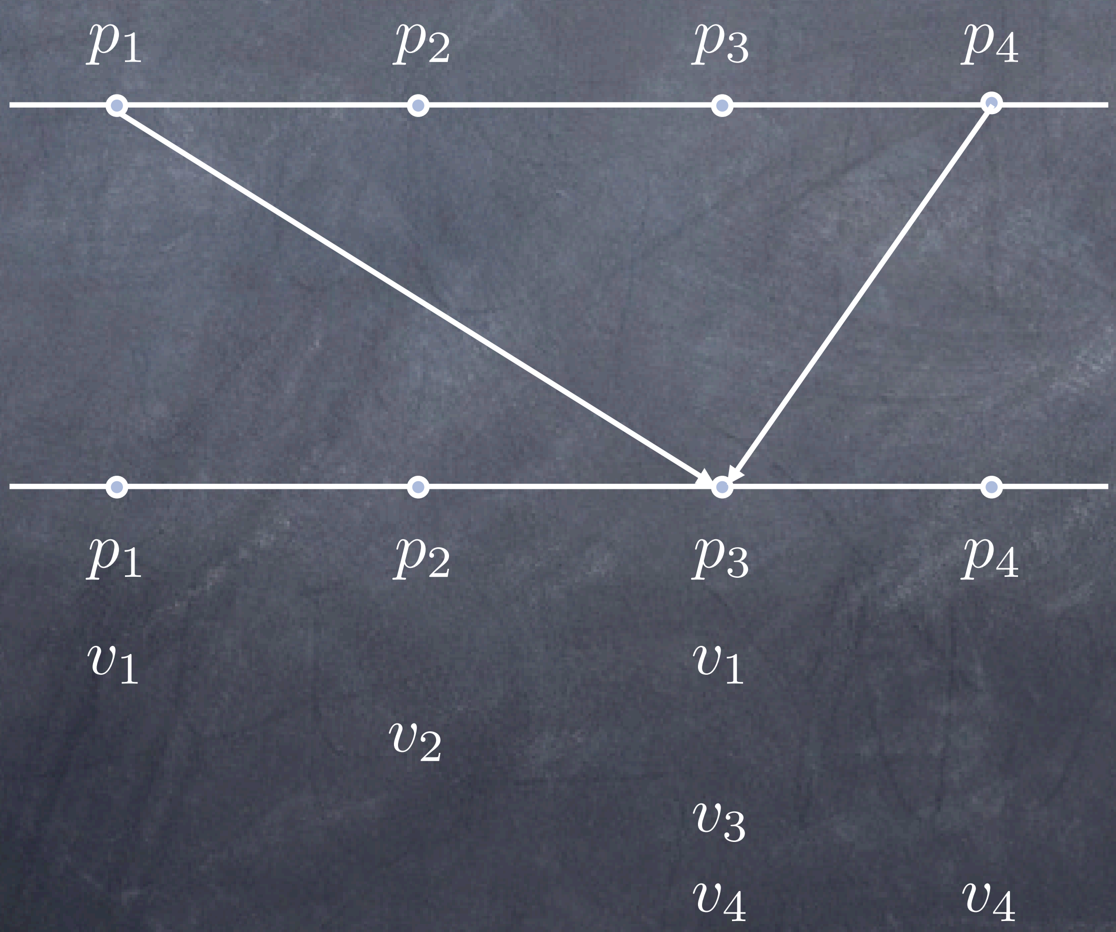


An execution

Suppose $v_1 = v_3 = v_4$ at the end of round 1

Can p_3 decide?



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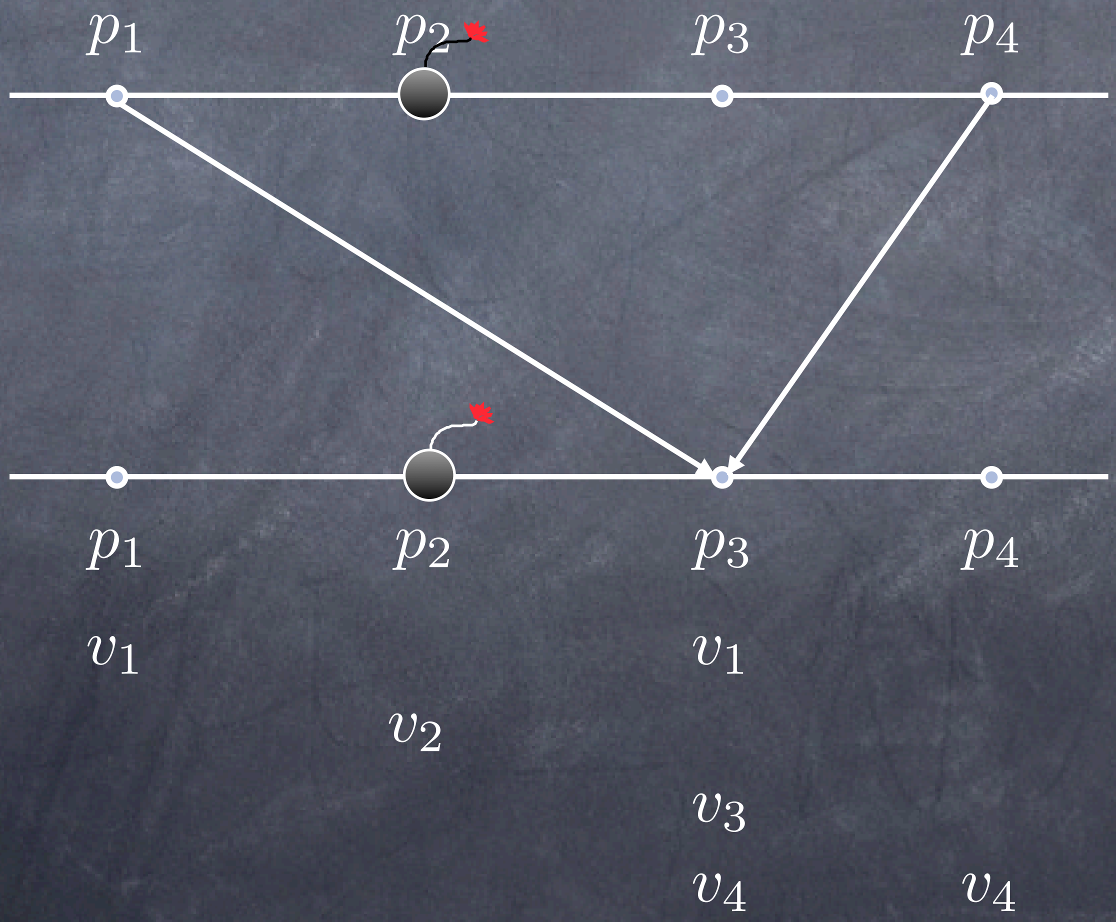

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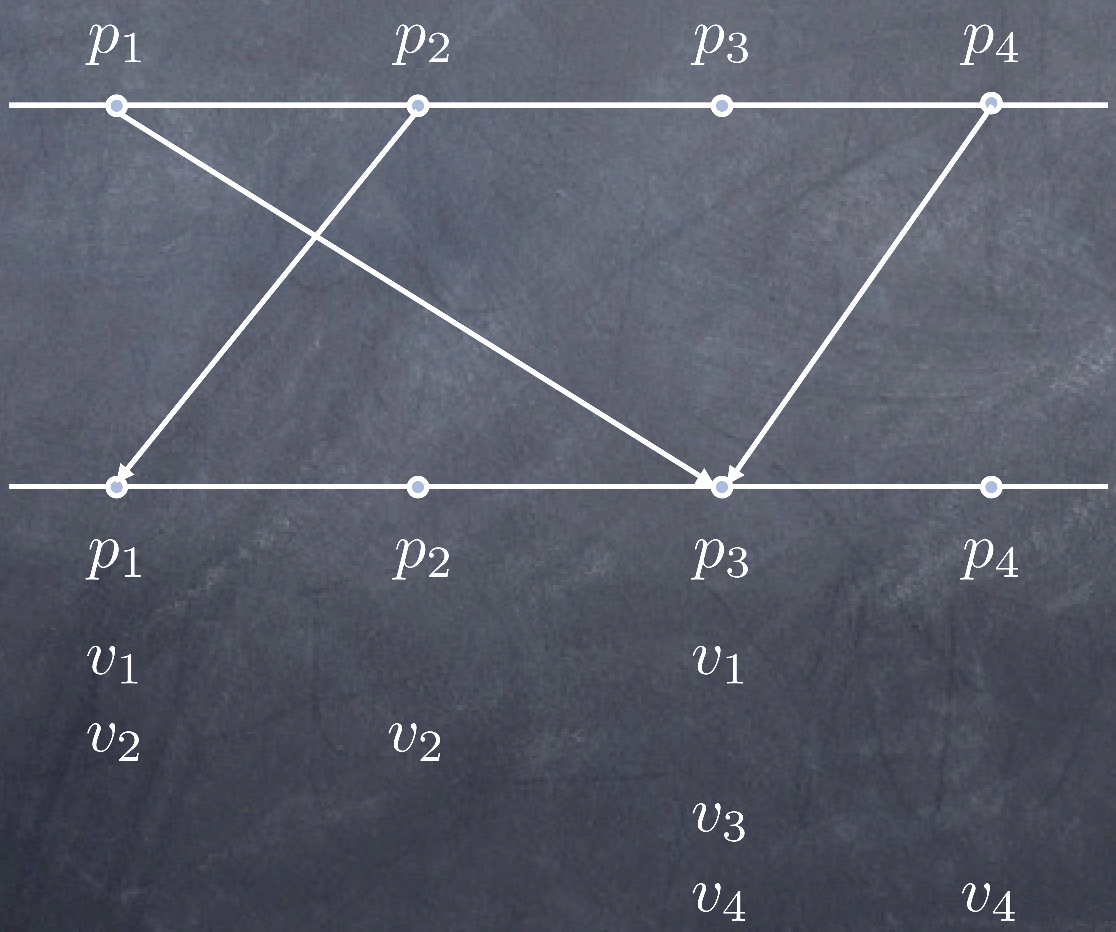

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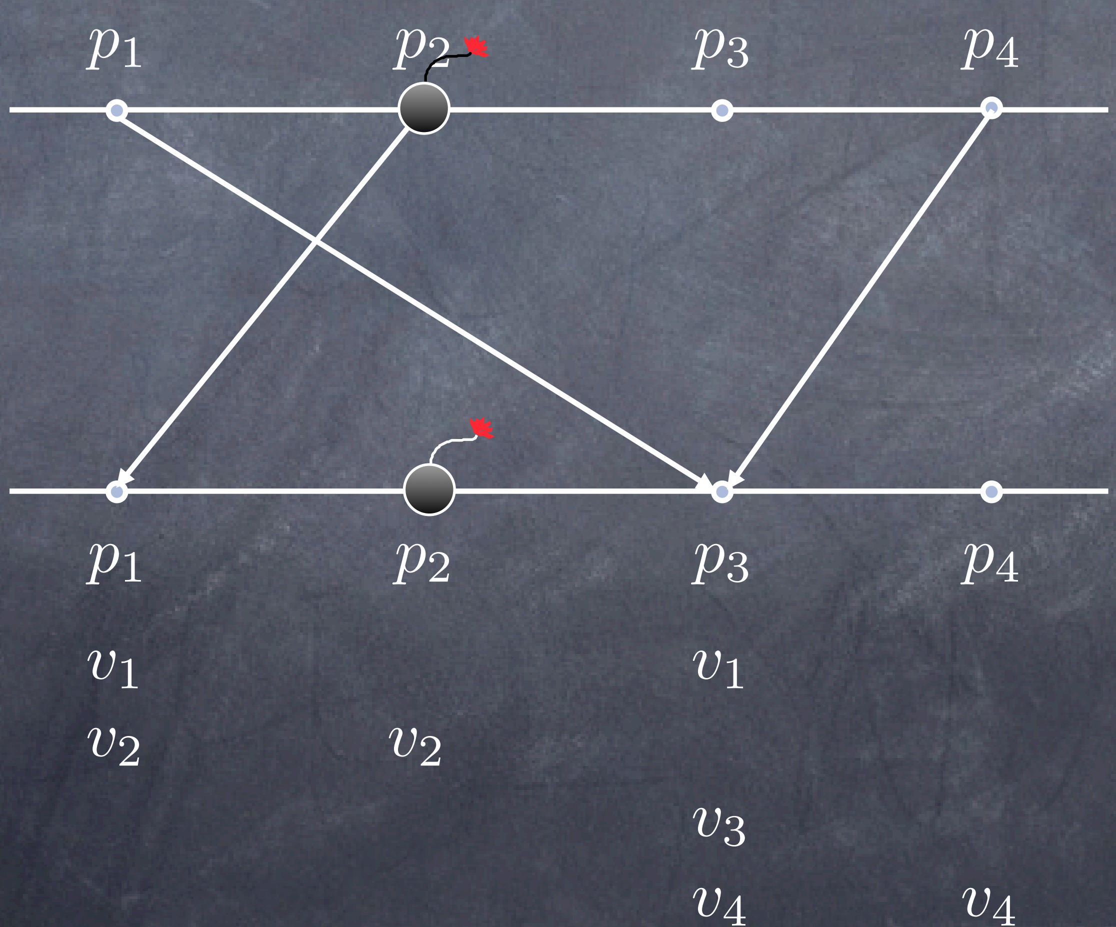

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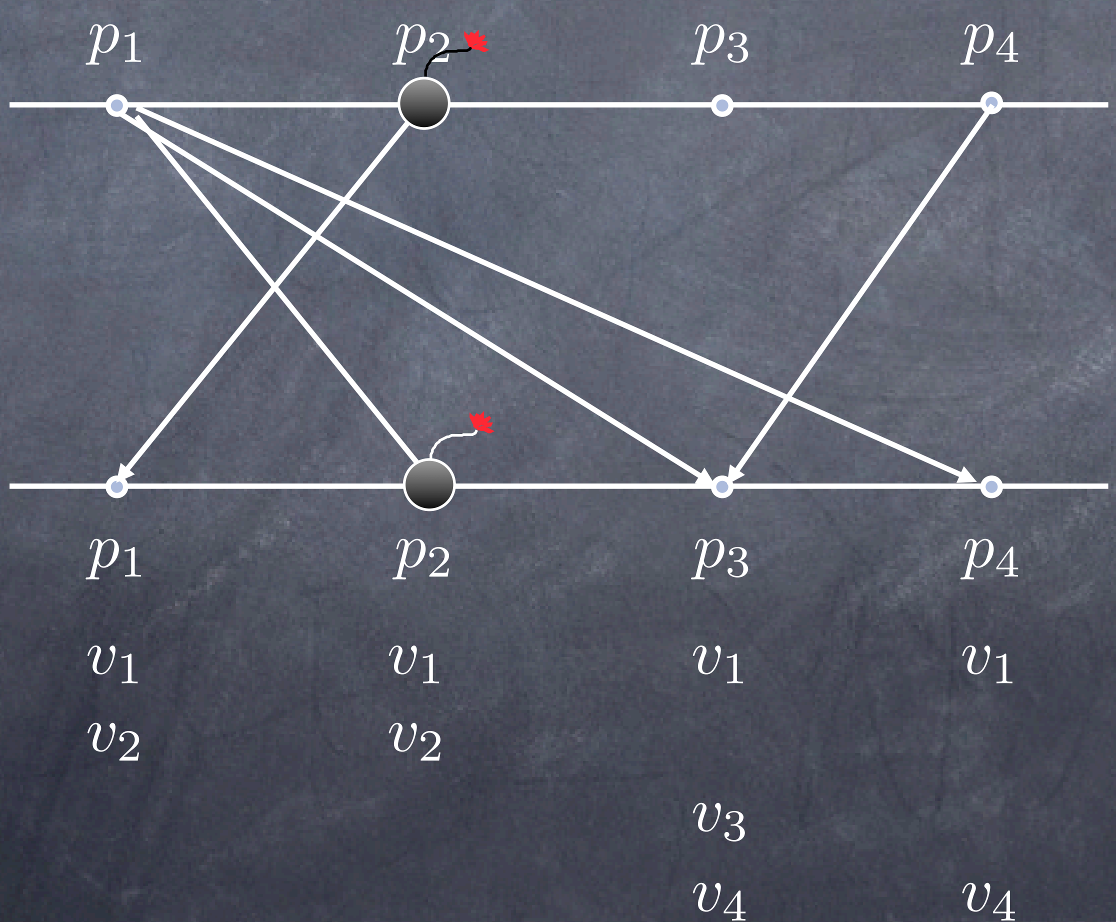

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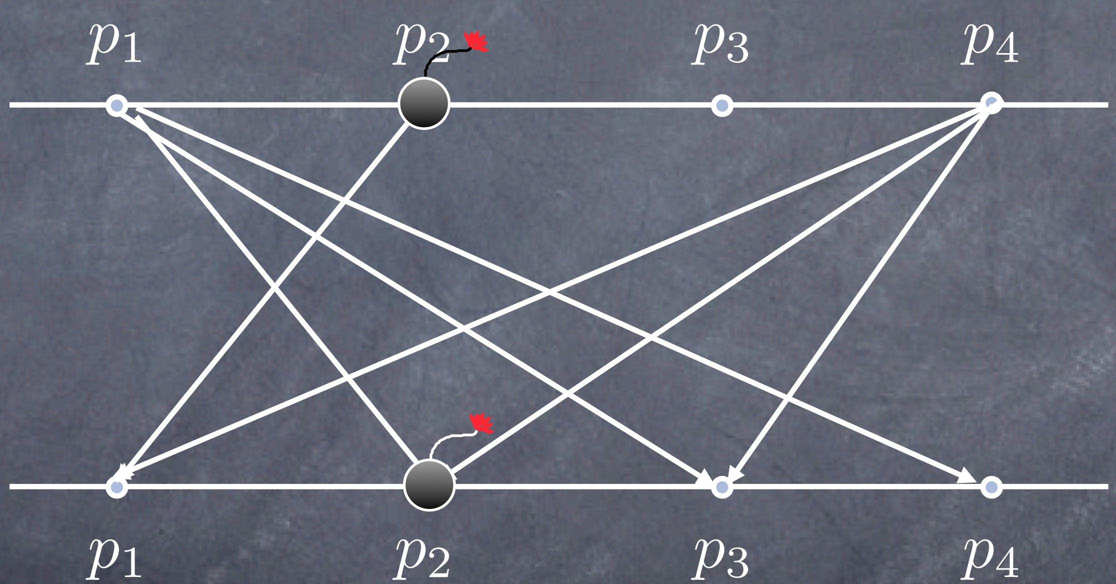

v_1	v_1	v_1	v_1
v_2	v_2		
		v_3	
v_4		v_4	v_4

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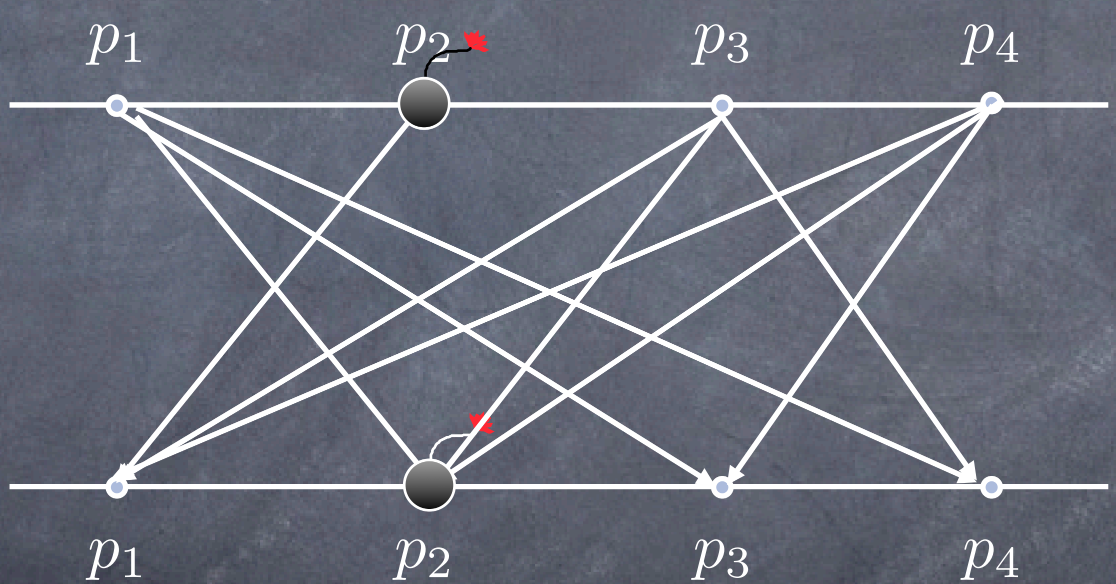

p_1	p_2	p_3	p_4
v_1	v_1	v_1	v_1
v_2	v_2		
v_3		v_3	v_3
v_4		v_4	v_4

Echoing values

- A process that receives a proposal in round 1, relays it to others during round 2.

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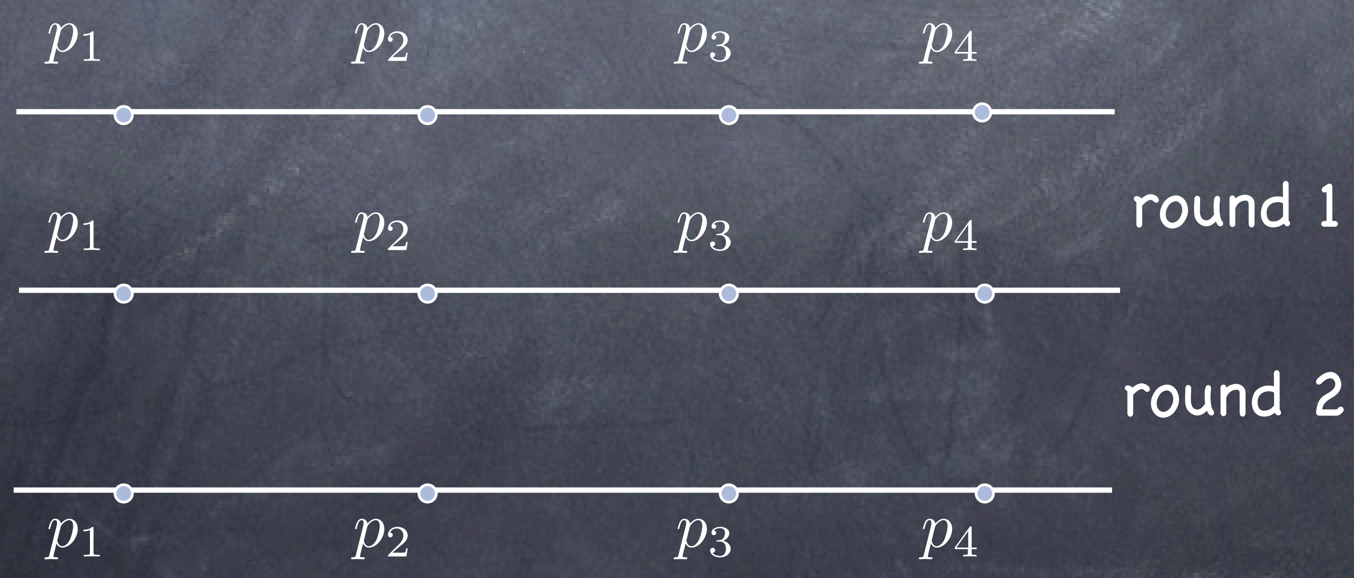

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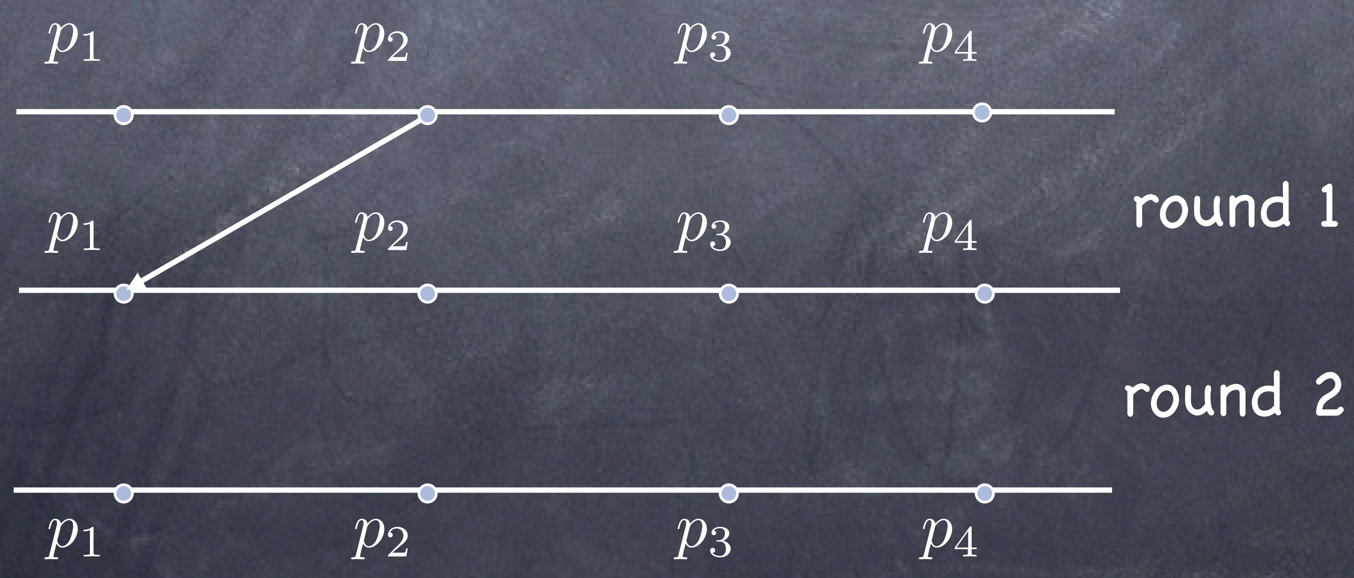

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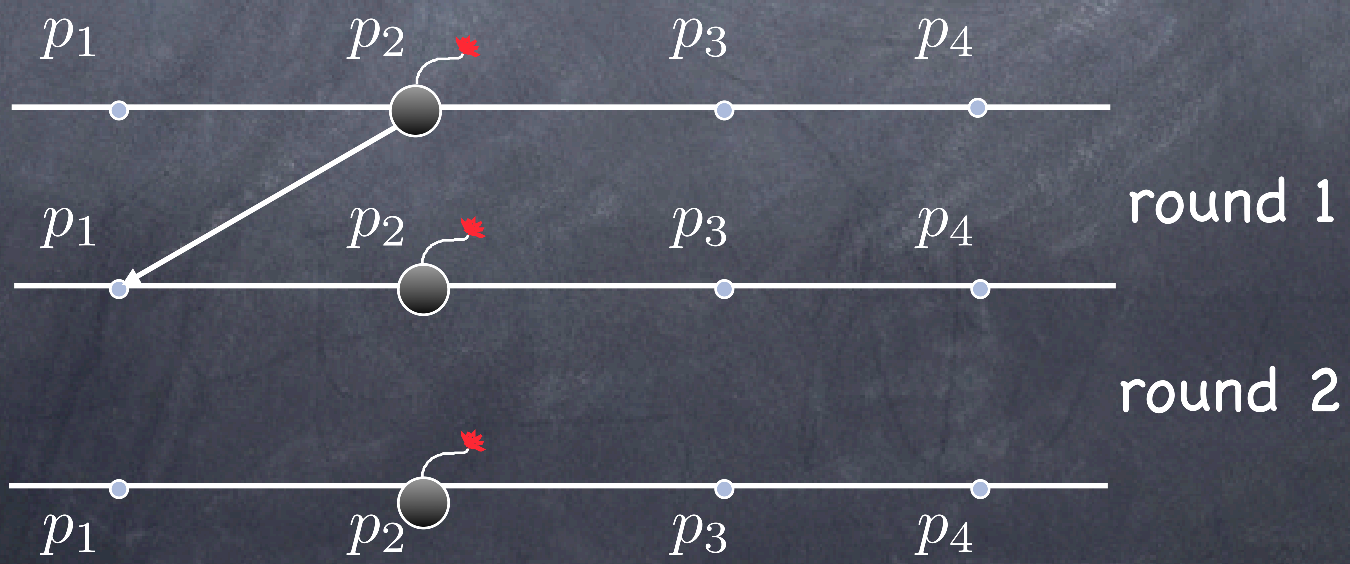

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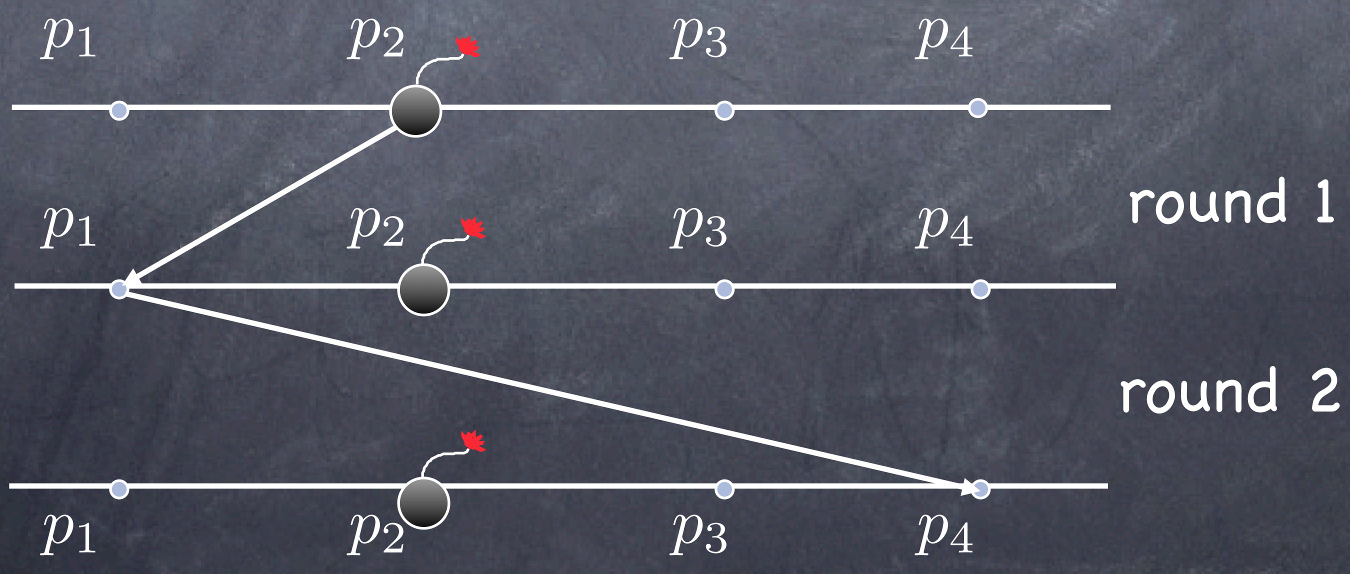

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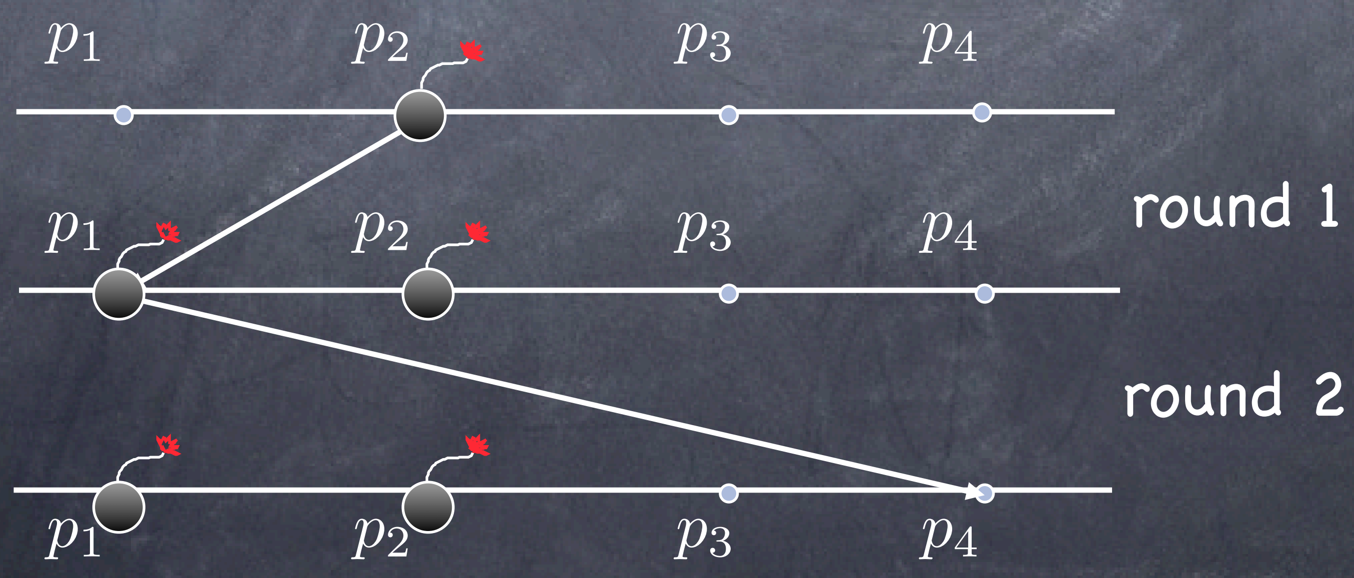


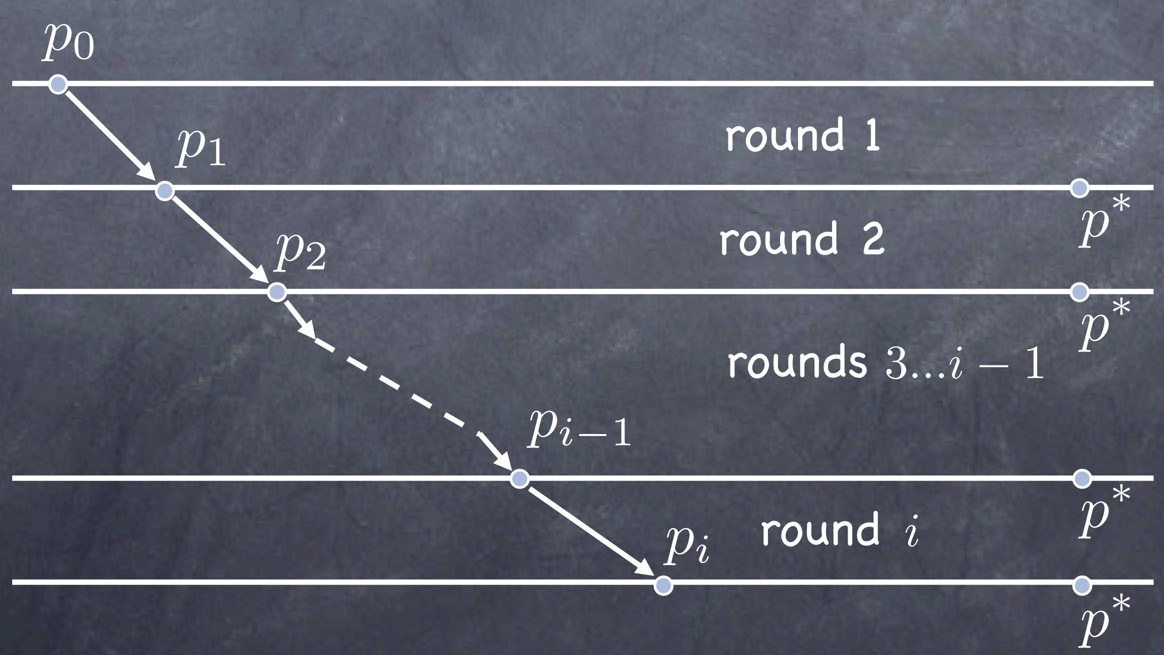
What is going on

- A correct process p^* has not received all proposals by the end of round i. Can p^* decide?
- Another process may have received the missing proposal at the end of round i and be ready to relay it in round $i+1$

Dangerous Chains

Dangerous chain

The last process in the chain is correct, all others are faulty



Living dangerously

How many rounds can a dangerous chain span?

- f faulty processes
- at most $f+1$ nodes in the chain
- spans at most f rounds

It is safe to decide by the end of round $f+1$!

The Algorithm

Code for process p_i:

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V$: p_i has not already sent $v\}$ to all

2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do

3: receive S_j from p_j

4: V := $V \cup S_j$

decide(x) occurs as follows:

5: if $k = f+1$ then

6: decide min(V)

Termination and Integrity

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Only if it was proposed:

- To be decided upon, must be in V at round f+1
- if value = v_i, then it is proposed in round 1
- else, suppose received in round k. By induction:
- k = 1:
  - by Uniform Integrity of underlying send and receive, it must have been sent in round 1
  - by the protocol and because only crash failures, it must have been proposed
- Induction Hypothesis: all values received up to round k = j have been proposed
- k = j+1
  - sent in round j+1 (Uniform Integrity of send and synchronous model)
  - must have been part of V of sender at end of round j
  - by protocol, must have been received by sender by end of round j
  - by induction hypothesis, must have been proposed

Validity

```
Initially V = {v_i}

To execute propose(v_i)
    round k, 1 ≤ k ≤ f+1
      send {v ∈ V : p_i has not already sent v} to all
      for all j, 0 ≤ j ≤ n−1, j ≠ i do
        receive S_j from p_j
        V := V ∪ S_j

decide(x) occurs as follows:
  if k = f+1 then
    decide min(V)
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Validity

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$V := V \cup S_j$

decide(x) occurs as follows:

if $k = f+1$ then

decide min(V)

- Suppose every process proposes v^*
- Since only crash model, only v^* can be sent
- By Uniform Integrity of send and receive, only v^* can be received
- By protocol, $V = \{ v^* \}$
- min(V) = v^*
- decide(v^*)

Agreement

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Lemma 1

For any $r \geq 1$, if a process p receives a value v in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that $p_r = p$, p_0 is v's proponent, and in each round p_{k-1} sends v and p_k receives it. Furthermore, all processes in the sequence are distinct.

Proof

By induction on the length of the sequence

Agreement

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decide(x) occurs as follows:

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Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

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Agreement follows from Lemma 2, since min is a deterministic function

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Proof:

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- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \leq f$, then p^* sends x in round $r+1 \leq f+1$; every correct process receives x and adds x to its V in round $r+1$

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- If $r \leq f$, then p^* sends x in round $r+1 \leq f+1$; every correct process receives x and adds x to its V in round $r+1$
- What if $r = f+1$?
- By Lemma 1, there exists a sequence of distinct processes $p_0, \ldots, p_{f+1} = p^*$
- Consider processes $p_0, \ldots, p_f$
- $f+1$ processes; only f faulty
- one of $p_0, \ldots, p_f$ is correct, and adds x to its V before p^* does it in round r

CONTRADICTION!

Terminating Reliable Broadcast

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TRB for benign failures

Sender in round 1:

```
send m to all
```

Process p in round k, 1 ≤ k ≤ f+1

```
if delivered m in round k-1 and p ≠ sender then
  send m to all
  halt
receive round k messages
if received m then
  deliver(m)
  if k = f+1 then halt
else if k = f+1
  deliver(SF)
  halt
```

Terminates in $f+1$ rounds

How can we do better?

- find a protocol whose round complexity is proportional to t –the number of failures that actually occurred– rather than to f –the max number of failures that may occur

Early stopping: the idea

- Suppose processes can detect the set of processes that have failed by the end of round i
- Call that set $faulty(p, i)$
- If $|faulty(p, i)| < i$ there can be no active dangerous chains, and p can safely deliver SF

Early Stopping: The Protocol

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

if p = sender **then** value := m **else** value:= ?

Process p in round $k, 1 \leq k \leq f+1$

send value **to** all
if value ≠ ? **and** delivered m in round $k-1$ **then** halt
receive round k values **from** all
$faulty(p, k) := faulty(p, k-1) \cup \{ q \mid p$ received no value from q in round $k \}$
if received value v ≠ ? **then**
    value := v
    deliver value
else if $k = f+1$ **or** $|faulty(p, k)| < k$ **then**
    value := SF
    deliver value
    if $k = f+1$ **then** halt

Termination

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

if p = sender then value := m else value:= ?

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else if $k = f+1$ or $|faulty(p,k)| < k$ then
    value := SF
    deliver value
    if $k = f+1$ then halt

- If in any round a process receives a value, then it delivers the value in that round
- If a process has received only "?" for $f+1$ rounds, then it delivers SF in round $f+1$

Validity

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

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    value := SF
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- If the sender is correct then it sends m to all in round 1
- By Validity of the underlying send and receive, every correct process will receive m by the end of round 1
- By the protocol, every correct process will deliver m by the end of round 1

Agreement - 1

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
if p = sender then value := m else value:= ?
```

Process p in round $k, 1 \leq k \leq f+1$

```
send value to all
if value ≠ ? and delivered m in round k−1 then halt
receive round k values from all
faulty(p,k) := faulty(p,k−1) ∪ {q | p
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if received value v ≠ ? then
    value := v
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else if k = f+1 or |faulty(p,k)| < k then
    value := SF
    deliver value
    if k = f+1 then halt
```

Lemma 1:

For any $r \geq 1$, if a process p delivers $m \neq$ SF in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that p_0 = sender, $p_r = p$, and in each round $k, 1 \leq k \leq r$, p_{k-1} sent m and p_k received it. Furthermore, all processes in the sequence are distinct, unless $r = 1$ and $p_0 = p_1 =$ sender

Lemma 2:

For any $r \geq 1$, if a process p sets value to SF in round r, then there exist some $j \leq r$ and a sequence of distinct processes $q_j, q_{j+1}, \ldots, q_r = p$ such that q_j only receives "?" in rounds 1 to j, $|faulty(q_j, j)| < j$, and in each round $k, j+1 \leq k \leq r$, q_{k-1} sends SF to q_k and q_k receives SF

Agreement - 2

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

```
1:   if p = sender then value := m else value:= ?
```

Process p in round $k, 1 \leq k \leq f+1$

```
2:   send value to all
3:   if value ≠ ?  and delivered m in round k−1 then halt
4:   receive round k values from all
5:   faulty(p, k) := faulty(p, k − 1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ?  then
7:       value := v
8:       deliver value
9:   else if k = f+1 or |faulty(p, k)| < k then
10:      value := SF
11:      deliver value
12:      if k = f+1 then halt
```

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Agreement - 2

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

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    if $k = f+1$ then halt

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Proof

By contradiction

Suppose p sets value = m and q sets value = SF

By Lemmas 1 and 2 there exist

$$p_0, \ldots, p_r$$

$$q_j, \ldots, q_r$$

with the appropriate characteristics

Since q_j did not receive m from process p_{k-1} $1 \leq k \leq j$ in round k

q_j must conclude that $p_0, \ldots, p_{j-1}$ are all faulty processes

But then, $|faulty(q_j, j)| \geq j$

CONTRADICTION

Agreement - 3

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

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Agreement - 3

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    value := SF
    deliver value
    if k = f+1 then halt
```

Proof

If no correct process ever receives m, then every correct process delivers SF in round f + 1

Let r be the earliest round in which a correct process delivers value ≠ SF

$r \leq f$

- By Lemma 3, no (correct) process can set value differently in round r
- In round $r + 1 \leq f + 1$, that correct process sends its value to all
- Every correct process receives and delivers the value in round $r + 1 \leq f + 1$

$r = f + 1$

- By Lemma 1, there exists a sequence $p_0, \ldots, p_{f+1} = p_r$ of distinct processes
- Consider processes $p_0, \ldots, p_f$
  - f + 1 processes; only f faulty
  - one of $p_0, \ldots, p_f$ is correct-- let it be p_c
  - To send v in round c + 1, p_c must have set its value to v and delivered v in round c < r

CONTRADICTION

Integrity

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

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- At most one m
  - Failures are benign, and a process executes at most one deliver event before halting
- If $m \neq$ SF, only if m was broadcast
  - From Lemma 1 in the proof of Agreement

A Lower Bound

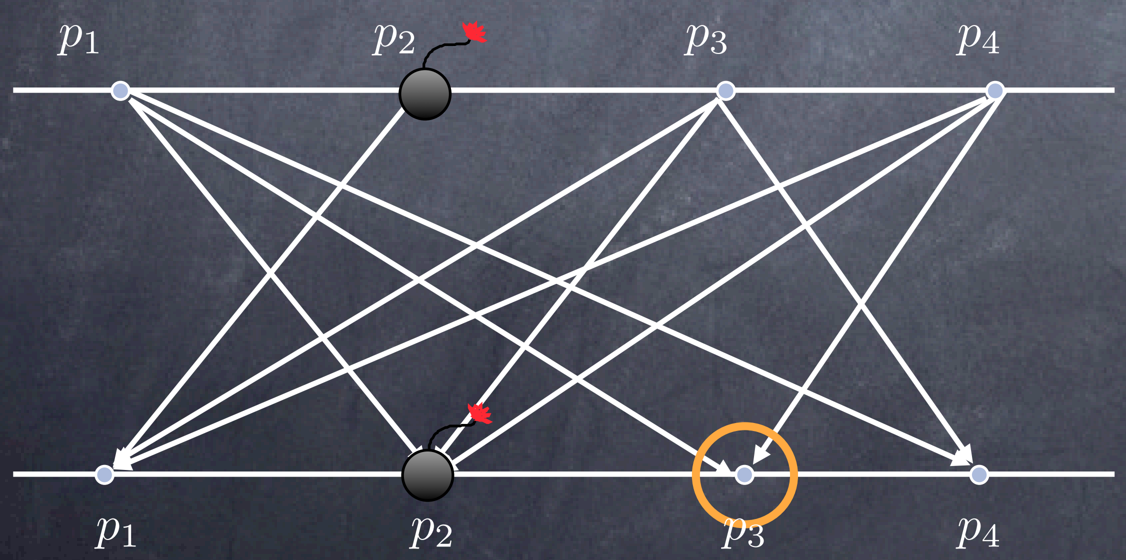
Theorem

There is no algorithm that solves the consensus problem in fewer than $f+1$ rounds in the presence of f crash failures, if $n \geq f+2$

We consider a special case $(f=1)$ to study the proof technique

Views

Let α be an execution. The view of process p_i in α, denoted by $\alpha|p_i$, is the subsequence of computation and message receive events that occur in p_i together with the state of p_i in the initial configuration of α



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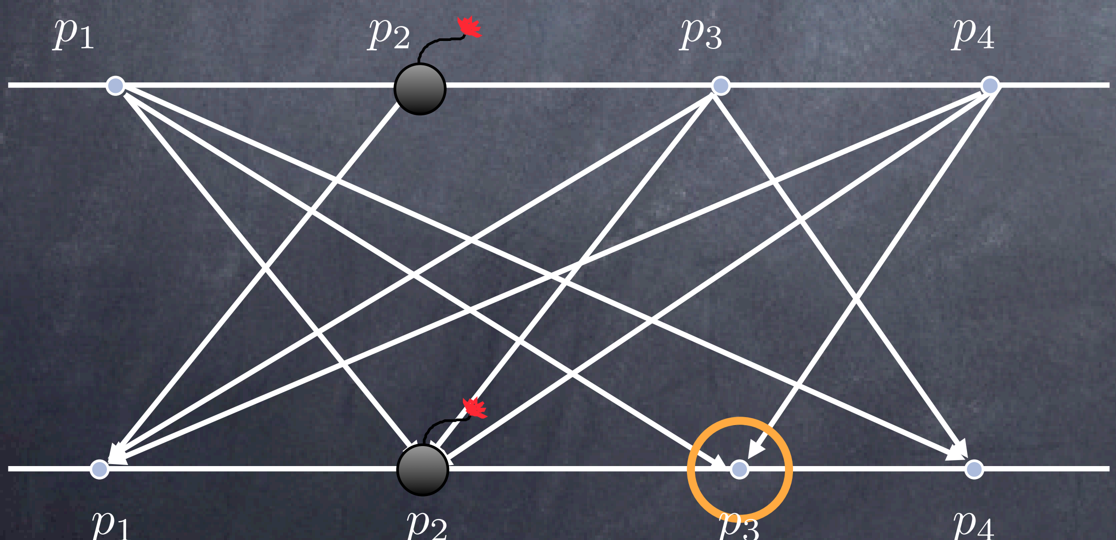

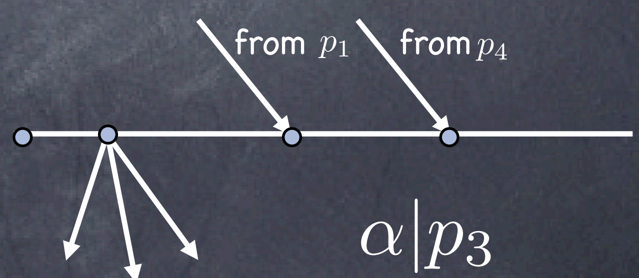

Similarity

Definition Let α_1 and α_2 be two executions of consensus and let p_i be a correct process in both α_1 and α_2.
α_1 is similar to α_2 with respect to p_i, denoted $\alpha_1 \sim_{p_i} \alpha_2$ if

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Lemma If $\alpha_1 \sim_{p_i} \alpha_2$ and p_i is correct, then dec(α_1) = dec(α_2)

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Lemma If $\alpha_1 \sim_{p_i} \alpha_2$ and p_i is correct, then dec(α_1) = dec(α_2)

The transitive closure of $\alpha_1 \sim_{p_i} \alpha_2$ is denoted $\alpha_1 \approx \alpha_2$.

We say that $\alpha_1 \approx \alpha_2$ if there exist executions $\beta_1, \beta_2, \ldots, \beta_{k+1}$ such that

$$\alpha_1 = \beta_1 \sim_{p_{i_1}} \beta_2 \sim_{p_{i_2}} \ldots, \sim_{p_{i_k}} \beta_{k+1} = \alpha_2$$

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Lemma If $\alpha_1 \approx \alpha_2$ then

dec(α_1) = dec(α_2)

Single-Failure Case

There is no algorithm that solves consensus in fewer than two rounds in the presence of one crash failure, if $n \geq 3$

The Idea

By contradiction

- Consider a one-round execution in which each process proposes 0. What is the decision value?
- Consider another one-round execution in which each process proposes 1. What is the decision value?
- Show that there is a chain of similar executions that relate the two executions.

So what?

α^is

Definition

α^i is the execution of the algorithm in which

- no failures occur
- only processes $p_0, \ldots, p_{i-1}$ propose 1

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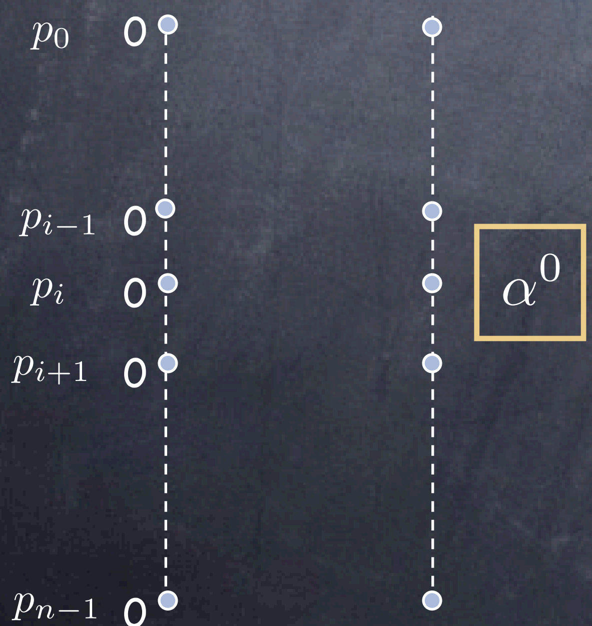

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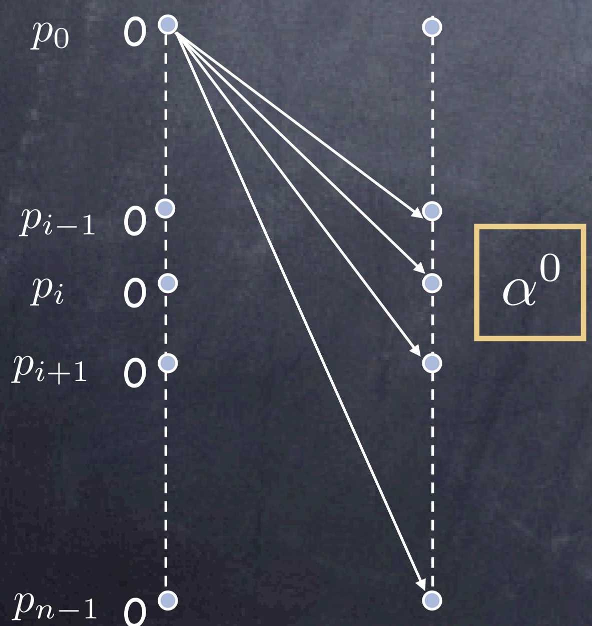

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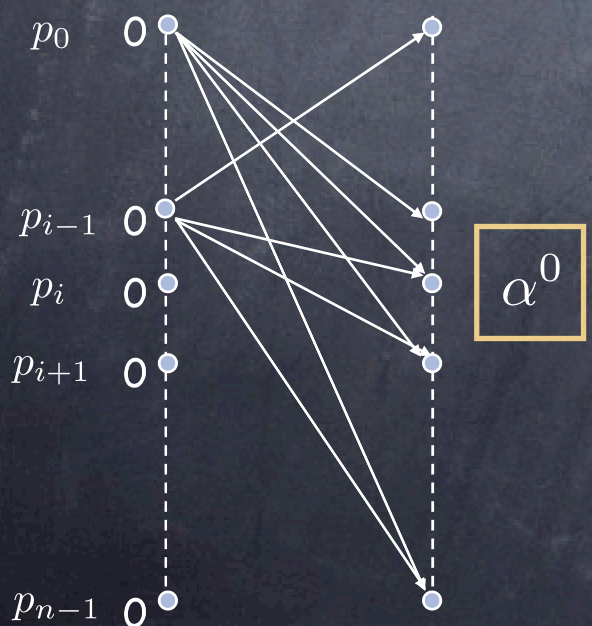

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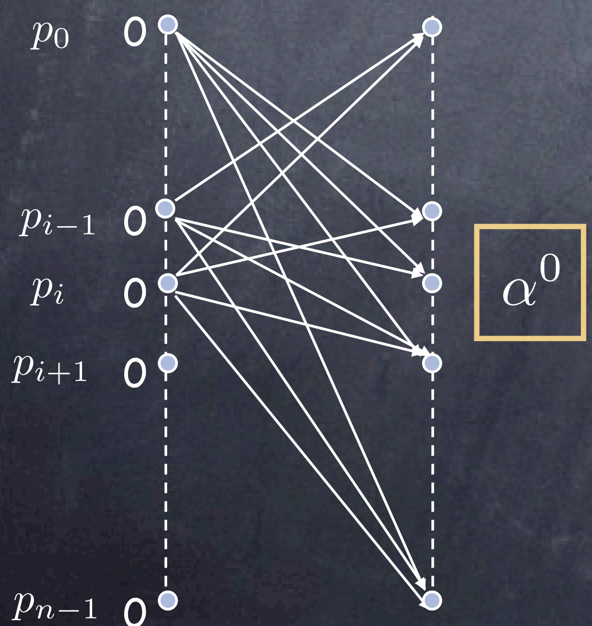

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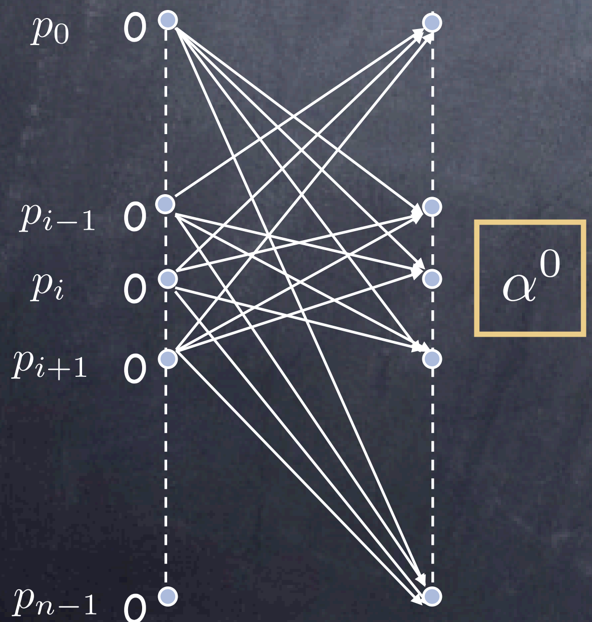

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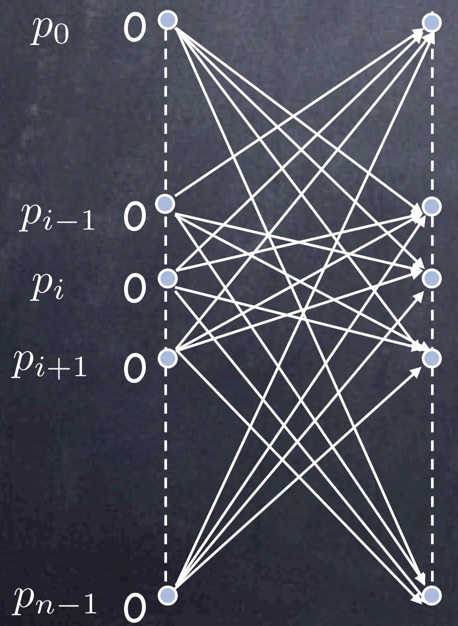

$\boxed{\alpha^0}$

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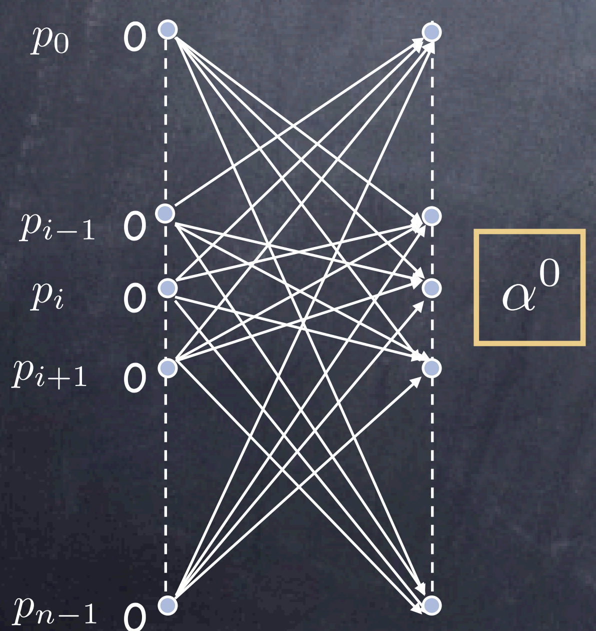

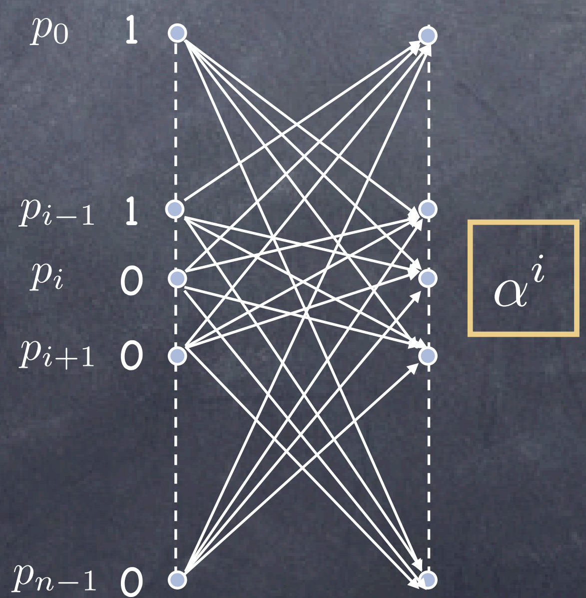

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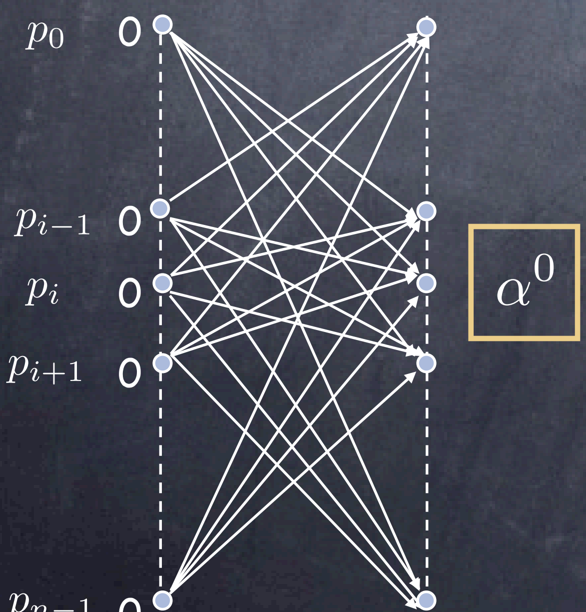

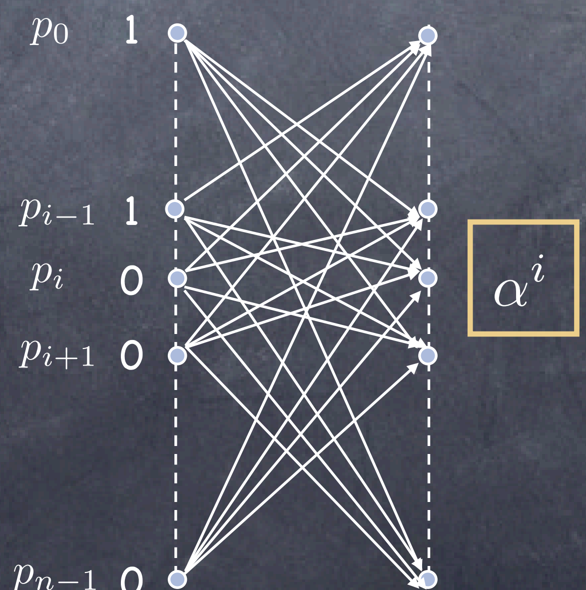

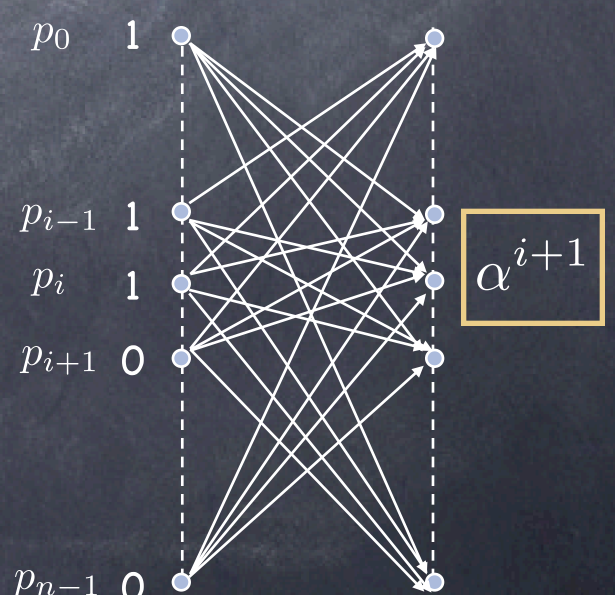

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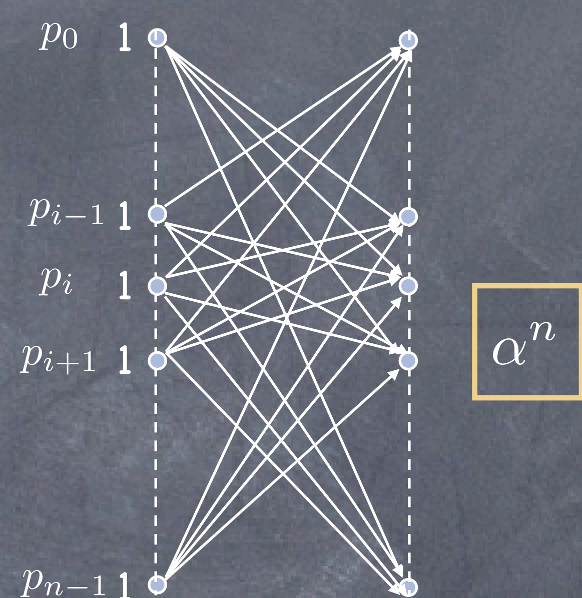
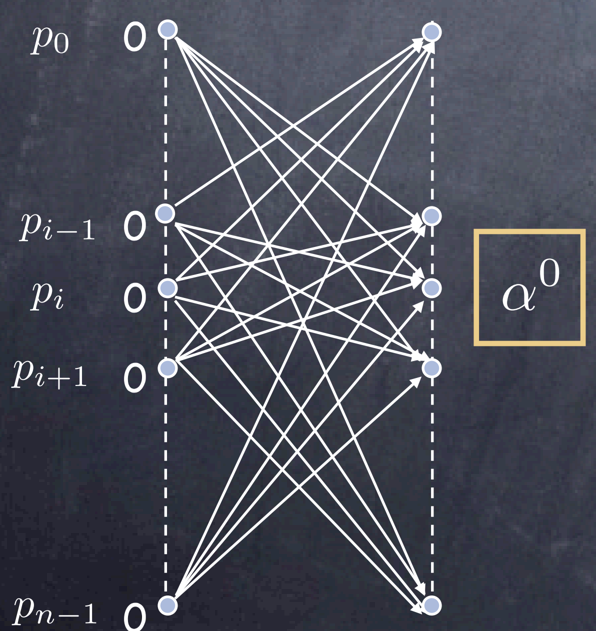
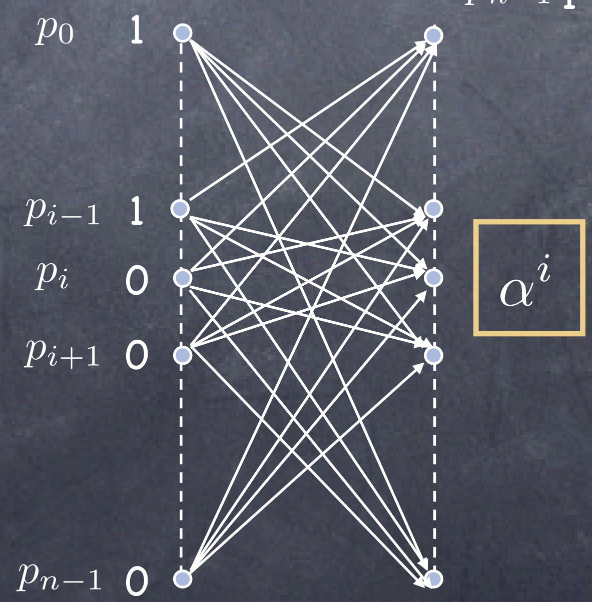
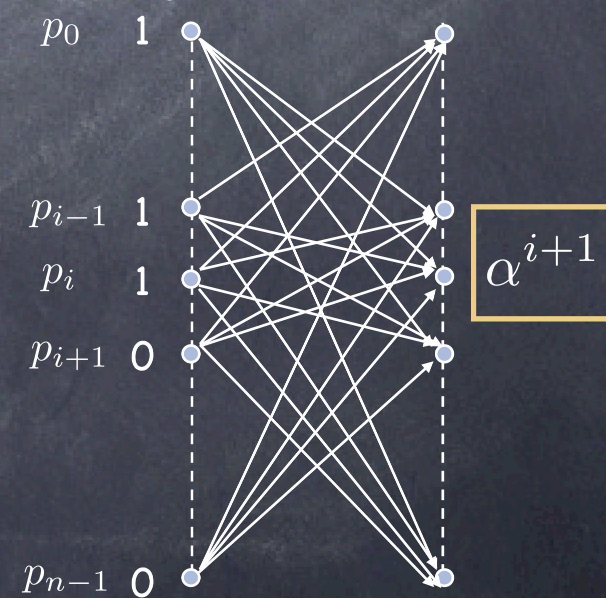








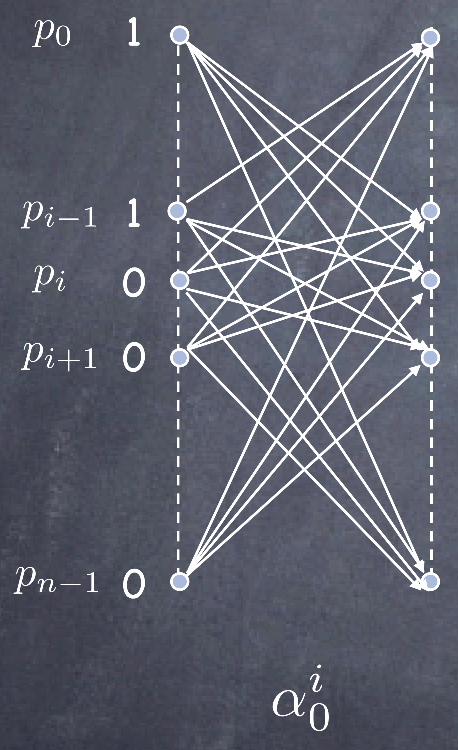
Adjacent α^is are similar!

Starting from α^i, we build a set of executions α^i_j where $0 \leq j \leq n-1$ as follows:

α^i_j is obtained from α^i after removing the messages that p_i sends to the j-th highest numbered processors (excluding itself)

The executions

The executions



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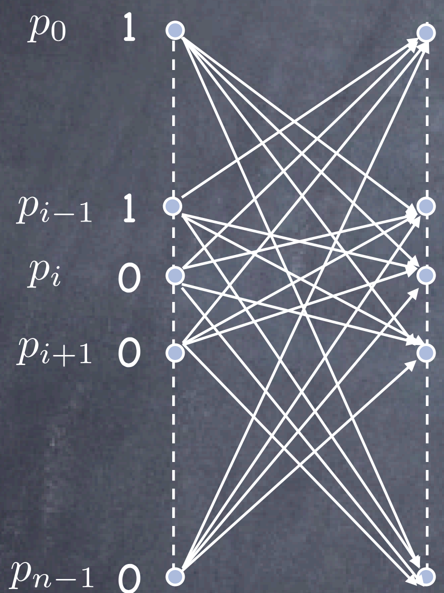
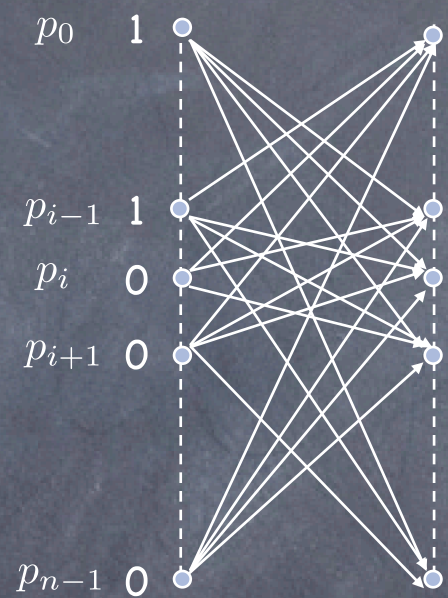
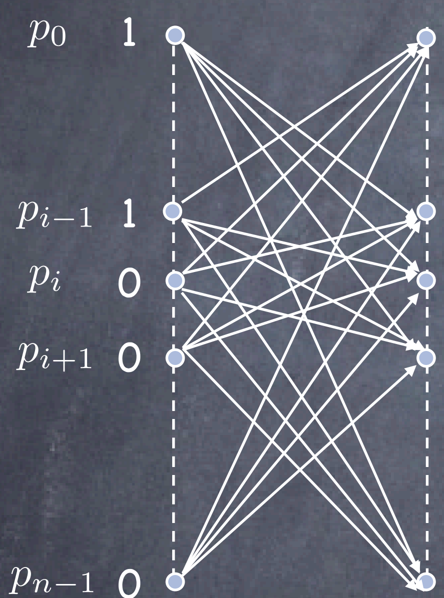


α_0^i



α_1^i

The executions



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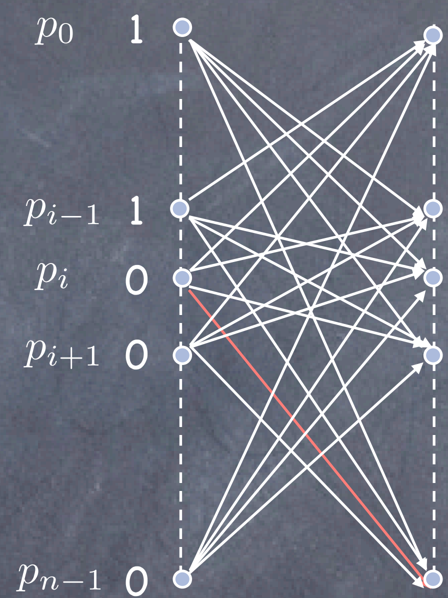

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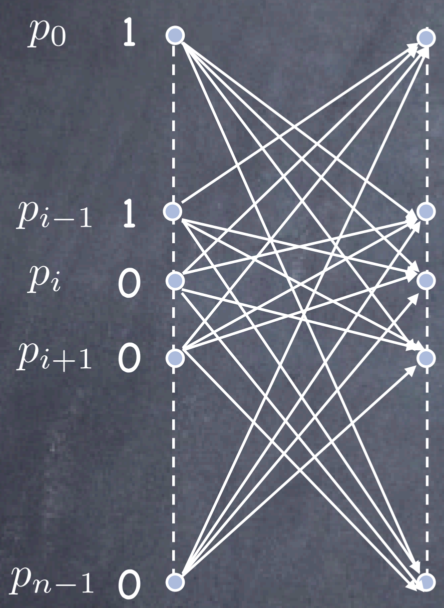
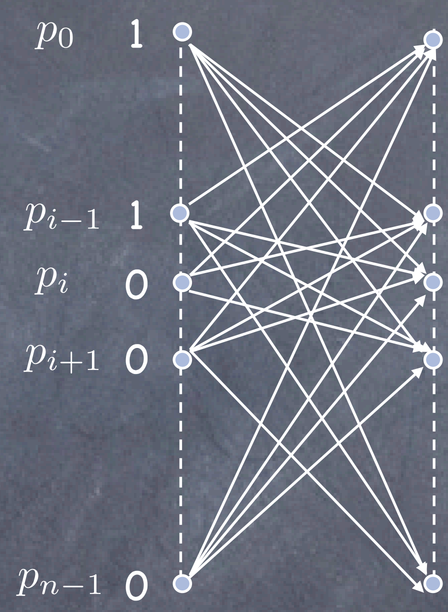
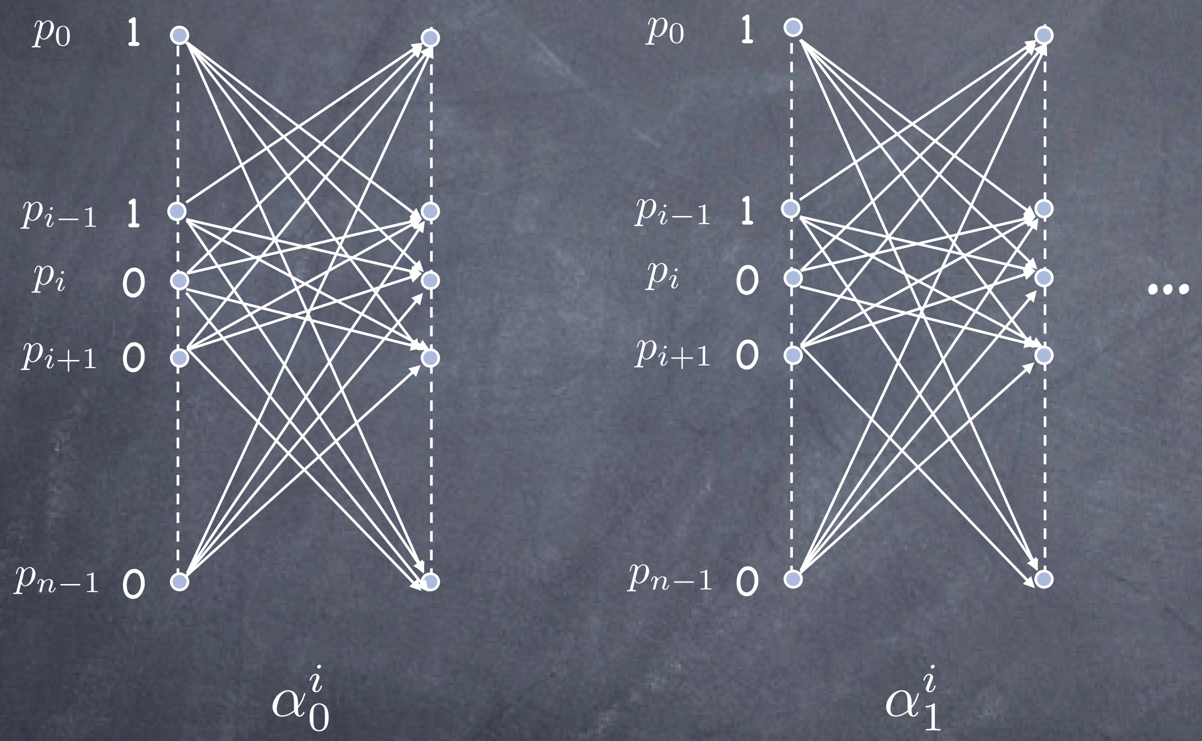


α_0^i

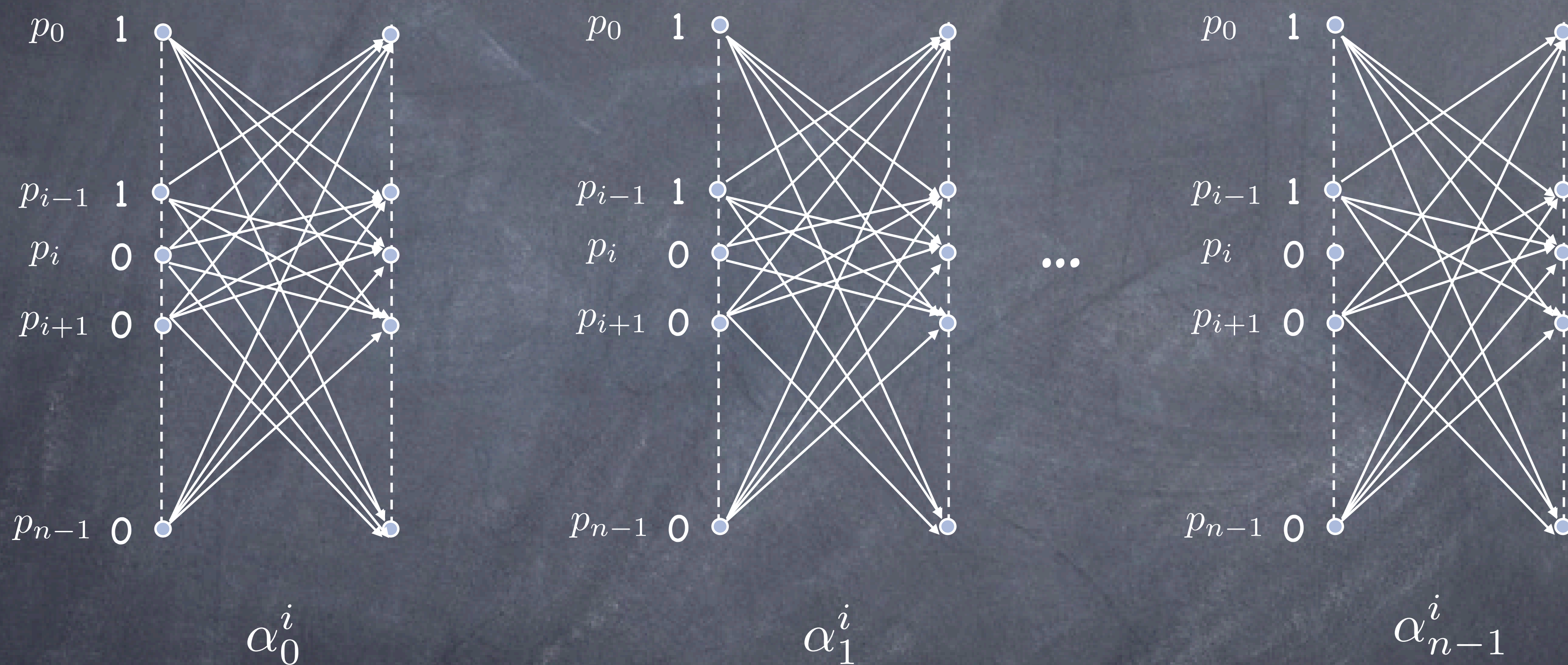


α_1^i

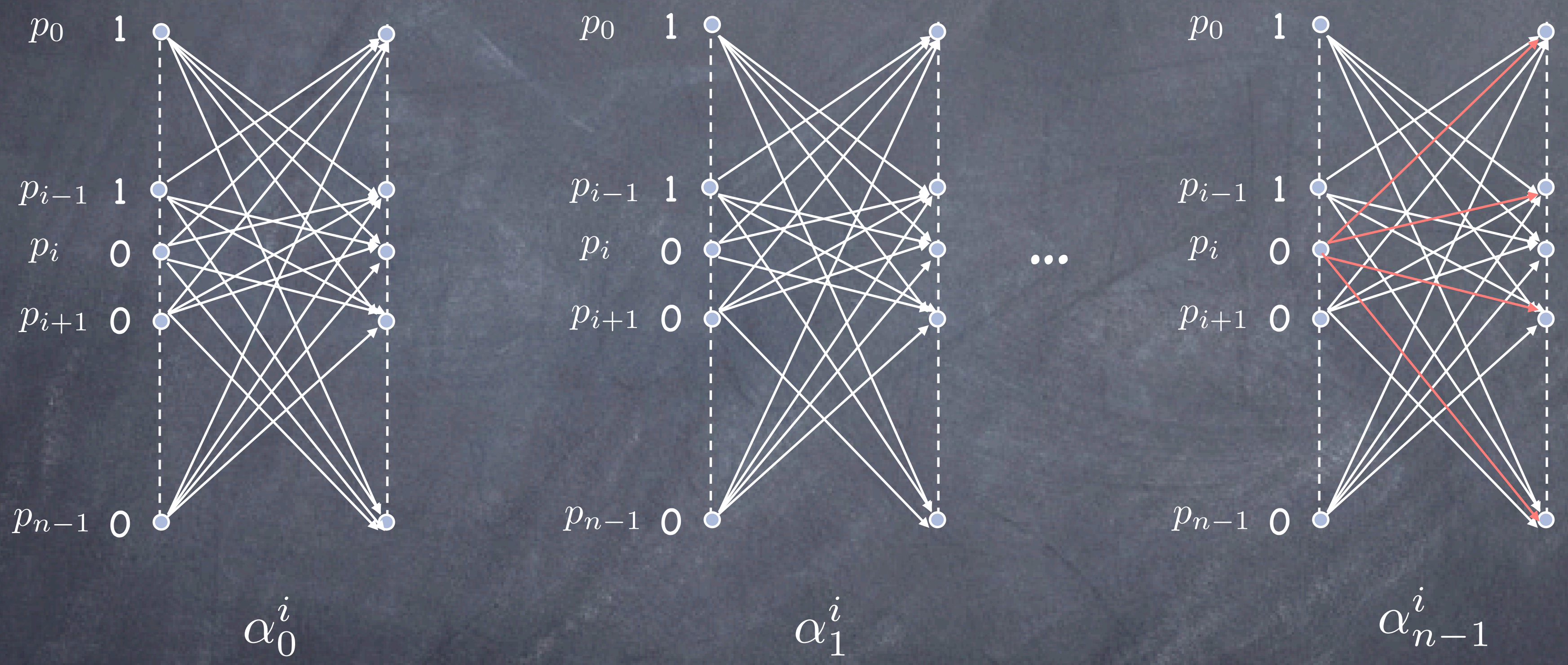
The executions



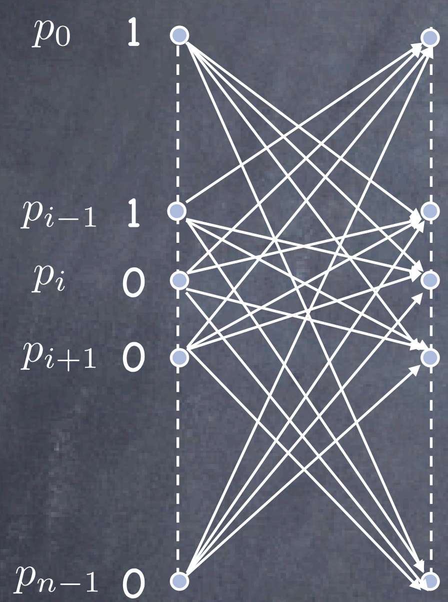
The executions



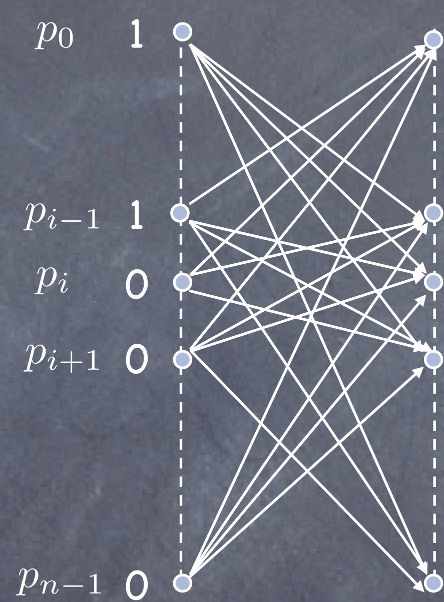
The executions



The executions

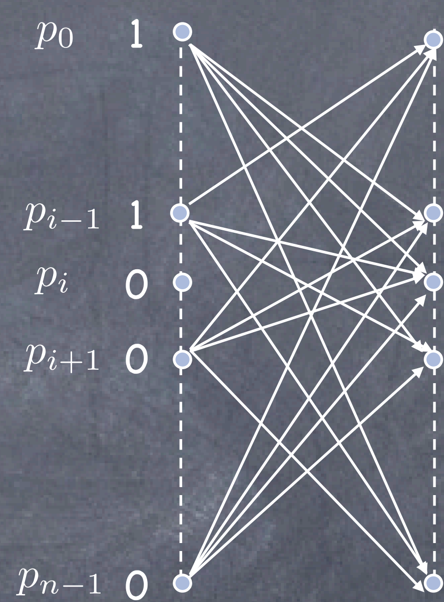


α_0^i



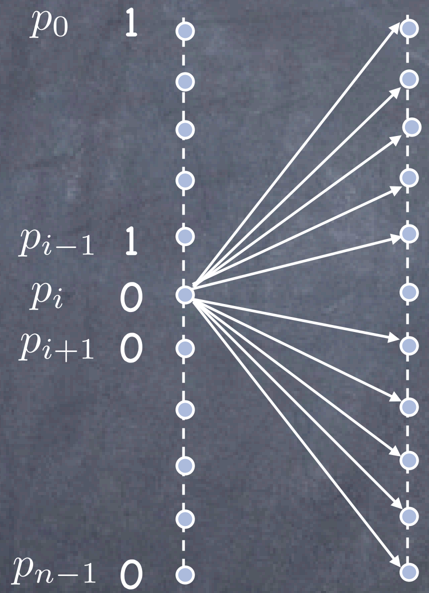
α_1^i

...



α_{n-1}^i

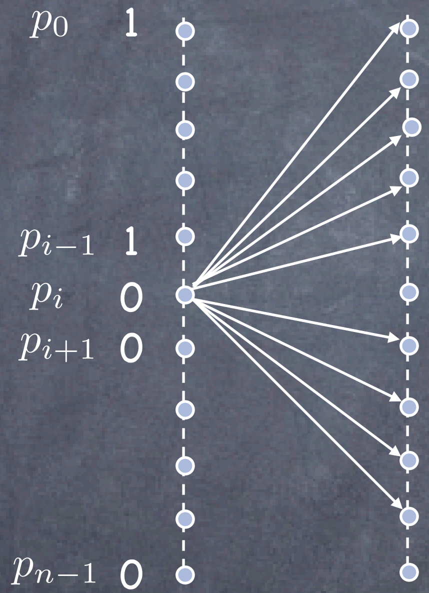
Indistinguishability



α^i

α_0^i

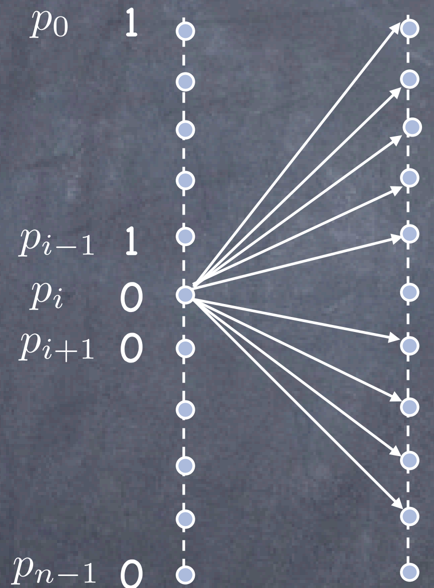
Indistinguishability



α^i

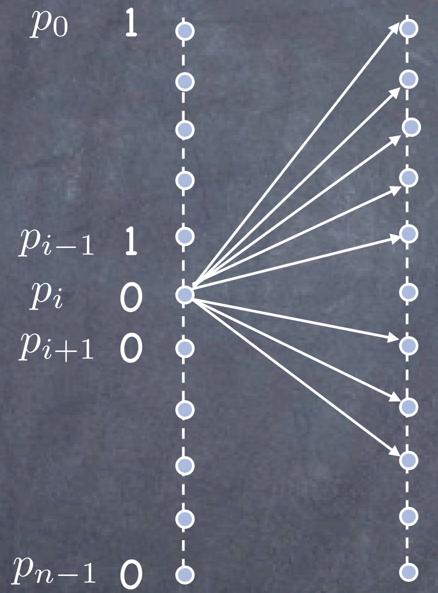
α_1^i

Indistinguishability



$$\alpha^i \approx \alpha_1^i$$

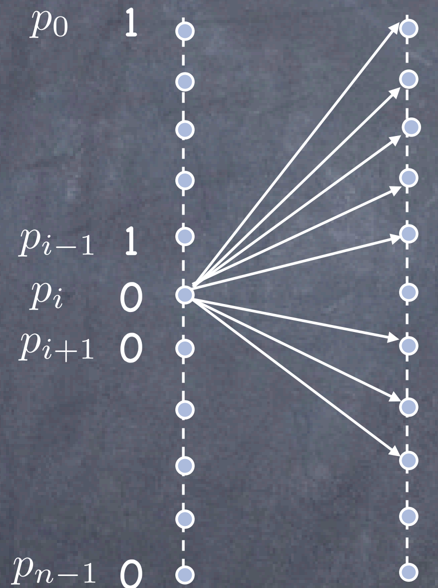
Indistinguishability



α^i

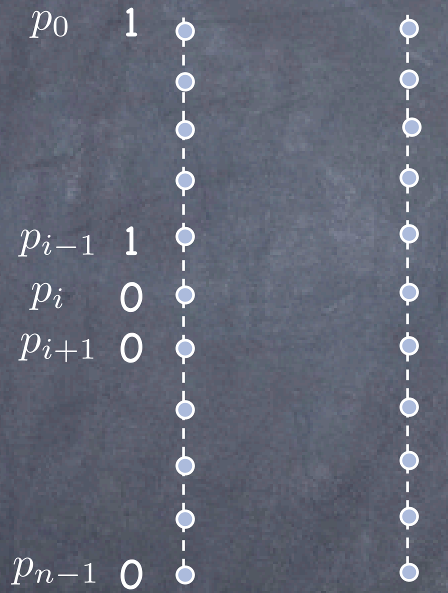
α_2^i

Indistinguishability



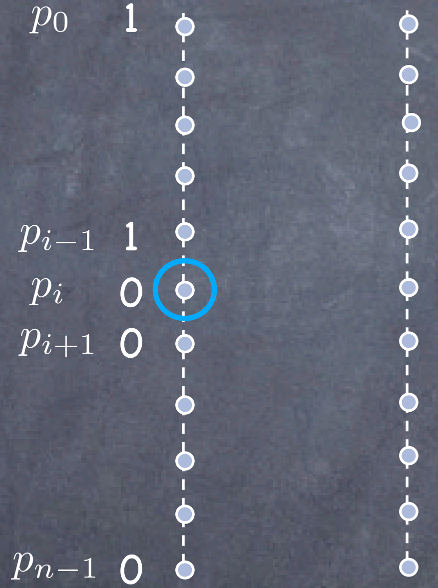
$$\alpha^i \approx \alpha_2^i$$

Indistinguishability



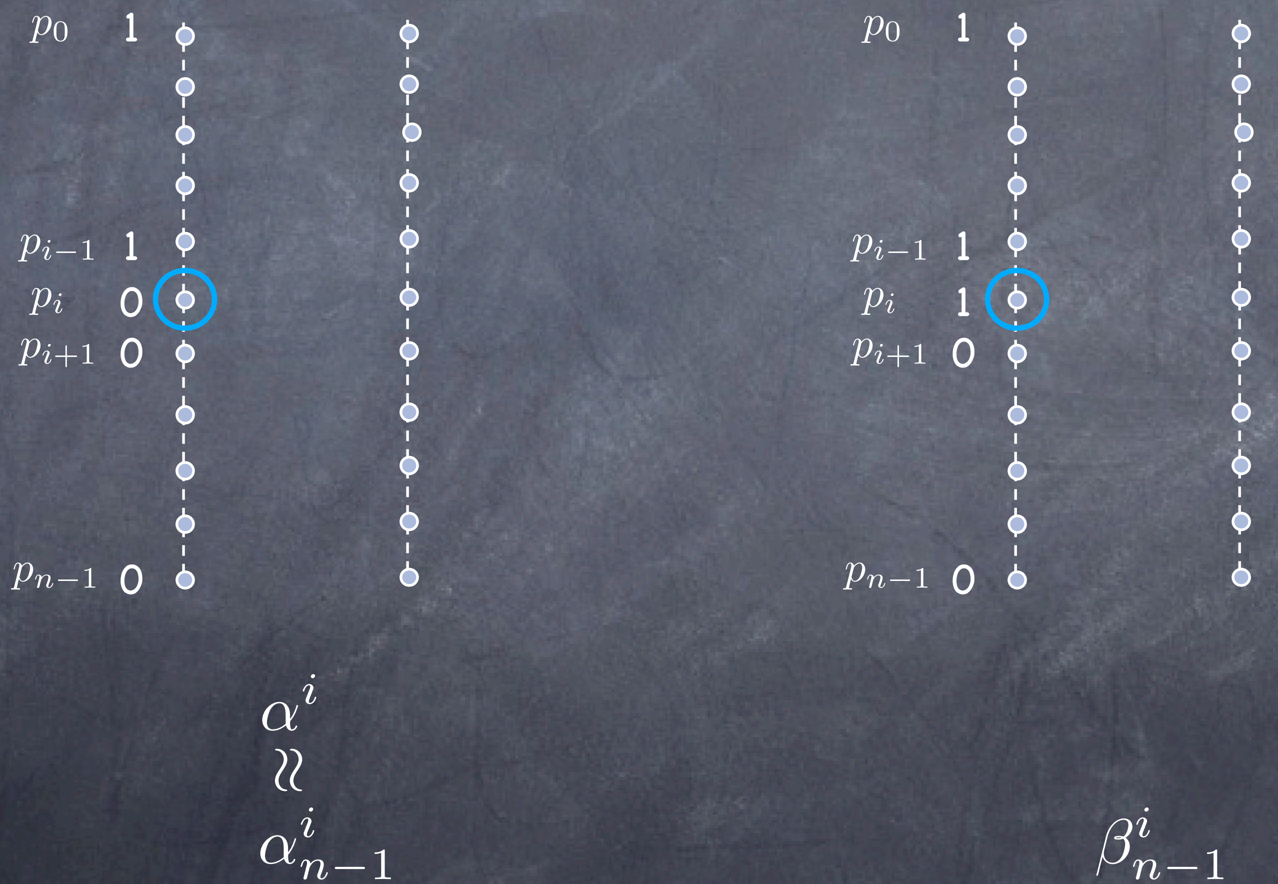
$$\alpha^i \approx \alpha^i_{n-1}$$

Indistinguishability

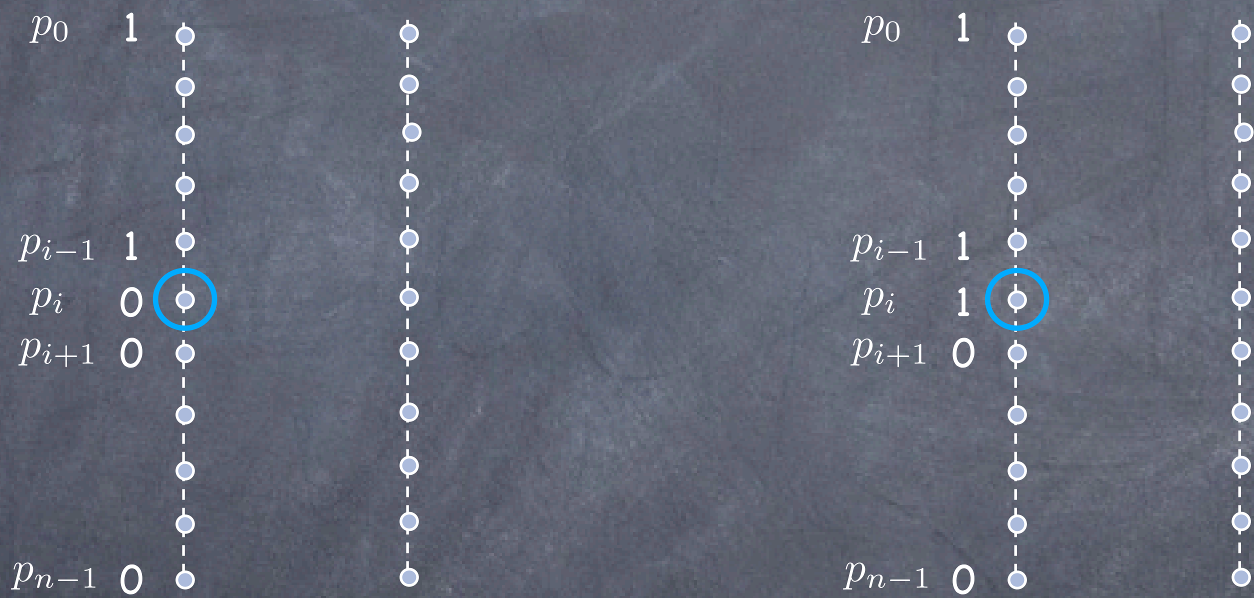


$$\alpha^i \approx \alpha^i_{n-1}$$

Indistinguishability

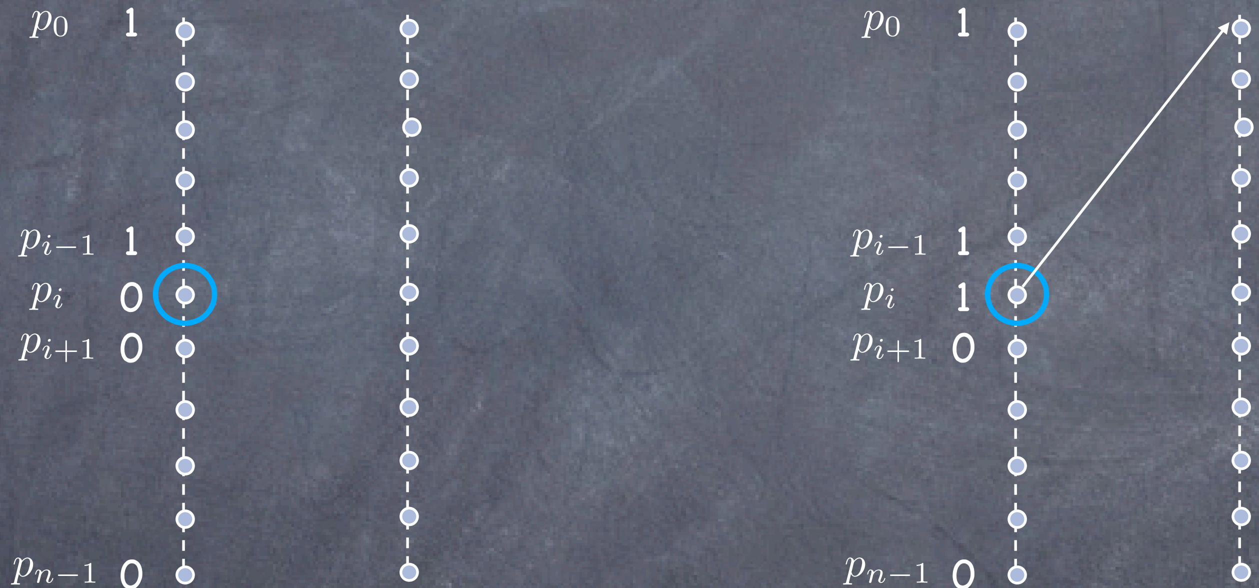


Indistinguishability



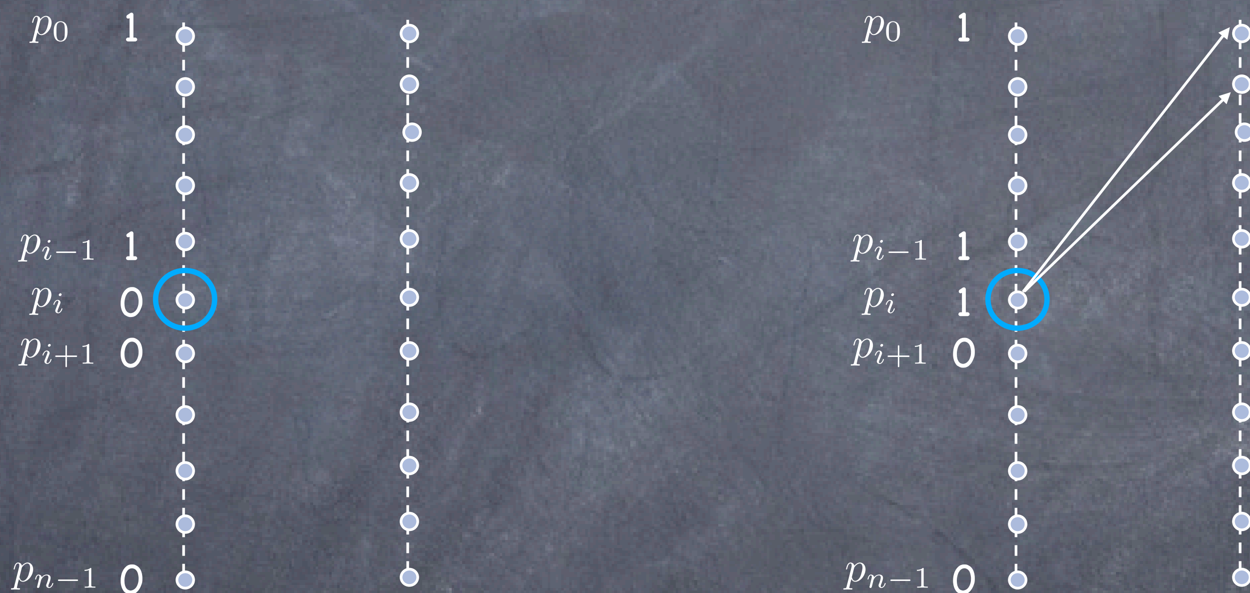
$$\alpha^i \approx \alpha^i_{n-1} \approx \beta^i_{n-1}$$

Indistinguishability



$$\alpha^i \wr \alpha^i_{n-1} \approx \beta^i_{n-2}$$

Indistinguishability

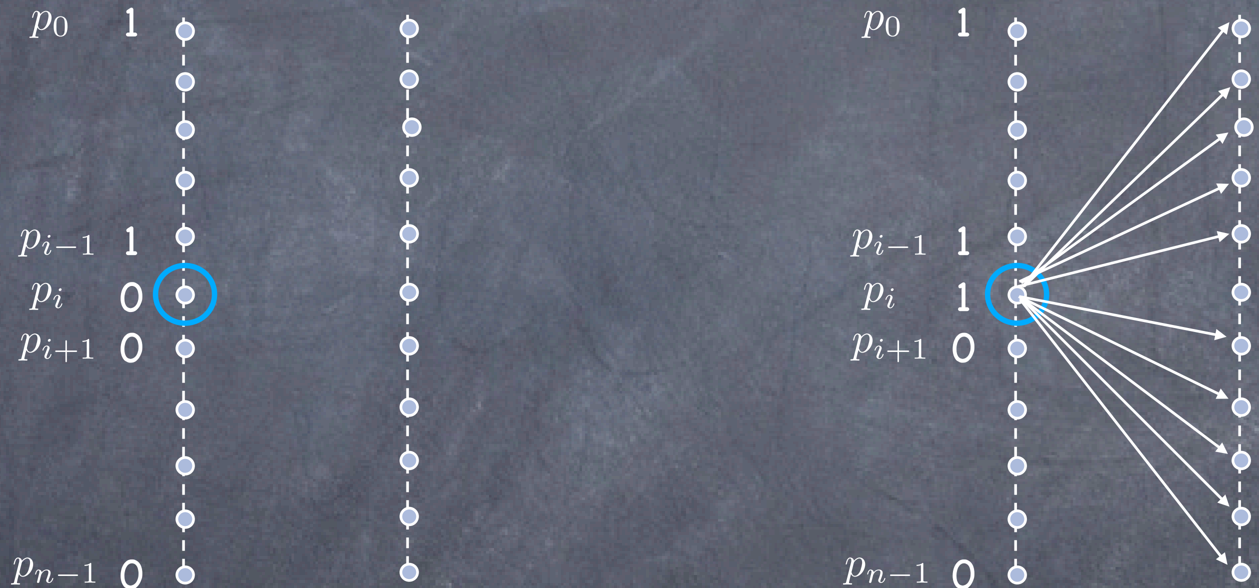


α^i

$\wr$

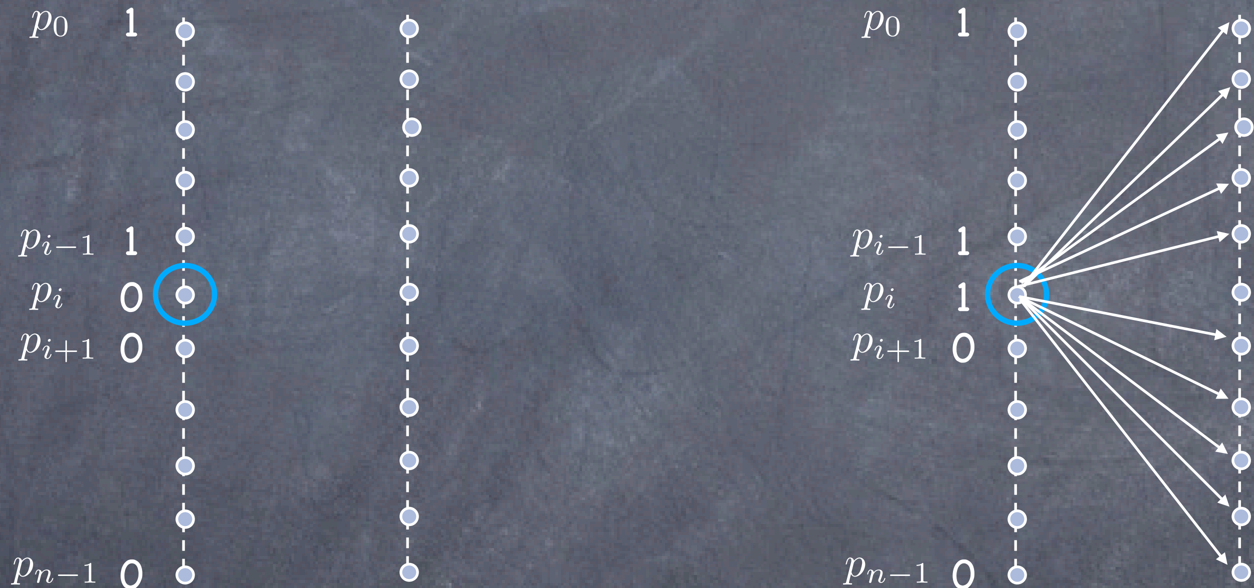
$\alpha^i_{n-1} \approx \beta^i_{n-3}$

Indistinguishability



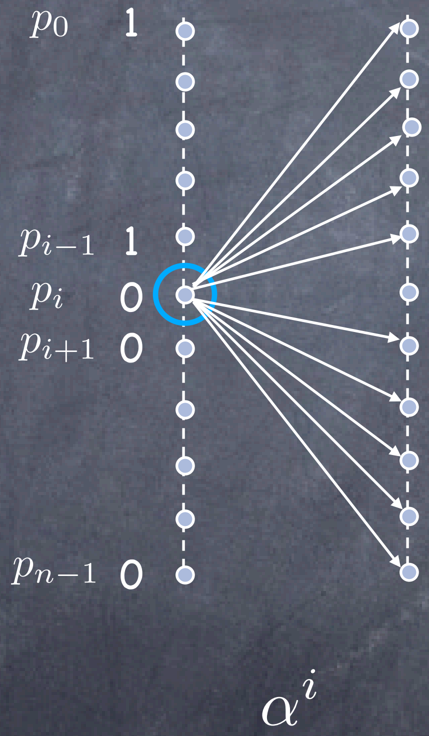
$$\alpha^i \wr\wr \alpha^i_{n-1} \approx \beta^i_0$$

Indistinguishability

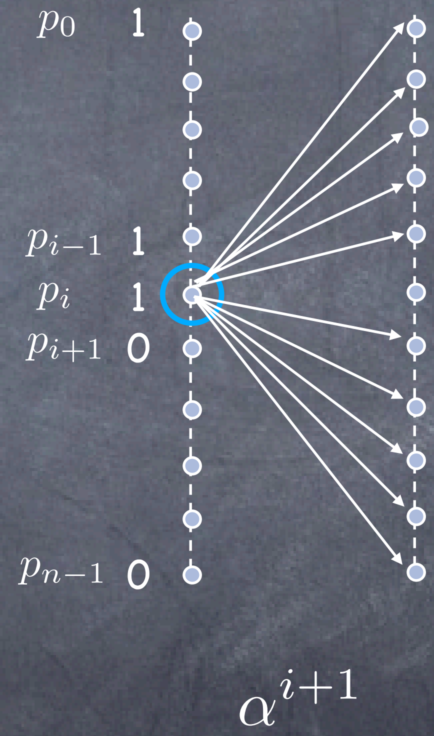


$$\alpha^i \approx \alpha^i_{n-1} \approx \beta^i_0 \approx \alpha^{i+1}$$

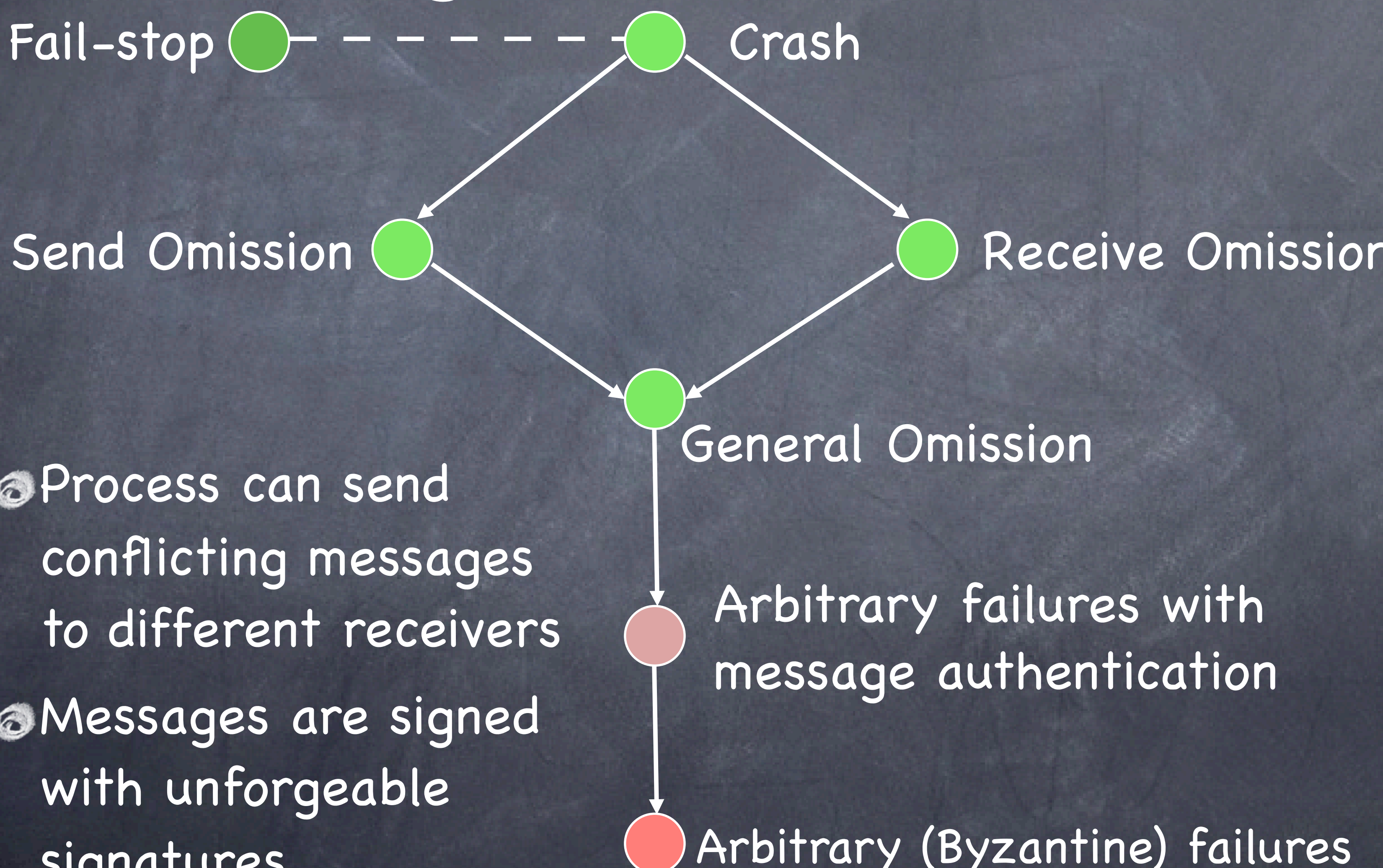
Indistinguishability



$\approx$



Arbitrary failures with message authentication



- Process can send conflicting messages to different receivers
- Messages are signed with unforgeable signatures

Valid messages

A valid message m has the following form:

in round 1:

$m : s_{id}$ (m is signed by the sender)

in round r > 1, if received by p from q :

$m : p_1 : p_2 : \ldots : p_r$ where

- p_1 = sender; $p_r = q$
- $p_1, \ldots, p_r$ are distinct from each other and from p
- message has not been tampered with

AFMA: The Idea

- A correct process p discards all non-valid messages it receives
- If a message is valid,
  - it "extracts" the value from the message
  - it relays the message, with its own signature appended
- At round $f+1$:
  - if it extracted exactly one message, p delivers it
  - otherwise, p delivers SF

AFMA: The Protocol

Initialization for process p :

  if p = sender and p wishes to broadcast m then

    extracted := relay := $\{m\}$

Process p in round $k, 1 \le k \le f+1$

  for each $s \in$ relay

    send $s : p$ to all

  receive round k messages from all processes

  relay := $\emptyset$

  for each valid message received $s = m : p_1 : p_2 : \ldots : p_k$

    if $m \notin$ extracted then

      extracted := extracted $\cup \{m\}$

      relay := relay $\cup \{s\}$

At the end of round $f+1$

  if $\exists m$ such that extracted = $\{m\}$ then

    deliver m

  else deliver SF

Termination

Initialization for process p:
  if p = sender and p wishes to broadcast m then
    extracted := relay := $\{m\}$

Process p in round k, $1 \leq k \leq f+1$
  for each $s \in$ relay
    send $s : p$ to all
  receive round k messages from all processes
  relay := $\emptyset$
  for each valid message received $s = m : p_1 : p_2 : \ldots : p_k$
    if $m \notin$ extracted then
      extracted := extracted $\cup \{m\}$
      relay := relay $\cup \{s\}$

At the end of round $f+1$
  if $\exists m$ such that extracted = $\{m\}$ then
    deliver m
  else deliver SF

In round $f+1$, every correct process delivers either m or SF and then halts

Agreement

Initialization for process p:

if p = sender and p wishes to broadcast m then

extracted := relay := $\{m\}$

Process p in round k, $1 \leq k \leq f+1$

for each $s \in$ relay

send $s : p$ to all

receive round k messages from all processes

relay := $\emptyset$

for each valid message received $s = m : p_1 : p_2 : \ldots : p_k$

if $m \notin$ extracted then

extracted := extracted $\cup \{m\}$

relay := relay $\cup \{s\}$

At the end of round $f+1$

if $\exists m$ such that extracted = $\{m\}$ then

deliver m

else deliver SF

Lemma. If a correct process extracts m, then every correct process eventually extracts m

Proof

Let r be the earliest round in which some correct process extracts m. Let that process be p.

- if p is the sender, then in round 1 p sends a valid message to all.

All correct processes extract that message in round 1

- otherwise, p has received in round r a message

$$m : p_1 : p_2 : \ldots : p_r$$

- Claim: $p_1, p_2, \ldots, p_r$ are all faulty
  - true for $p_1 = s$
  - Suppose $p_j, 1 \leq j \leq r$, were correct
    - p_j signed and relayed message in round j
    - p_j extracted message in round $j-1$

CONTRADICTION

- If $r \leq f, p$ will send a valid message

$$m : p_1 : p_2 : \ldots : p_r : p$$

in round $r+1 \leq f+1$ and every correct process will extract it in round $r+1 \leq f+1$

- If $r = f+1$, by Claim above, $p_1, p_2, \ldots, p_{f+1}$ faulty
  - At most f faulty processes
  - CONTRADICTiON

Validity

Initialization for process p:
  if p = sender and p wishes to broadcast m then
    extracted := relay := $\{m\}$

Process p in round k, $1 \leq k \leq f+1$
  for each $s \in$ relay
    send $s : p$ to all
  receive round k messages from all processes
  relay := $\emptyset$
  for each valid message received $s = m : p_1 : p_2 : \ldots : p_k$
    if $m \notin$ extracted then
      extracted := extracted $\cup \{m\}$
      relay := relay $\cup \{s\}$

At the end of round $f+1$
    if $\exists m$ such that extracted = $\{m\}$ then
      deliver m
    else deliver SF

Validity

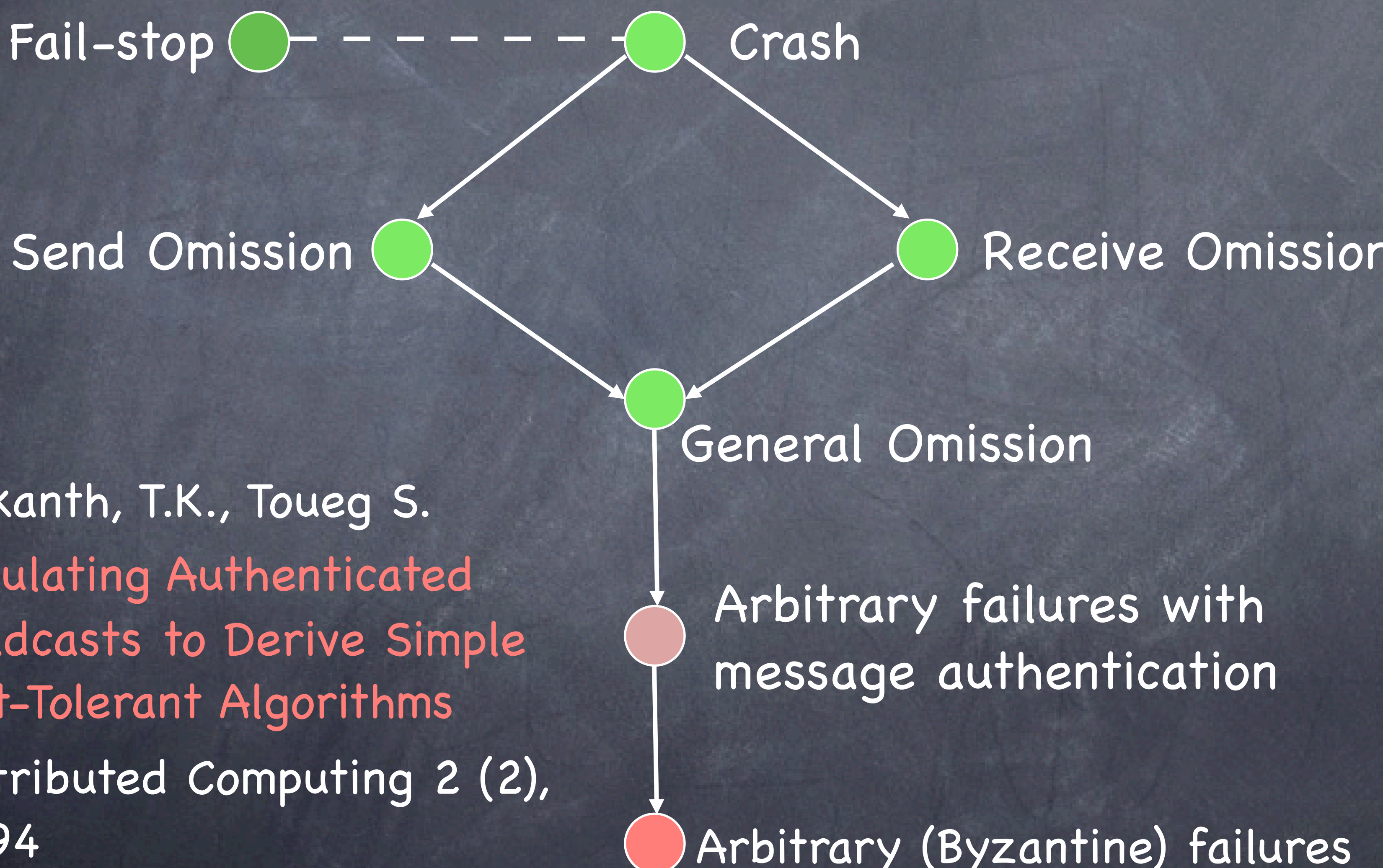
Initialization for process p:
  if p = sender and p wishes to broadcast m then
    extracted := relay := $\{m\}$

Process p in round k, $1 \leq k \leq f+1$
  for each $s \in$ relay
    send $s : p$ to all
  receive round k messages from all processes
  relay := $\emptyset$
  for each valid message received $s = m : p_1 : p_2 : \ldots : p_k$
    if $m \notin$ extracted then
      extracted := extracted $\cup \{m\}$
      relay := relay $\cup \{s\}$

At the end of round $f+1$
  if $\exists m$ such that extracted = $\{m\}$ then
    deliver m
  else deliver SF

From Agreement and the observation that the sender, if correct, delivers its own message.

TRB for arbitrary failures



Srikanth, T.K., Toueg S.

Simulating Authenticated Broadcasts to Derive Simple Fault-Tolerant Algorithms

Distributed Computing 2 (2), 80-94

AF: The Idea

- Identify the essential properties of message authentication that made AFMA work
- Implement these properties without using message authentication

AF: The Approach

- Introduce two primitives

  broadcast(p, m, i) (executed by p in round i)
  accept(p, m, i) (executed by q in round $j \geq i$)

- Give axiomatic definitions of broadcast and accept
- Derive an algorithm that solves TRB for AF using these primitives
- Show an implementation of these primitives that does not use message authentication

Properties of broadcast and accept

- Correctness If a correct process p executes broadcast(p, m, i) in round i, then all correct processes will execute accept(p, m, i) in round i
- Unforgeability If a correct process q executes accept(p, m, i) in round $j \geq i$, and p is correct, then p did in fact execute broadcast(p, m, i) in round i
- Relay If a correct process q executes accept(p, m, i) in round $j \geq i$, then all correct processes will execute accept(p, m, i) by round $j+1$

AF: The Protocol - 1

sender s in round 0:

0: extract m

sender s in round 1:

1: broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$

2: if p extracted m in round $k-1$ and $p \neq$ sender then

4: broadcast(p, m, k)

5: if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in rounds 1 through k (where (i) q_i distinct from each other and from p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$) and p has not previously extracted m then

6: extract m

7: if $k = f+1$ then

8: if in the entire execution p has extracted exactly one m then

9: deliver m

10: else deliver SF

11: halt

Termination

sender s in round 0:

0: extract m

sender s in round 1:

1: broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$

2: if p extracted m in round $k-1$ and $p \neq$ sender then

4: broadcast(p, m, k)

5: if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in rounds 1 through k
(where (i) q_i distinct from each other and from p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$)
and p has not previously extracted m then

6: extract m

7: if $k = f+1$ then

8: if in the entire execution p has extracted exactly one m then

9: deliver m

10: else deliver SF

11: halt

In round $f+1$, every correct process delivers either m or SF and then halts

Agreement - 1

sender s in round 0:
extract m
sender s in round 1:
broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$
if p extracted m in round $k-1$ and $p \neq$ sender then
    broadcast(p, m, k)
if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in
    rounds 1 through k
        (where (i) q_i distinct from each other and from
        p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$)
    and p has not previously extracted m then
        extract m
if $k = f+1$ then
    if in the entire execution p has extracted exactly
            one m then
        deliver m
    else deliver SF
    halt

Lemma
If a correct process extracts m, then
every correct process eventually extracts m

Agreement - 1

```
sender s in round 0:
    extract m
sender s in round 1:
    broadcast(s, m, 1)

Process p in round k, 1 ≤ k ≤ f+1
    if p extracted m in round k−1 and p ≠ sender then
        broadcast(p, m, k)
    if p has executed at least k accept(q_i, m, j_i)  1 ≤ i ≤ k in
        rounds 1 through k
            (where (i) q_i distinct from each other and from
             p, (ii) one q_i is s, and (iii) 1 ≤ j_i ≤ k )
      and p has not previously extracted m then
            extract m
    if k = f+1 then
        if in the entire execution p has extracted exactly
                    one m then
            deliver m
        else deliver SF
        halt
```

Lemma

If a correct process extracts m, then every correct process eventually extracts m

Agreement - 1

sender s in round 0:
0: extract m
sender s in round 1:
1: broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$
2: if p extracted m in round $k-1$ and $p \neq$ sender then
4: broadcast(p, m, k)
5: if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in rounds 1 through k
(where (i) q_i distinct from each other and from p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$)
and p has not previously extracted m then
6: extract m
7: if $k = f+1$ then
8: if in the entire execution p has extracted exactly one m then
9: deliver m
10: else deliver SF
11: halt

Lemma

If a correct process extracts m, then every correct process eventually extracts m

Proof

Let r be the earliest round in which some correct process extracts m. Let that process be p.

- if r = 0, then p = s and p will execute broadcast(s,m,1) in round 1. By CORRECTNESS, all correct processes will execute **accept** (s,m,1) in round 1 and extract m
- if r > 0, the sender is faulty. Since p has extracted m in round r, p has accepted at least r triples with properties (i), (ii), and (iii) by round r
  - r ≤ f By RELAY, all correct processes will have accepted those r triples by round r + 1
  - p will execute broadcast(p,m,r + 1) in round r + 1
  - By CORRECTNESS, any correct process other than p, q_1, q_2,...,q_r will have accepted r + 1 triples (q_k,m,j_k), 1 ≤ j_k ≤ r + 1, by round r + 1
  - q_1, q_2,...,q_r,p are all distinct
  - every correct process other than q_1, q_2,...,q_r,p will extract m
  - p has already extracted m; what about q_1, q_2,...,q_r?

Agreement - 2

sender s in round 0:

extract m

sender s in round 1:

broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$

if p extracted m in round $k-1$ and $p \neq$ sender then

  broadcast(p, m, k)

if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in rounds 1 through k (where (i) q_i distinct from each other and from p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$) and p has not previously extracted m then

  extract m

if $k = f+1$ then

  if in the entire execution p has extracted exactly one m then

    deliver m

  else deliver SF

  halt

Claim: $q_1, q_2, \ldots, q_r$ are all faulty

- Suppose q_k were correct
- p has accepted(q_k, m, j_k) in round $j_k \leq r$
- By UNFORGEABILITY, q_k executed broadcast (q_k, m, j_k) in round j_k
- q_k extracted m in round $j_{k-1} < r$

CONTRADICTION

- Case 2: $r = f+1$
  - Since there are at most f faulty processes, some process q_l in $q_1, q_2, \ldots, q_{f+1}$ is correct
  - By UNFORGEABILITY, q_l executed broadcast(q_l, m, j_l) in round $j_l \leq r$
  - q_l has extracted m in round $j_{l-1} < f+1$

CONTRADICTION

Validity

sender s in round 0:
extract m
sender s in round 1:
broadcast$(s, m, 1)$

Process p in round $k, 1 \leq k \leq f+1$
if p extracted m in round $k-1$ and $p \neq$ sender then
broadcast(p, m, k)
if p has executed at least k accept(q_i, m, j_i) $1 \leq i \leq k$ in rounds 1 through k
(where (i) q_i distinct from each other and from p, (ii) one q_i is s, and (iii) $1 \leq j_i \leq k$)
and p has not previously extracted m then
extract m
if $k = f+1$ then
if in the entire execution p has extracted exactly one m then
deliver m
else deliver SF
halt

- A correct sender executes broadcast$(s, m, 1)$ in round 1
- By <u>CORRECTNESS</u>, all correct processes execute accept$(s, m, 1)$ in round 1 and extract m
- In order to extract a different message m', a process must execute accept$(s, m', 1)$ in some round $i \leq f+1$
- By <u>UNFORGEABILITY</u>, and because s is correct, no correct process can extract $m' \neq m$
- All correct processes will deliver m

Implementing broadcast and accept

- A process that wants to broadcast m, does so through a series of witnesses
  - Sends m to all
  - Each correct process becomes a witness by relaying m to all
- If a process receives enough witness confirmations, it accepts m

Can we rely on witnesses?

- Only if not too many faulty processes!
- Otherwise, a set of faulty processes could fool a correct process by acting as witnesses of a message that was never broadcast
- How large can be f with respect to n?

Byzantine Generals

- One General G, a set of Lieutenants L_i
- General can order Attack (A) or Retreat (R)
- General may be a traitor; so may be some of the Lieutenants

* * *

I. If G is trustworthy, every trustworthy L_i must follow G's orders

II. Every trustworthy L_i must follow same battleplan

The plot thickens...

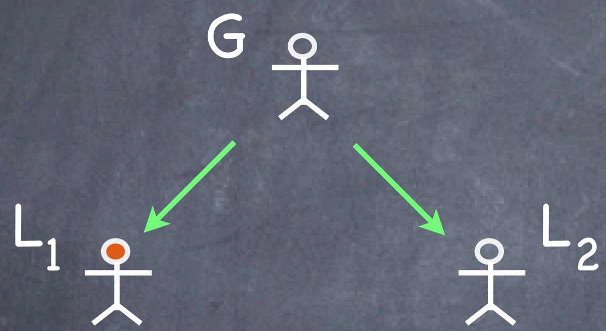
One traitor

The plot thickens...

One traitor

The plot thickens...

One traitor



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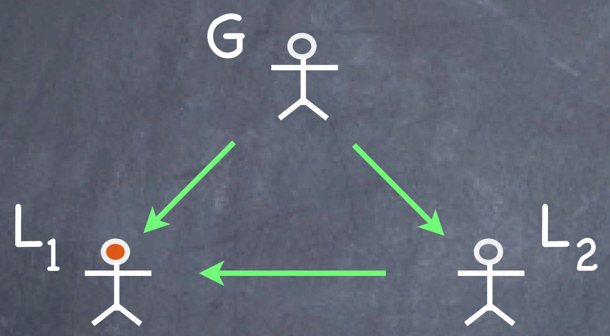

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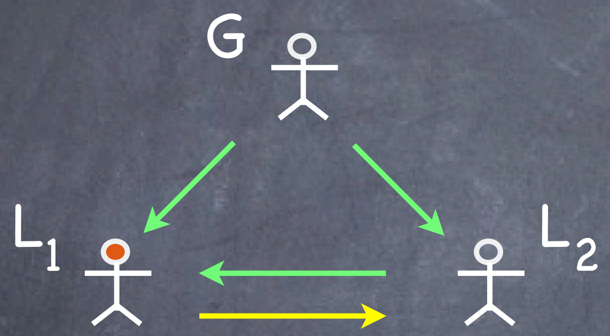

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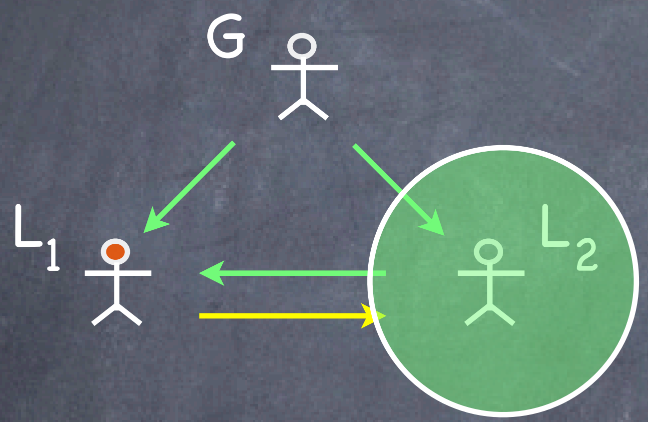

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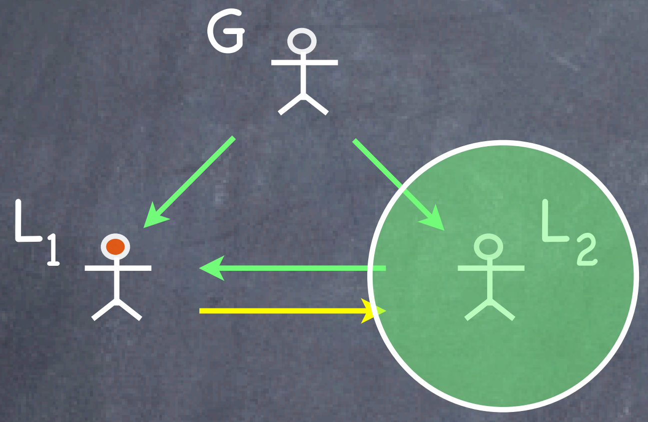
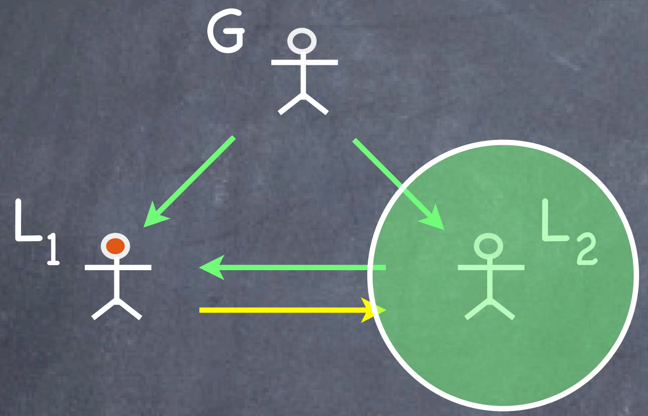
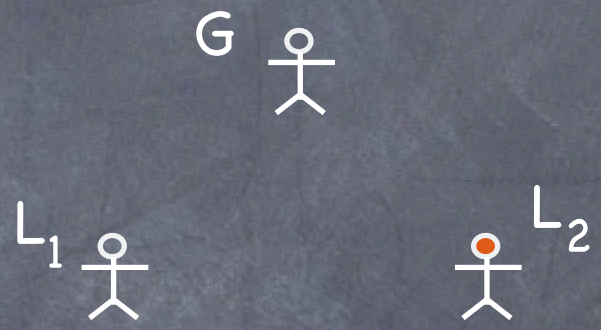


The plot thickens...

One traitor





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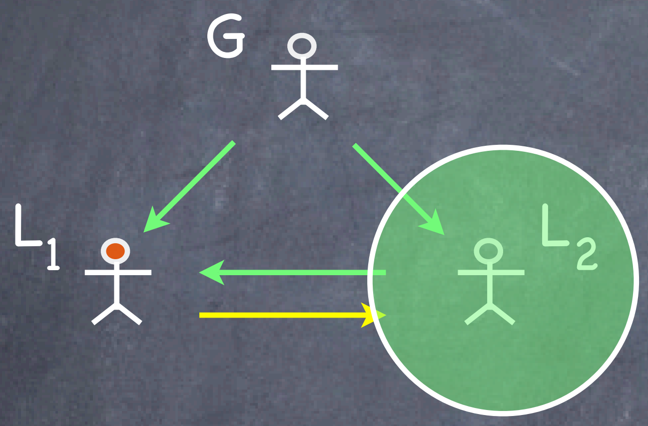

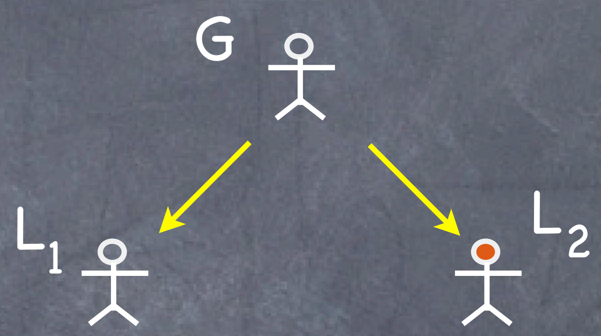

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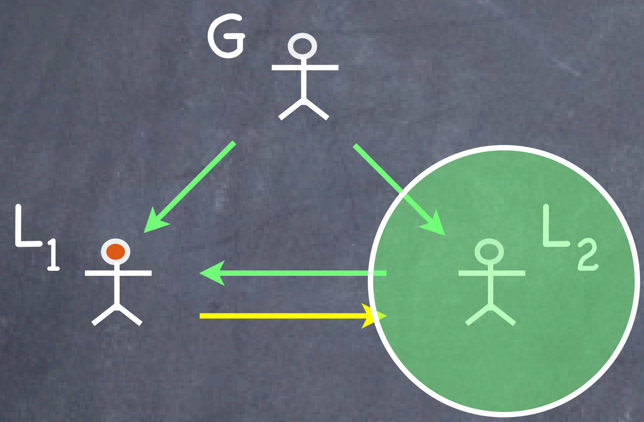

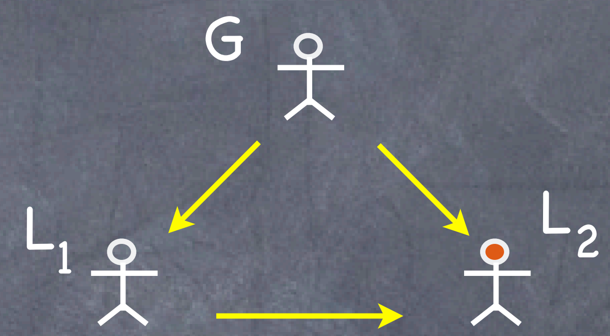

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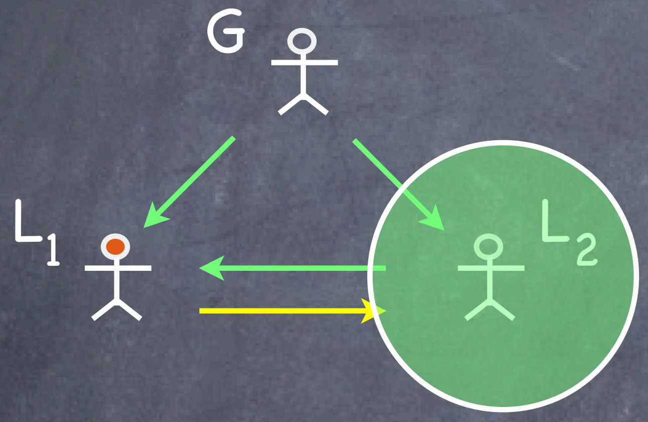

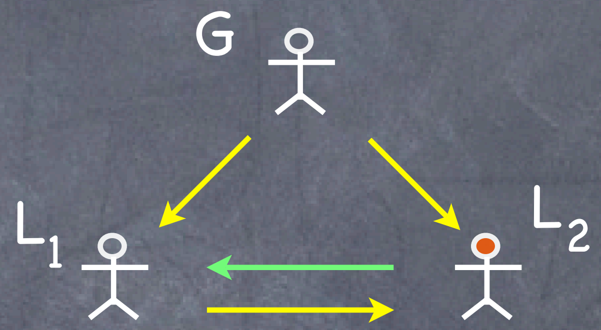

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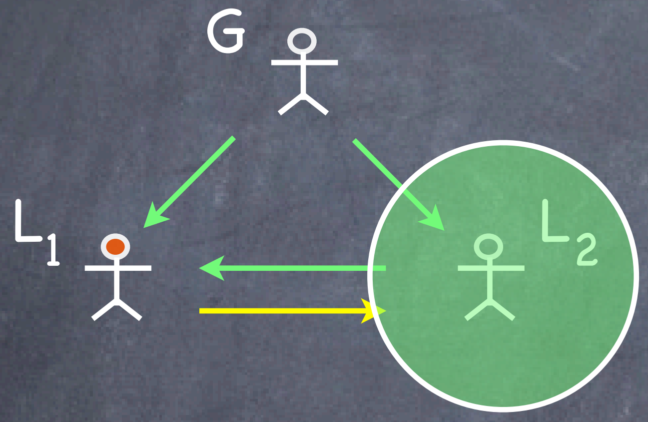

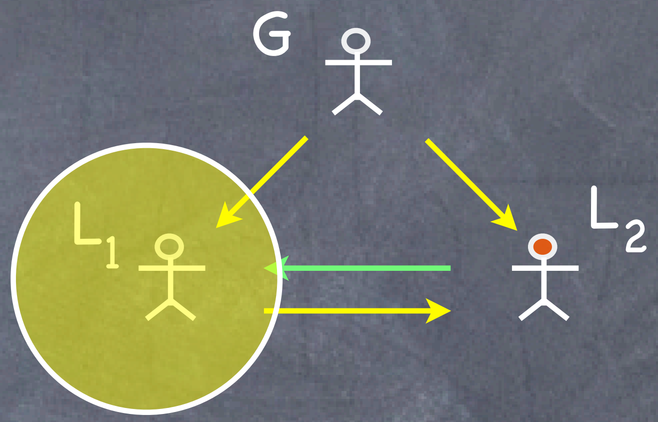

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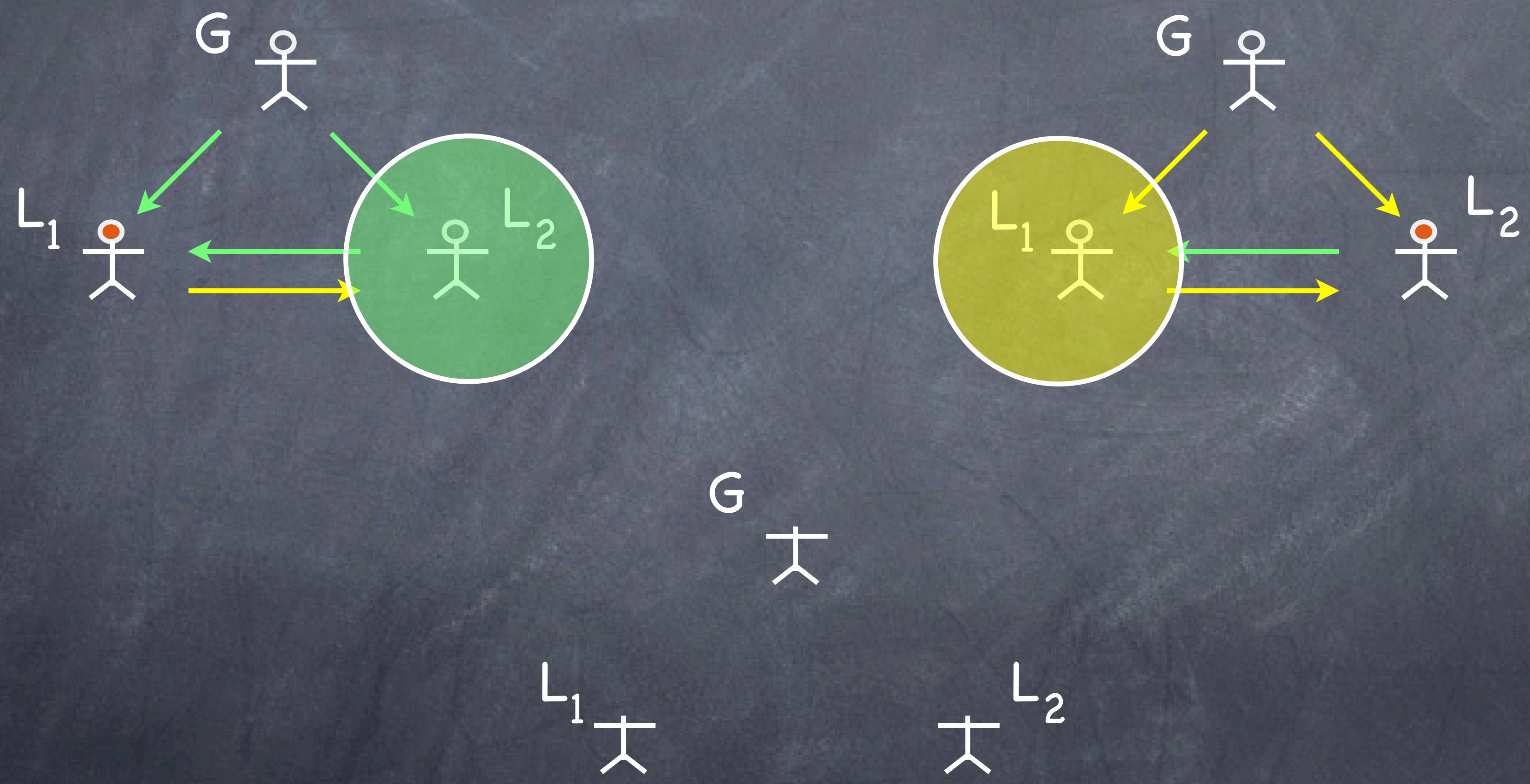

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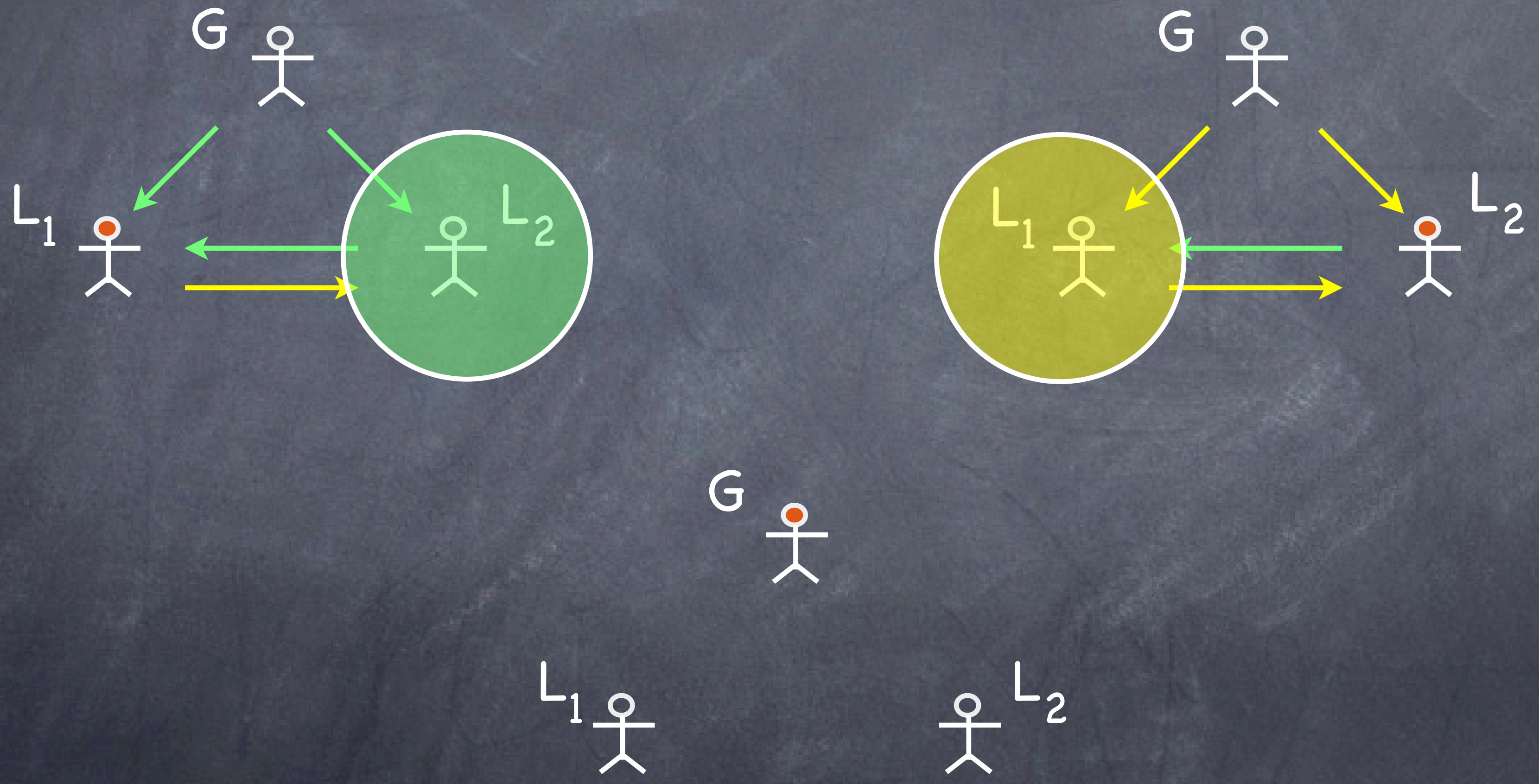

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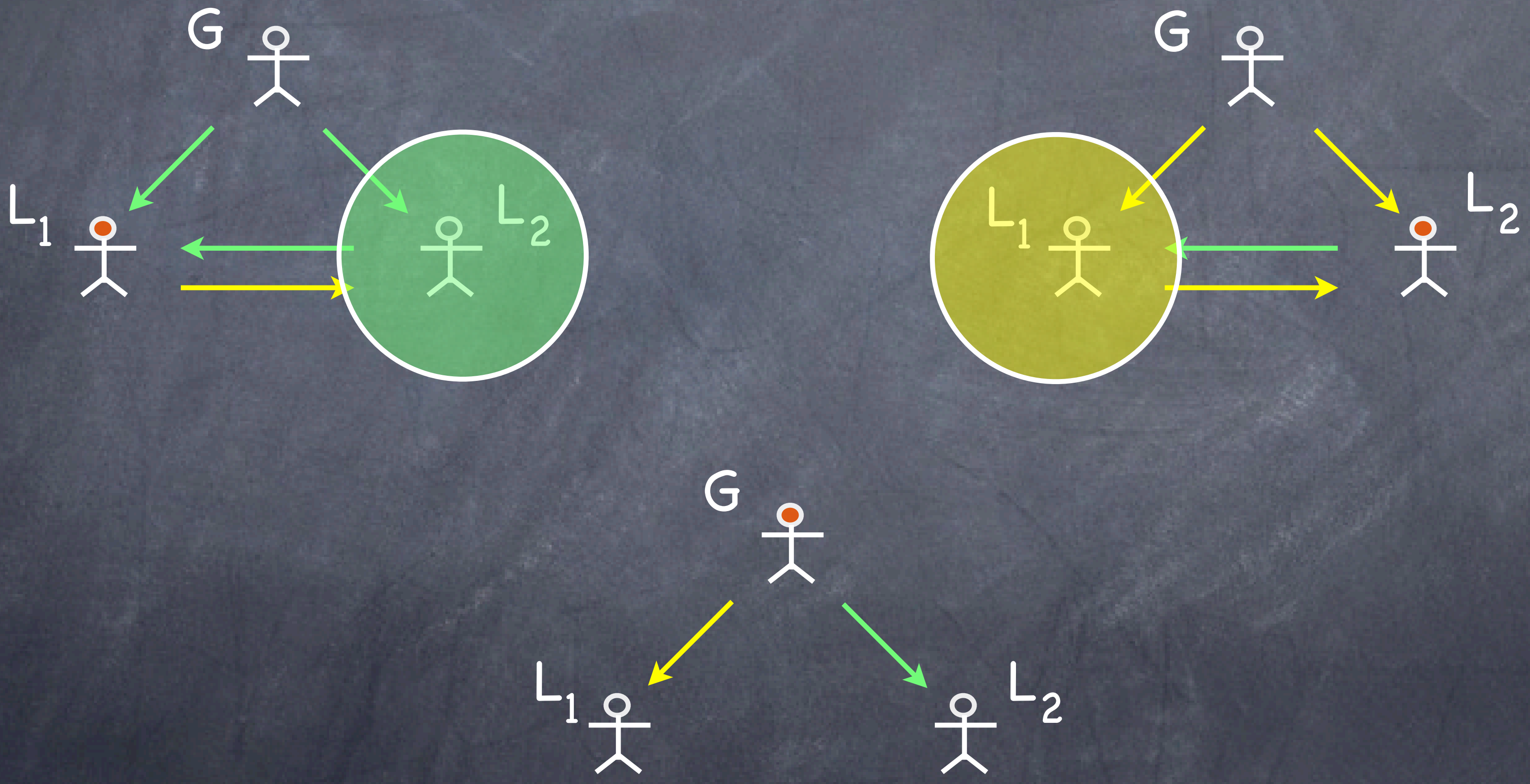

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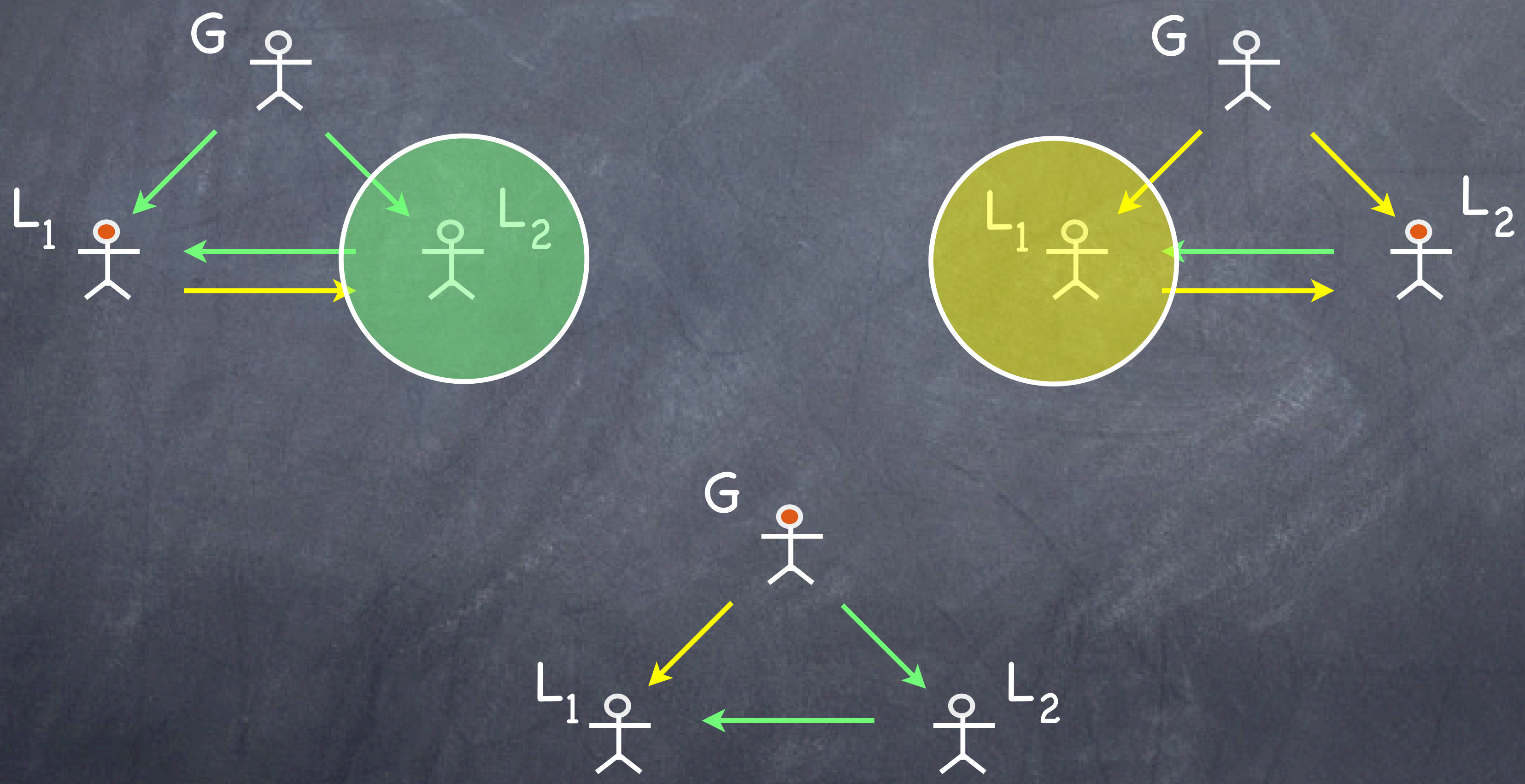

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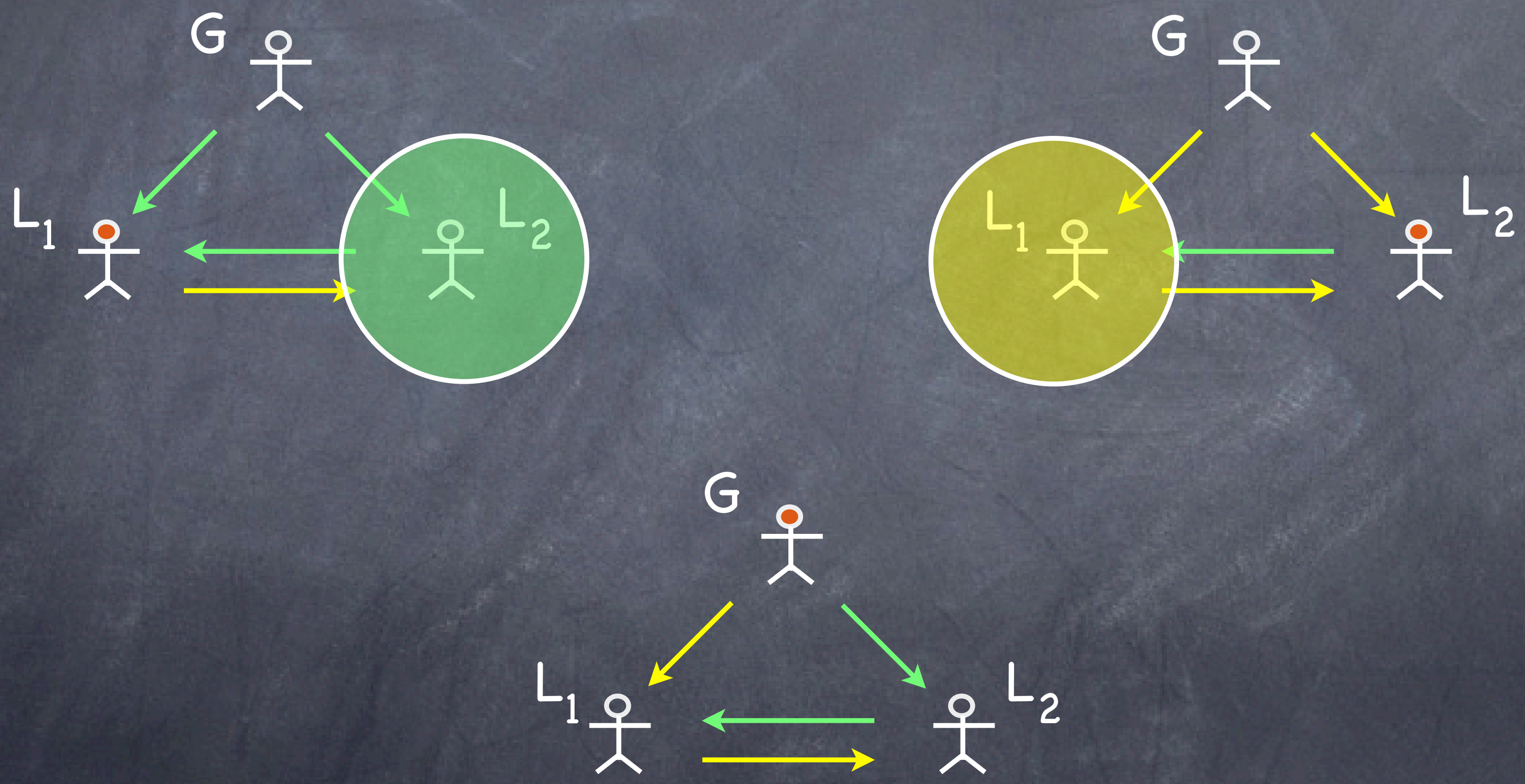

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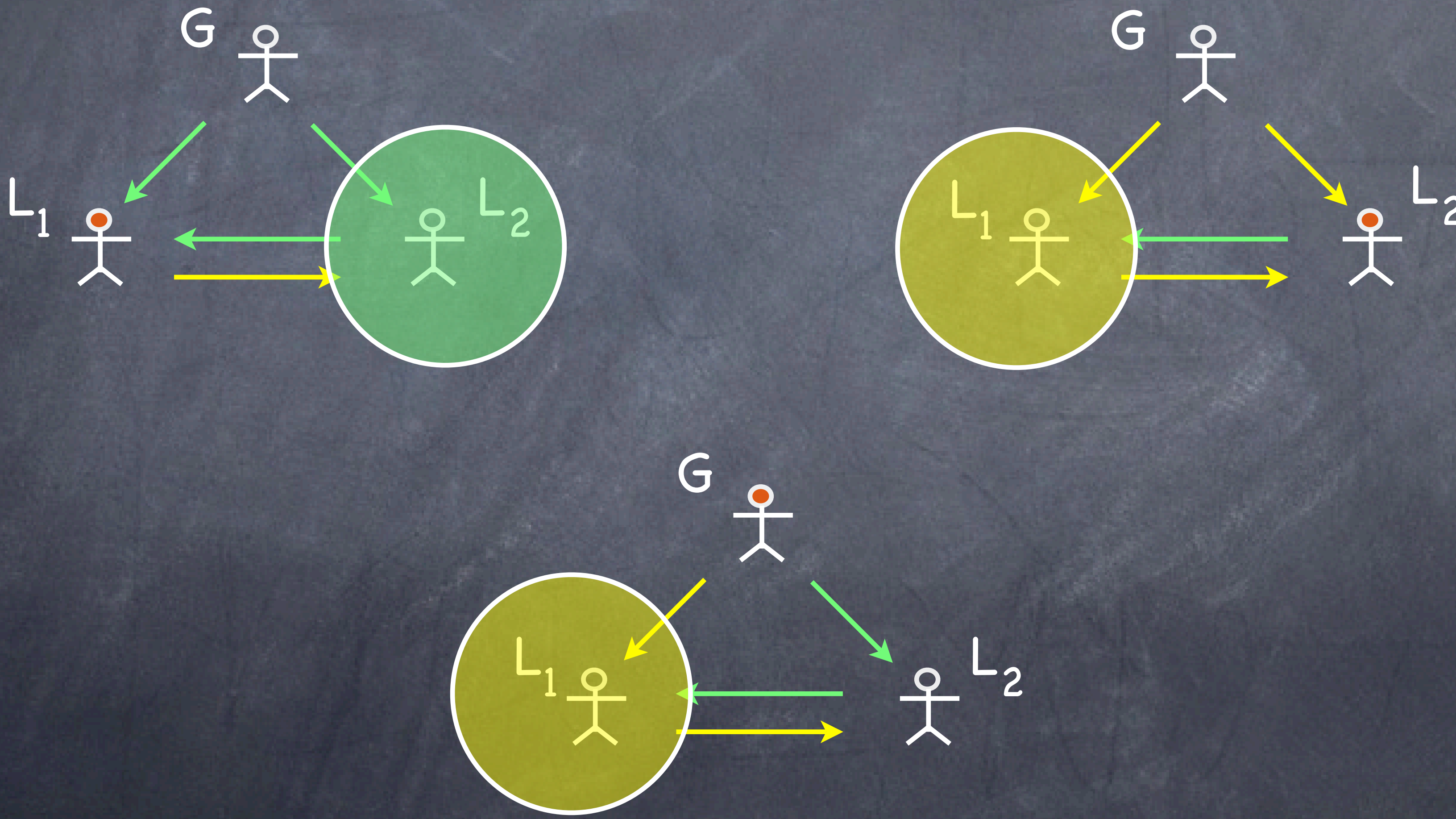

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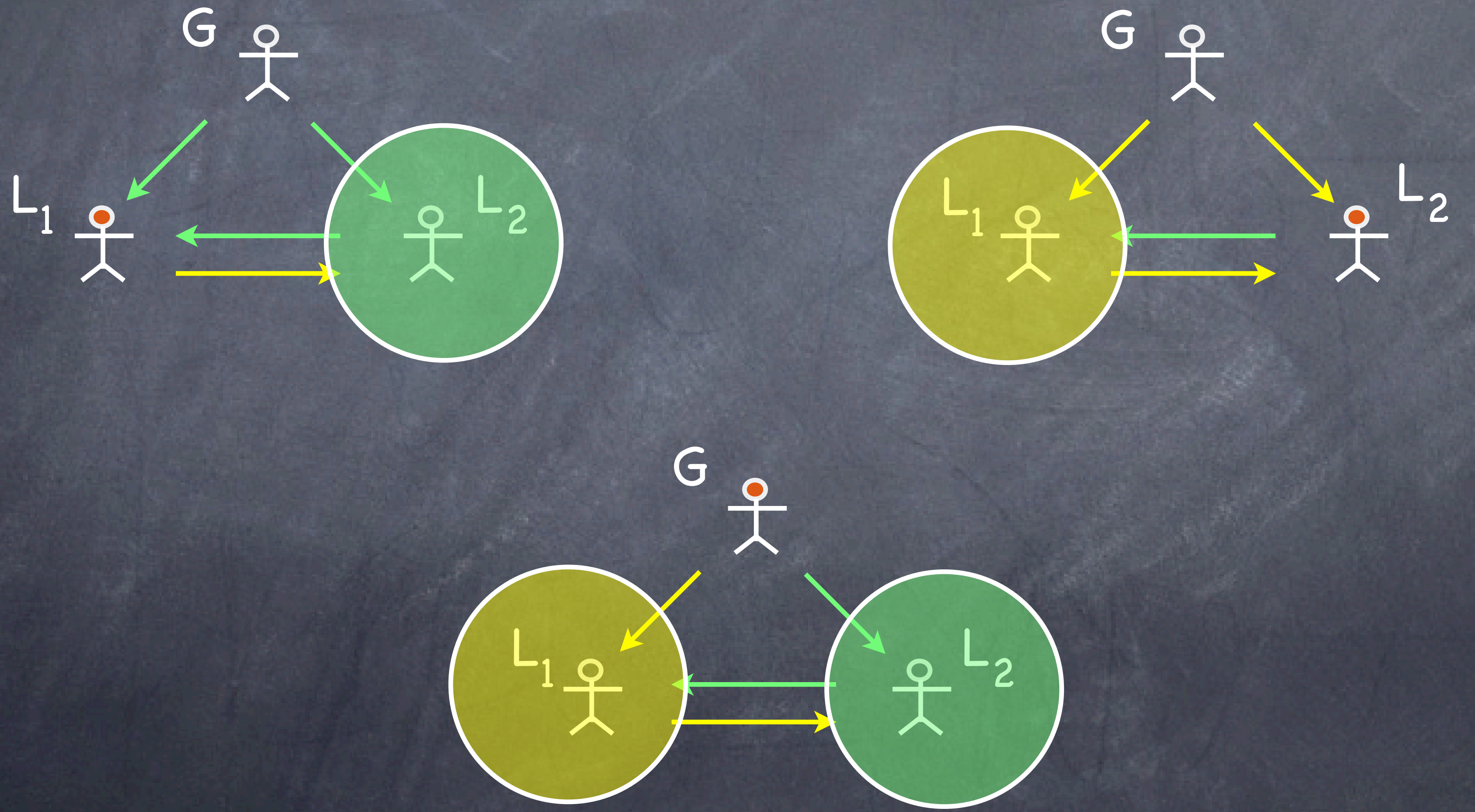

A Lower Bound

Theorem

There is no algorithm that solves TRB for Byzantine failures if $n \leq 3f$

(Lamport, Shostak, and Pease, The Byzantine Generals Problem, ACM TOPLAS, 4 (3), 382-401, 1982)

Back to the protocol...

- To broadcast a message in round r, p sends $(init, p, m, r)$ to all
- A confirmation has the form $(echo, p, m, r)$
- A witness sends $(echo, p, m, r)$ if either:
  - it receives $(init, p, m, r)$ from p directly or
  - it receives confirmations for (p, m, r) from at least $f+1$ processes (at least one correct witness)
- A process accepts (p, m, r) if it has received $n - f$ confirmations (as many as possible...)
- Protocol proceeds in rounds. Each round has 2 phases

Implementation of broadcast and accept

Phase $2r-1$

1: p sends $(init, p, m, r)$ to all

Phase $2r$

2: if q received $(init, p, m, r)$ in phase $2r-1$ then

3: q sends $(echo, p, m, r)$ to all /* q becomes a witness */

4: if q receives $(echo, p, m, r)$ from at least $n-f$ distinct processes in phase $2r$ then

5: q accepts (p, m, r)

Phase $j > 2r$

6: if q has received $(echo, p, m, r)$ from at least $f+1$ distinct processes in phases $(2r, 2r+1, \ldots, j-1)$ then

7: q sends $(echo, p, m, r)$ to all processes /* q becomes a witness */

8: if q has received $(echo, p, m, r)$ from at least $n-f$ processes in phases $(2r, 2r+1, \ldots, j)$ then

9: q accepts (p, m, r)

Implementation of broadcast and accept

Phase $2r-1$

p sends $(init, p, m, r)$ to all

Phase $2r$

if q received $(init, p, m, r)$ in phase $2r-1$ then

  q sends $(echo, p, m, r)$ to all   /* q becomes a witness */

if q receives $(echo, p, m, r)$ from at least $n-f$ distinct processes in phase $2r$ then

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Phase $j > 2r$

if q has received $(echo, p, m, r)$ from at least $f+1$ distinct processes in phases $(2r, 2r+1, \ldots, j-1)$ then

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if q has received $(echo, p, m, r)$ from at least $n-f$ processes in phases $(2r, 2r+1, \ldots, j)$ then

  q accepts (p, m, r)

Is termination a problem?

The implementation is correct

Theorem

If $n > 3f$, the given implementation of broadcast(p, m, r) and accept(p, m, r) satisfies Unforgeability, Correctness, and Relay

Assumption

Channels are authenticated

Correctness

If a correct process p executes broadcast(p, m, r) in round r, then all correct processes will execute accept(p, m, r) in round r

Correctness

If a correct process p executes broadcast(p, m, r) in round r, then all correct processes will execute accept(p, m, r) in round r

If p is correct then

- p sends $(init, p, m, r)$ to all in round r (phase $2r-1$)
- by Validity of the underlying send and receive, every correct process receives $(init, p, m, r)$ in phase
- every correct process becomes a witness
- every correct process sends $(echo, p, m, r)$ in phase $2r$
- since there are at least $n-f$ correct processes, every correct process receives at least $n-f$ echoes in phase $2r$
- every correct process executes accept(p, m, r) in phase $2r$ (in round r)

Unforgeability - 1

If a correct process q executes accept(p, m, r) in round $j \geq r$, and p is correct, then p did in fact execute broadcast(p, m, r) in round r

- Suppose q executes accept(p, m, r) in round j
- q received $(echo, p, m, r)$ from at least $n - f$ distinct processes by phase k, where $k = 2j - 1$ or $k = 2j$
- Let k' be the earliest phase in which some correct process q' becomes a witness to (p, m, r)

Unforgeability - 1

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- Let k' be the earliest phase in which some correct process q' becomes a witness to (p, m, r)

Case 1: $k' = 2r - 1$

- q' received $(init, p, m, r)$ from p
- since p is correct, it follows that p did execute broadcast(p, m, r) in round r

Case 2: $k' > 2r - 1$

- q' has become a witness by receiving $(echo, p, m, r)$ from $f+1$ distinct processes
- at most f are faulty; one is correct
- this process was a witness to (p, m, r) before phase k'

CONTRADICTION

The first correct process receives $(init, p, m, r)$ from p!

Unforgeability -2

- For q to accept, some correct process must become witness.
- Earliest correct witness q' becomes so in phase $2r-1$, and only if p did indeed executed broadcast(p, m, r)
- Any correct process that becomes a witness later can only do so if a correct process is already a witness.
- For any correct process to become a witness, p must have executed broadcast(p, m, r)

Relay

If a correct process q executes accept(p, m, r) in round $j \geq r$, then all correct processes will execute accept(p, m, r) by round $j + 1$

Relay

If a correct process q executes accept(p, m, r) in round $j \geq r$, then all correct processes will execute accept(p, m, r) by round $j + 1$

- Suppose correct q executes accept(p, m, r) in round j (phase $k = 2j - 1$ or $k = 2j$)
- q received at least $n - f$ $(echo, p, m, r)$ from distinct processes by phase k
- At least $n - 2f$ of them are correct.
- All correct procs received $(echo, p, m, r)$ from at least $n - 2f$ correct processes by phase k
- From $n > 3f$, it follows that $n - 2f \geq f + 1$. Then, all correct processes become witnesses by phase k
- All correct processes send $(echo, p, m, r)$ by phase $k + 1$
- Since there are at least $n - f$ correct processes, all correct processes will accept(p, m, r) by phase $k + 1$ (round $2j$ or $2j + 1$)

Taking a step back...

- Specified Consensus and TRB
- In the synchronous model :
  - solved Consensus and TRB for General Omission failures
  - proved lower bound on rounds required by TRB
  - solved TRB for AFMA
  - proved lower bound on replication for solving TRB with AF
  - solved TRB with AF

Ordered Broadcasts for Benign Failures

FIFO Order

If a process broadcasts a message m before it broadcasts a message m', then no correct process delivers m' unless it has previously delivered m

Uniform FIFO Order

If a process broadcasts a message m before it broadcasts a message m', then no process (correct or faulty) delivers m' unless it has previously delivered m

Causal Order

If the broadcast of a message m causally precedes the broadcast of a message m', then no correct process delivers m' unless it has previously delivered m

Uniform Causal Order

If the broadcast of a message m causally precedes the broadcast of a message m', then no process (correct or faulty) delivers m' unless it has previously delivered m .

From FIFO to Causal

Local Order

If a process broadcasts a message m and a process delivers m before broadcasting m′, then no correct process delivers m′ unless it previously delivered m

Causal Order = FIFO Order + Local Order

Total Order

If correct processes p and q both deliver messages m and m′, then p delivers m before m′ if and only if q delivers m before m′

Uniform Total Order

If correct or faulty processes p and q both deliver messages m and m′, then p delivers m before m′ if and only if q delivers m before m′

A Modular Approach to Broadcast Protocols

(Hadzilakos & Toueg)