Lecture 3

1 Generics and Type Inference

Figure 5.1 shows two functions that calculate (1) the length of a list of integers and (2) the length of a list of strings. These two functions are almost identical. The only difference between them is in the type of element in the list. We can abstract away the differences by writing a generic length function:

```scala
def length[A](lst : List[A]): Int = lst match {
  case Nil => 0
  case _ :: tail => 1 + length(tail)
}
```

In the code above, \( A \) is a type argument. It truly is an argument to the function, just like \( lst \). i.e., when \( length \) is invoked, we need to specify \( A \):

```scala
length[Int](List(1, 2, 3))
length[String](List("A", "B", "C", "D"))
```

However, \( A \) holds a type instead of a value.

Scala lets you elide type arguments in most cases. So, we could also write:

```scala
length(List(1, 2, 3))
length(List("A", "B", "C", "D"))
```

But, it is important to note that under the hood, Scala figures out that we meant \( A \) to be \( Int \) on the first line and \( String \) on the second line and inserts the type arguments for us. In fact, we relied on this in the definition of \( length \): Scala expands the recursive call \( length(tail) \) to \( length[A](tail) \). Note that Scala isn’t just “guessing” the right type. It uses the type of the value-arguments to infer the missing type argument.

However, Scala’s type inference is not perfect. It saves you typing, but don’t expect it to figure out exactly what you mean. It can make mistakes. For example, if we write \( List(1, "2", 3) \), Scala infers that it has type \( List[\text{Any}] \). But, this is almost certainly not what we meant! We probably meant to construct a list of integers or strings. If we instead wrote \( List[Int](1, "2", 3) \), Scala would have signalled a type error.

```scala
def ilength(lst : List[Int]): Int = lst match {
  case Nil => 0
  case _ :: tail => 1 + ilength(tail)
}

(def slength(lst : List[String]): Int = lst match {
  case Nil => 0
  case _ :: tail => 1 + slength(tail)
})
```

(a) Length of a list of integers. (b) Length of a list of integers.

Figure 5.1: Two very similar length functions.
1.1 Generic Data Structures

Figure 5.4 In the last lecture, we build a data structure that represents binary search trees with strings for values. We can write a similar data structure with integer-values, as shown in fig. 5.4. But, this will force us to redefine the functions we wrote earlier to proceed SBinTrees. Instead, we can write a generic type for binary trees where the values have some unknown type $A$:

```scala
sealed trait BinTree[A]
case class Leaf[A]() extends BinTree[A]
case class Node[A](lhs: BinTree[A], key: Int, value: A, rhs: BinTree[A]) extends BinTree[A]
```

Here are some examples of binary trees with values of different types:

- This binary tree has integer values:
  ```scala
  Node(Leaf(), 10, 500, Leaf())
  ```

- This binary tree has string values:
  ```scala
  Node(Leaf(), 10, "five hundred", Leaf())
  ```

- This binary tree has boolean values:
  ```scala
  Node(Leaf(), 10, true, Leaf())
  ```

We can easily update the `insert`, `find`, and `size` functions from the last lecture to work with generic binary trees.

2 Higher-Order Functions on Lists

2.1 The Map Function

Study the four functions in fig. 5.5. The function `incrs` adds one to every element in a list of integers, the function `lengths` calculates the length of every string in a list, the function `doubles` doubles every number in a list, and the function `negates` negates every number in a list. Hopefully, you’ve noticed that these four functions have a lot of in common. The only difference between them is the operation that they perform on the head of the list.

Here is a variant of `doubles` that makes the operation explicit, by moving it into a separate function:
Figure 5.5: Four different functions that transform elements of a list.

\[
\text{def incrs(list: List[Int]): List[Int] =}
\begin{align*}
\text{lst match} & \\
\text{case Nil => Nil} & \\
\text{case h :: t => h + 1 :: incrs(t)} &
\end{align*}
\]

(a) A function that increments all numbers in a list.

\[
\text{def lengths(list: List[String]): List[Int] =}
\begin{align*}
\text{lst match} & \\
\text{case Nil => Nil} & \\
\text{case h :: t => h.length :: lengths(t)} &
\end{align*}
\]

(b) A function that calculates the lengths of all strings in a list.

\[
\text{def doubles(list: List[Int]): List[Int] =}
\begin{align*}
\text{lst match} & \\
\text{case Nil => Nil} & \\
\text{case h :: t => h * 2 :: doubles(t)} &
\end{align*}
\]

(c) A function that doubles all numbers in a list.

\[
\text{def negates(list: List[Int]): List[Int] =}
\begin{align*}
\text{lst match} & \\
\text{case Nil => Nil} & \\
\text{case h :: t => -h :: negates(t)} &
\end{align*}
\]

(d) A function that negates all numbers in a list.

In this version, we’re simply applying a function \( f \) to \( h \), where \( f \) is the doubling function. *You should apply the same refactoring to the other functions.* E.g., instead of directly writing `head.length`, refactor the function so that this expression is in a helper function.

Once we’ve re-written the operation as \( f(h) \), all three functions look identical: the only difference is that each refers to a different function. Instead of writing three functions that are almost identical, we can write one function that takes \( f \) as an argument:

\[
\text{def map(f: Int => Int, lst: List[Int]): List[Int] = lst match}
\begin{align*}
\text{case Nil => Nil} & \\
\text{case h :: t => f(h) :: map(f, t)} &
\end{align*}
\]

With this function, we can make our examples much more succinct:

\[
\text{def doubles(lst: List[Int]) =} \\
\text{map(f, lst)}
\]

\[
\text{def incrs(lst: List[Int]) =} \\
\text{map(f, lst)}
\]

\[
\text{def lengths(lst: List[String]) =} \\
\text{map(f, lst)}
\]

Unfortunately, the definition of \texttt{lengths} above does not type-check. Scala reports two type errors and they are both very informative:

```
<console>:15: error: type mismatch;
```
The `map` function only works of lists of integers. However, if you look at the definition of `map` closely, you'll see that all the `Int`-specific code has been factored out into `f`. We can make the function even more generic by introducing two type-parameters:

```
def map[A,B](f: A => B, lst : List[A]): List[B] = lst match {
  case Nil => Nil
  case h :: t => f(h) :: map(f, t)
}
```

The type-parameter `A` is the type of the elements in `lst` and the type-parameter `B` is the type elements in the produced list. In the code above, we've made all the type-arguments explicit. But, as we've discussed before, we can rely on type-inference to fill them in:

```
def map[A,B](f: A => B, lst : List[A]): List[B] = lst match {
  case Nil => Nil
  case h :: t => f(h) :: map(f, t)
}
```

### 2.2 The Filter Function

Another common pattern when programming with lists is to select certain elements that have some property. For example, here is a function that consumes a list, and produces a new list that only contains the even numbers:

```
def filterEven(lst : List[Int]): List[Int] = lst match {
  case Nil => Nil
  case head :: tail =>
    (head % 2 == 0) match {
      case true => head :: filterEven(tail)
      case false => filterEven(tail)
    }
}
```

This is a very common pattern too. For example, we could write a function to select the odd numbers or the prime numbers. If we had a list of strings, we could select all the strings with length 5 or all the strings that represent English-language words. All these functions have the same shape: they test the value of `head` in some way. If the test succeeds, `head` is added in the output list. But, if the test fails, it is excluded.

Following the same strategy we used to derive `map`, we first package the `head`-test into a function:

```
def f(n: Int): Boolean = head % 2
```

```
def filterEven(lst : List[Int]): List[Int] = lst match {
  case Nil => Nil
  case head :: tail =>
    f(head) match {
      case true => head :: filterEven(tail)
      case false => filterEven(tail)
    }
}
```
Now that the pattern is clearer, we generalize `filterEven` to take \( f \) as an argument:

```scala
def filter(f: Int => Boolean, lst: List[Int]): List[Int] = lst match {
  case Nil => Nil
  case head :: tail =>
    f(head) match {
      case true => head :: filter(f, tail)
      case false => filter(f, tail)
    }
}

def filterEven(lst: List[Int]): List[Int] = {
  def f(n: Int): Boolean = n % 2
  filter(f, lst)
}
```

Finally, just as we did for `map`, we can generalize the type of `filter` so that it can be applied to `List[A]`:

```scala
def filter[A](f: A => Boolean, lst: List[A]): List[A] = lst match {
  case Nil => Nil
  case head :: tail =>
    f(head) match {
      case true => head :: filter(f, tail)
      case false => filter(f, tail)
    }
}
```

### 2.3 Sorting

You’ve written sorting functions that sort lists in ascending or descending order. But, the following higher-order function can be used to sort any lists of any type, given an arbitrary “less than” operator.

```scala
def insert[A](lessThan: (A, A) => Boolean, x: A, lst: List[A]): List[A] = lst match {
  case Nil => List(x)
  case hd :: tl => {
    if (lessThan(x, hd)) {
      x :: hd :: tl
    } else {
      hd :: insert(lessThan, x, tl)
    }
  }
}

def sort[A](lessThan: (A, A) => Boolean, lst: List[A]) = lst match {
  case Nil => Nil
  case hd :: tl => insert(lessThan, hd, sort(lessThan, tl))
}

def sortAscending(lst: List[Int]): Int = {
  def f(x: Int, y: Int): Boolean = x < y
  sort(f, lst)
}

def sortDescending(lst: List[Int]): Int = {
  def f(x: Int, y: Int): Boolean = x > y
  sort(f, lst)
}
```
def sortByLengthAscending(lst: List[String]): Int = {
    def f(x: String, y: String): Boolean = x.length < y.length
    sort(f, lst)
}

3 Builtin Methods on Lists

All the higher-order list-processing functions that we've defined are already built-in to Scala as methods on lists.

- We can map over a list using the map method:
  ```scala
def f(n: Int): Int = n + 1
assert(List(10, 20, 30).map(f) == List(11, 21, 22))
```

- We can filter a list using the filter method:
  ```scala
def f(n: Int): Boolean = n % 2 == 0
assert(List(0, 1, 2, 3, 4, 5).map(f) == List(0, 2, 4))
```

- We can sort a list using the sortWith method:
  ```scala
def f(x: Int, y: Int): Boolean = x > y
assert(List(3, 7, 9, 11).sortWith(f) == List(11, 9, 7, 3))
```

There are several other higher-order functions that you can discover from the documentation or the Scala console.