**Theorem 2.** For any constants $\beta, \alpha > 0$, smoothing parameter $\gamma (\in (0, 1))$ and margin parameter $\gamma > 0$ there exists an adversarial strategy with expected loss bounded as:

$$
\mathbb{E}[\ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta(s))] + O(\sqrt{\frac{\log T}{T}}) + \frac{1}{4\gamma} \log T + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T}
$$

where $\mathcal{N}(\mu, \sigma^2)$ is the Laplace distribution.

Also yields a policy regret bound, against policy class derived from $\mathcal{F}$.

**Proof Ideas**

- **Full information bound:** If full information involves fixed score, can obtain bandit bound via importance weighting. E.g., EXP4

$$
\mathbb{E}[\ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta(s))] + 2\gamma \sqrt{\text{KL}(p||q)} + \mathbb{E}[\ell^*(\theta(s))] + 2\gamma \sqrt{\text{KL}(p||q)}
$$

We show existence of full-information algorithm with regret scaling with (1) local norms and (2) sequential complexity. Use adaptive technique of Foster et al. (2015). We show for $\mathcal{F} = \mathcal{A}$

$$
\mathbb{E}[\ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta(s))] + 2\gamma \sqrt{\text{KL}(p||q)} + \mathbb{E}[\ell^*(\theta(s))] + 2\gamma \sqrt{\text{KL}(p||q)}
$$

**Restricted Reduction**

- Use full-information algorithm with class $\mathcal{G} = \phi^* \circ \phi; F$, to obtain $p \in \mathcal{A}(S)$

**Theorem 4.** Assume $\mathcal{F}$ is parameterized by a compact convex set $\mathcal{F} \subseteq \mathcal{B}$. Then $\mathcal{F}$-Lipschitz $m$-th and $\mathcal{F}$-smooth $1$-sth order methods, noting $\mathcal{F}$-Lipschitz parameters

$$
\mathbb{E}[\sum_{t=1}^T \ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta^*(t))] + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T}
$$

Moreover the running time is $O\left(\frac{1}{\gamma^2} \right)$

**Hinge-LMC**

- **Bandit Multiclass:** First efficient $\mathcal{F}$-algorithm against a loss without $\mathcal{F}$-smoothness.

- **Realizability:** If $\mathcal{F}$-smoothness, then $\mathcal{F}$-Lipschitz parameters are effective. $\mathcal{F}$-smooth $1$-sth order methods, noting $\mathcal{F}$-Lipschitz parameters

$$
\mathbb{E}[\sum_{t=1}^T \ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta^*(t))] + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T}
$$

**Smooth-FTL and Lipschitz CB**

- **Oracle Efficient:** Makes log($T$) calls to hingo-LMC minimization oracle.

- **Lipschitz CB:** Also yields $\mathcal{F}$-algorithm for Lipschitz CB with p-dimensional content and finite action space. Use best known guarantees for Lipschitz CB.

- **Suboptimality:** Matches information-theoretic lower bounds, has a strictly suboptimal worst-case guarantee.

---


---

**Contextual Bandit Protocol**

- **Surrogate Loss Functions**

- **Settting**

- **Achievable Regret Bounds**

- **Lipschitz CB**

- **Proof of Theorem 2**

- **Hinge-LMC Algorithm**

- **Smooth-FTL and Lipschitz CB**

- **Oracle Efficient**

---

**Contextual-Bandits with Surrogate Losses:** Margin Bounds and Efficient Algorithms

Dylan Foster and Akshay Krishnamurthy

Microsoft Research

Cornell University

For full information, rate is $\Theta\left(\frac{1}{\gamma^2} \right)$. For any $\gamma$, $\mathcal{F}$-Lipschitz parameters

$$
\mathbb{E}[\sum_{t=1}^T \ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta^*(t))] + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T}
$$

Moreover the running time is $O\left(\frac{1}{\gamma^2} \right)$

**Hinge-LMC**

- **Bandit Multiclass:** First efficient $\mathcal{F}$-algorithm against a loss without $\mathcal{F}$-smoothness.

- **Realizability:** If $\mathcal{F}$-smoothness, then $\mathcal{F}$-Lipschitz parameters are effective. $\mathcal{F}$-smooth $1$-sth order methods, noting $\mathcal{F}$-Lipschitz parameters

$$
\mathbb{E}[\sum_{t=1}^T \ell^*(\theta(t))] \leq \mathbb{E}[\ell^*(\theta^*(t))] + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{1}{\gamma} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T} + \frac{1}{4\gamma} \frac{\log T}{T}
$$

**Smooth-FTL and Lipschitz CB**

- **Oracle Efficient:** Makes log($T$) calls to hingo-LMC minimization oracle.

- **Lipschitz CB:** Also yields $\mathcal{F}$-algorithm for Lipschitz CB with p-dimensional content and finite action space. Use best known guarantees for Lipschitz CB.

- **Suboptimality:** Matches information-theoretic lower bounds, has a strictly suboptimal worst-case guarantee.