Instructions. You will be randomly assigned groups to work on these problems in discussion section. List your group members on your worksheet and turn it in at the end of class. Write first and last names. Each group member should turn in their own paper.

1. **Network Flow.** Decide if the following statements are true or false and either provide a proof (if true) or a counter example (if false).

   (a) Let $G$ be an arbitrary flow network, with a source $s$, a sink $t$, and a positive integer capacity $c_e$ on every edge $e$. If $f$ is a maximum $s - t$ flow in $G$, then $f$ saturates every edge out of $s$ with flow (i.e., for all edges $e$ out of $s$, we have $f(e) = c_e$).

   (b) Let $G$ be an arbitrary flow network, with a source $s$, a sink $t$, and a positive integer capacity $c_e$ on every edge $e$; and let $(A, B)$ be a minimum $s - t$ cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then $(A, B)$ is still a minimum $s - t$ cut with respect to these new capacities $\{1 + c_e : e \in E\}$. 
2. **Network Flow - Ford-Fulkerson.**

(a) What is the maximum flow you can send from \( s \) to \( t \) in the following graph?

(b) Remove as many of the edges as you can without changing the maximum flow into \( t \).

(c) How small can you make all of the capacities without changing the overall flow?