

# Authenticated Encryption

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Based on <http://cseweb.ucsd.edu/~mihir/cse107/>

# Motivation

In practice we often want **both** privacy and authenticity.

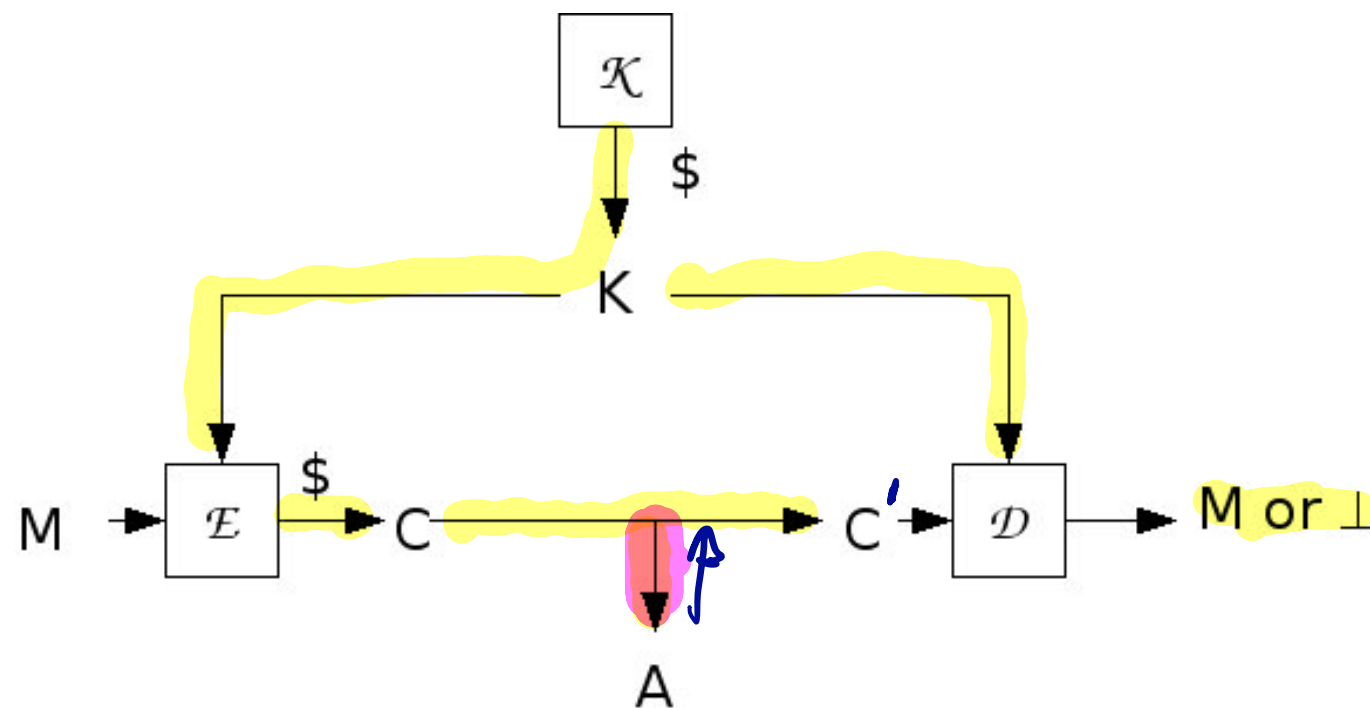
**Example:** A doctor wishes to send medical information  $M$  about Alice to the medical database. Then

- We want **data privacy** to ensure Alice's medical records remain **confidential**.
- We want **authenticity** to ensure the person sending the information is really the doctor and the information was **not modified** in transit.

We refer to this as **authenticated encryption**. *combines enc + macs.*

# Syntax

Syntactically, an authenticated encryption scheme is just a symmetric encryption scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  where



# Security

- The same notion of **privacy** applies, namely IND-CPA

# Security

- The same notion of **privacy** applies, namely IND-CPA
- For **authenticity**, the adversary's goal is to get the receiver to accept a "non-authentic" ciphertext (i.e., not actually transmitted by the sender)

IND-CPA

integrity of ciphertexts

# INT-CTXT

very similar to UF-CMA

Let  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  be a symmetric encryption scheme and  $A$  an adversary.

Game  $\text{INTCTXT}_{\mathcal{AE}}$

**procedure Initialize**

$K \xleftarrow{\$} \mathcal{K} ; S \leftarrow \emptyset$

**procedure Enc( $M$ )**

$C \xleftarrow{\$} \mathcal{E}_K(M)$

$S \leftarrow S \cup \{C\}$

Return  $C$

**procedure Finalize( $C^*$ )**

$M \leftarrow \mathcal{D}_K(C^*)$

if ( $C^* \notin S \wedge M \neq \perp$ ) then

    return true

Else return false

The int-ctxt advantage of  $A$  is

$$\text{Adv}_{\mathcal{AE}}^{\text{int-ctxt}}(A) = \Pr[\text{INTCTXT}_{\mathcal{AE}}^A \Rightarrow \text{true}]$$

# Integrity + Privacy

The goal of authenticated encryption is to provide both integrity and privacy. We will be interested in IND-CPA + INT-CTXT.

# Plain Encryption: CBC\$

It is IND-CPA assuming

$E$  is a good PRF

Alg  $\mathcal{E}_K(M)$

$C[0] \xleftarrow{\$} \{0, 1\}^n$

For  $i = 1, \dots, m$  do

$C[i] \leftarrow E_K(C[i-1] \oplus M[i])$

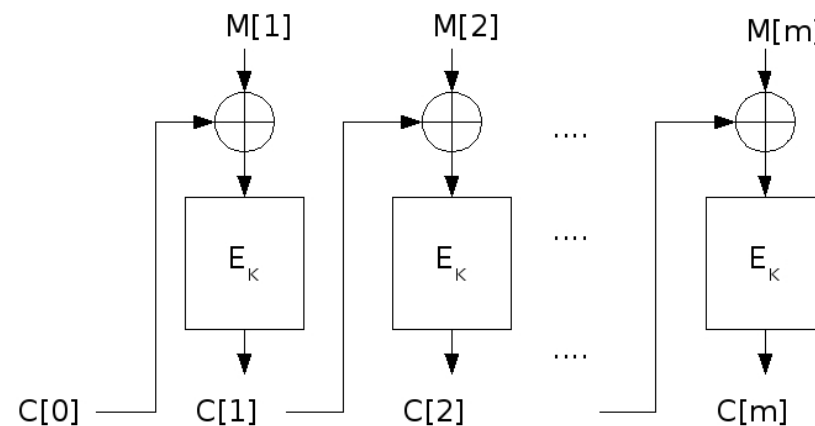
Return  $C$

Alg  $\mathcal{D}_K(C)$

For  $i = 1, \dots, m$  do

$M[i] \leftarrow E_K^{-1}(C[i]) \oplus C[i-1]$

Return  $M$



**Question:** Is CBC\$ encryption INT-CTXT secure?

NO!



# Plain Encryption Does Not Provide Integrity

⊥ "but" error or undefined

Alg  $\mathcal{E}_K(M)$

$C[0] \xleftarrow{\$} \{0, 1\}^n$   
For  $i = 1, \dots, m$  do  
     $C[i] \leftarrow E_K(C[i-1] \oplus M[i])$   
Return  $C$

Alg  $\mathcal{D}_K(C)$

For  $i = 1, \dots, m$  do  
     $M[i] \leftarrow E_K^{-1}(C[i]) \oplus C[i-1]$   
Return  $M$

adversary  $A$

$C[0]C[1]C[2] \xleftarrow{\$} \{0, 1\}^{3n}$   
Return  $C[0]C[1]C[2]$

} breaks INT-CTXT

Then

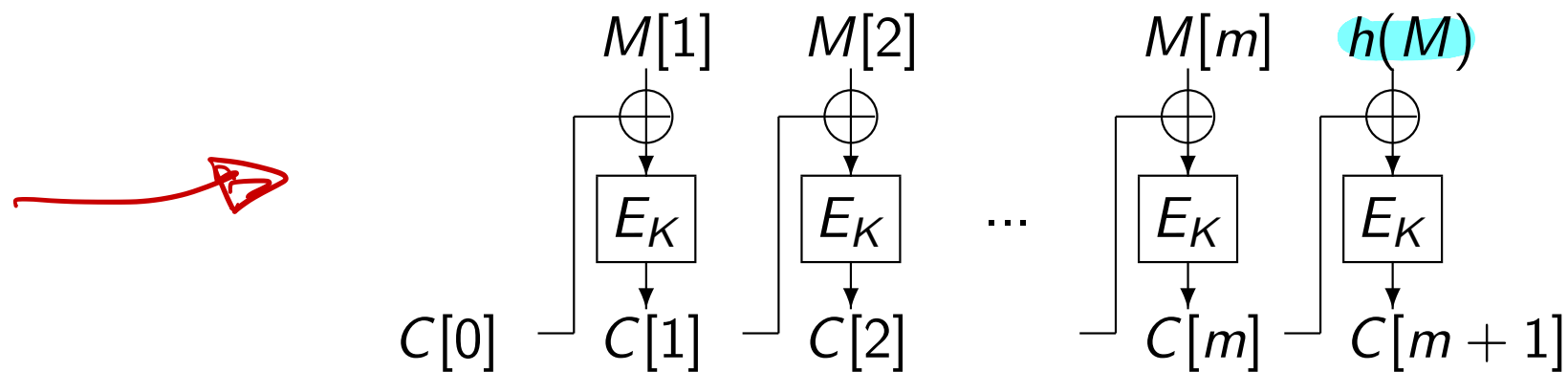
$$\text{Adv}_{\mathcal{SE}}^{\text{int-ctxt}}(A) = 1$$

This violates INT-CTXT.

A scheme whose decryption algorithm never outputs ⊥ cannot provide integrity!

# Encryption with Redundancy

*CBC with redundancy*

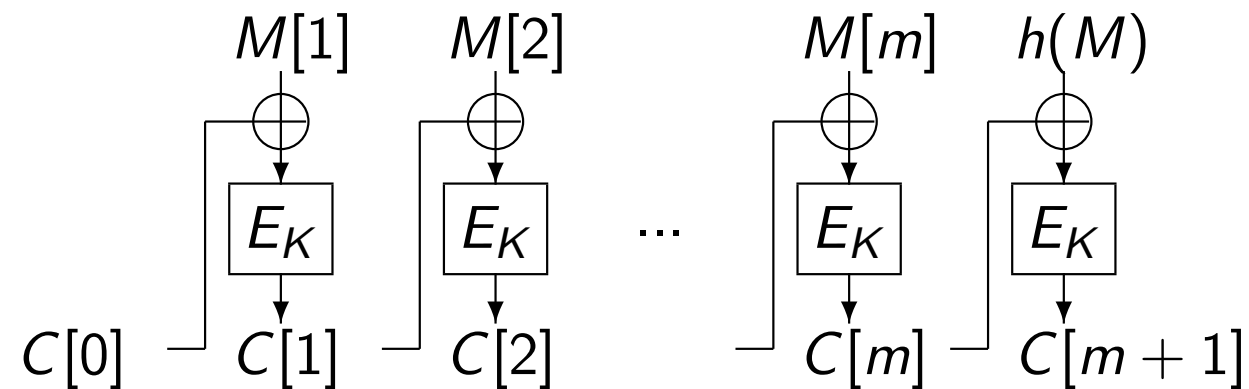


Here  $E: \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$  is our block cipher and  $h: \{0, 1\}^* \rightarrow \{0, 1\}^n$  is a “redundancy” function, for example

- $h(M[1] \dots M[m]) = 0^n$
- $h(M[1] \dots M[m]) = M[1] \oplus \dots \oplus M[m]$  ← checksum
- A CRC
- $h(M[1] \dots M[m])$  is the first  $n$  bits of  $\text{SHA1}(M[1] \dots M[m])$ .

The redundancy is verified upon decryption.

# Encryption with Redundancy



Let  $E: \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$  be our block cipher and  $h: \{0, 1\}^* \rightarrow \{0, 1\}^n$  a redundancy function. Let  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}', \mathcal{D}')$  be CBC\$ encryption and define the encryption with redundancy scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  via

**Alg**  $\mathcal{E}_K(M)$

$M[1] \dots M[m] \leftarrow M$

$M[m+1] \leftarrow h(M)$

$C \xleftarrow{\$} \mathcal{E}'_K(M[1] \dots M[m]M[m+1])$

return  $C$

**Alg**  $\mathcal{D}_K(C)$

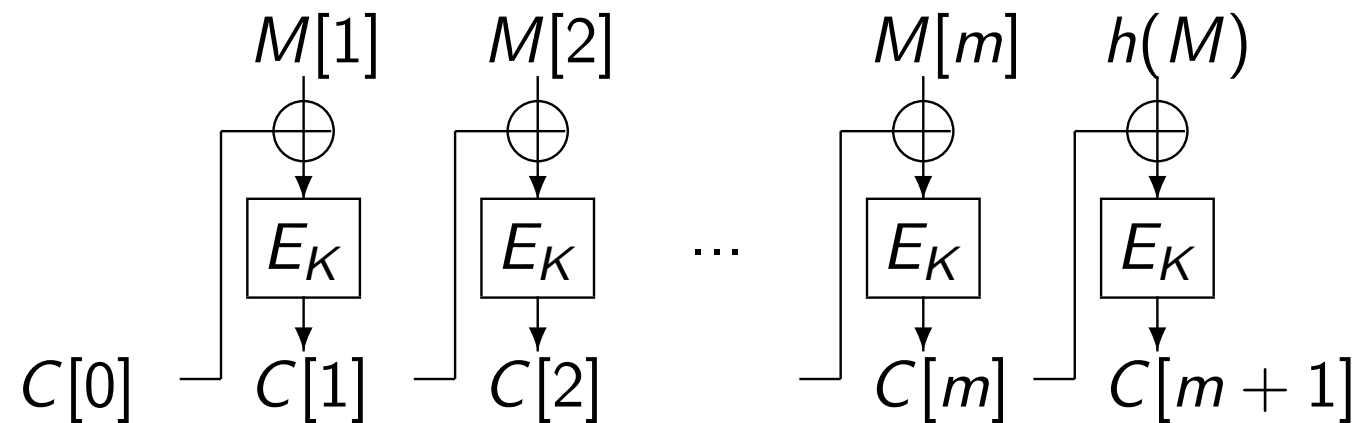
$M[1] \dots M[m]M[m+1] \leftarrow \mathcal{D}'_K(C)$

if  $(M[m+1] \equiv h(M))$  then

    return  $M[1] \dots M[m]$

else return  $\perp$

# Does it Work?



The adversary will have a hard time producing the last enciphered block of a new message.



intuition

Is the intuition correct?

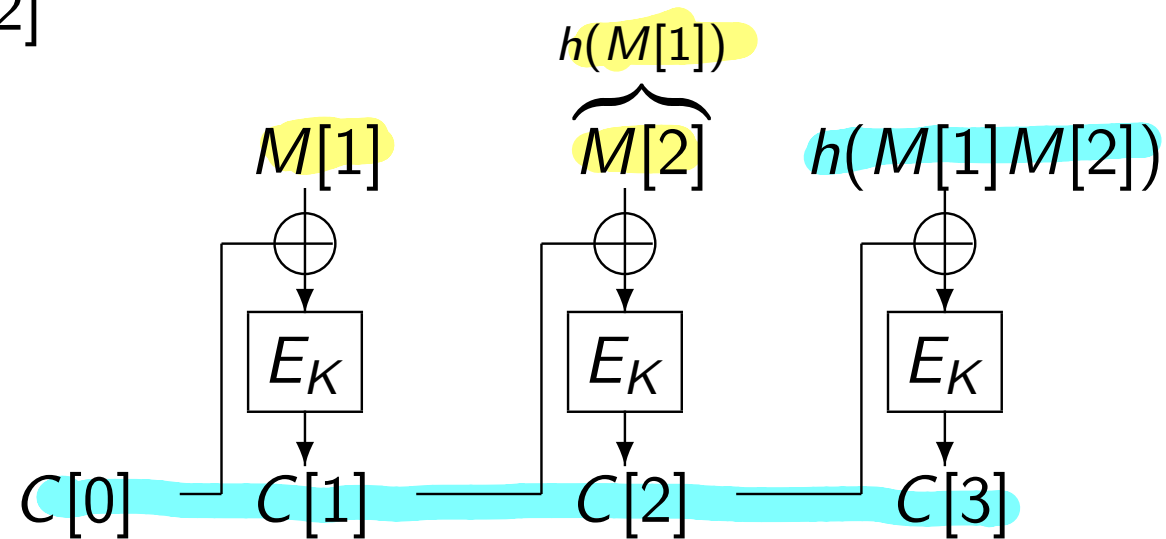
# Attacks

## adversary A

$M[1] \xleftarrow{\$} \{0, 1\}^n ; M[2] \leftarrow h(M[1])$

$C[0]C[1]C[2]C[3] \xleftarrow{\$} \mathbf{Enc}(M[1]M[2])$

Return  $C[0]C[1]C[2]$



This attack succeeds for any (not secret-key dependent) redundancy function  $h$ .

doesn't matter how "complicated"  
your redundancy fn. is.

# WEP Attack

A “real-life” rendition of this attack broke the 802.11 WEP protocol, which instantiated  $h$  as CRC and used a stream cipher for encryption [BGW].

What makes the attack easy to see is having a clear, strong and formal security model.

# Generic Composition

Build an authenticated encryption scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  by combining

- a given IND-CPA symmetric encryption scheme  $\mathcal{SE} = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$
- a given PRF  $F : \{0, 1\}^k \times \{0, 1\}^* \rightarrow \{0, 1\}^n$

how  
to  
combine?

	CBC\$-AES	CTR\$-AES	...
HMAC-SHA1	AE scheme		
CMAC			
ECBC			
⋮			

Want generic composition methods that work for arbitrary secure starting schemes.

# Generic Composition

Build an authenticated encryption scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  by combining

- a given IND-CPA symmetric encryption scheme  $\mathcal{SE} = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$
- a given PRF  $F : \{0, 1\}^k \times \{0, 1\}^* \rightarrow \{0, 1\}^n$

A key  $K = K_e || K_m$  for  $\mathcal{AE}$  always consists of a key  $K_e$  for  $\mathcal{SE}$  and a key  $K_m$  for  $F$ :

MAC  
(prf)

**Alg  $\mathcal{K}$**

$K_e \xleftarrow{\$} \mathcal{K}'; K_m \xleftarrow{\$} \{0, 1\}^k$

Return  $K_e || K_m$



# Generic Composition

The **order** in which the primitives are applied is important. Can consider

Method	Usage
→ Encrypt-and-MAC (E&M)	SSH →
→ MAC-then-encrypt (MtE)	SSL/TLS →
→ Encrypt-then-MAC (EtM)	IPSec →

# Encrypt-and-MAC

PRF

PRF == MAC

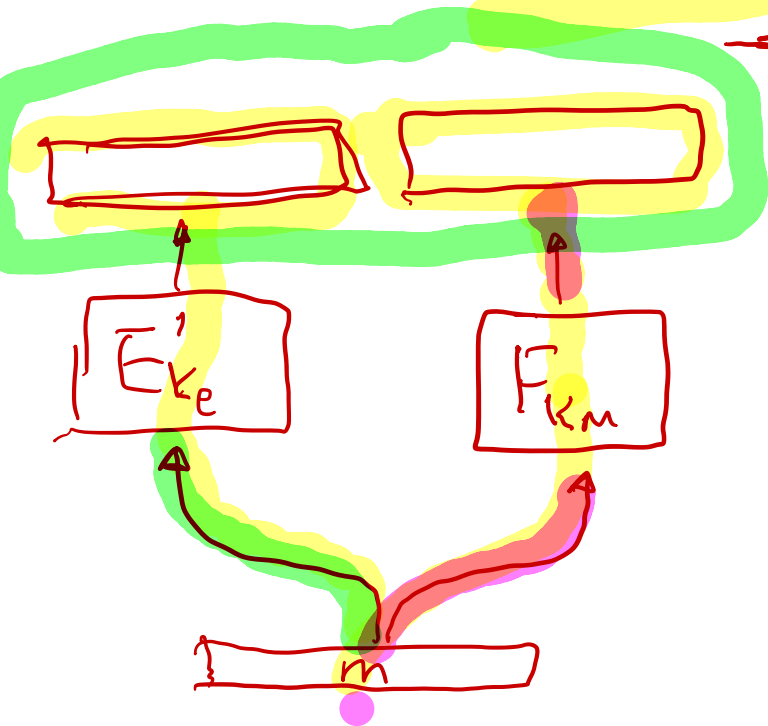
$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

**Alg**  $\mathcal{E}_{K_e || K_m}(M)$

$C' \xleftarrow{\$} \mathcal{E}'_{K_e}(M)$   
 $T \leftarrow F_{K_m}(M)$   
 Return  $C' || T$

**Alg**  $\mathcal{D}_{K_e || K_m}(C' || T)$

$M \leftarrow \mathcal{D}'_{K_e}(C')$   
 If  $(T = F_{K_m}(M))$  then return  $M$   
 Else return  $\perp$



Security	Achieved?
IND-CPA	NO!
INT-CTXT	NO!

why?  
 $F_{K_m}$  is deterministic so usual attack applies

ciphertext could have a "superfluous" bit

Decryption never uses last bit of ciphertext

Adversary  $A$   $E(LR(\cdot, i, b)) \parallel \mathcal{O}(\cdot, i)$

$c_1 \parallel t_1 \leftarrow \mathcal{O}(1^n, 1^n)$

$c_2 \parallel t_2 \leftarrow \mathcal{O}(1^n, 0^n)$

If  $t_1 = t_2 \dots$

---

Idea:  $\parallel$  For INT-LTXT adversary

Adversary  $A$   $ENC(\cdot)$

$c \parallel t \leftarrow ENC(0^n)$

Parse  $c$  as  $c' \parallel b$

superfluous bit

$c_{new} \leftarrow c' \parallel \bar{b} \leftarrow$  bitwise comp.

ret  $c_{new} \parallel t$

# MAC-then-Encrypt

→  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

**Alg**  $\mathcal{E}_{K_e||K_m}(M)$

$T \leftarrow F_{K_m}(M)$

$C \xleftarrow{\$} \mathcal{E}'_{K_e}(M||T)$

Return  $C$

**Alg**  $\mathcal{D}_{K_e||K_m}(C)$

$M||T \leftarrow \mathcal{D}'_{K_e}(C)$

If  $(T = F_{K_m}(M))$  then return  $M$

Else return  $\perp$

Security	Achieved?
IND-CPA	YES!
INT-CTXT	NO!

→ superfluous bits

encrypt

$m||t$

where  $t$  is tag of  $m$ .

# Encrypt-then-MAC

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

**Alg**  $\mathcal{E}_{K_e || K_m}(M)$

$C' \xleftarrow{\$} \mathcal{E}'_{K_e}(M)$

$T \leftarrow F_{K_m}(C')$

Return  $C' || T = C$

**Alg**  $\mathcal{D}_{K_e || K_m}(C' || T)$

$M \leftarrow \mathcal{D}'_{K_e}(C')$

If  $(T = F_{K_m}(C'))$  then return  $M$

Else return  $\perp$

Security	Achieved?
IND-CPA	YES
INT-CTXT	YES

# Two keys?

→ 1 physical key ~ → { 2 synthetic keys

We have used separate keys  $K_e, K_m$  for the encryption and message authentication. However, these can be derived from a single key  $K$  via  $K_e = F_K(0)$  and  $K_m = F_K(1)$ , where  $F$  is a PRF such as a block cipher, the CBC-MAC or HMAC.

Trying to directly use the same key for the encryption and message authentication is error-prone, but works if done correctly.

→ CMAC (OMAC)

one-key MAC  
no re-keying

# Generic Composition in Practice

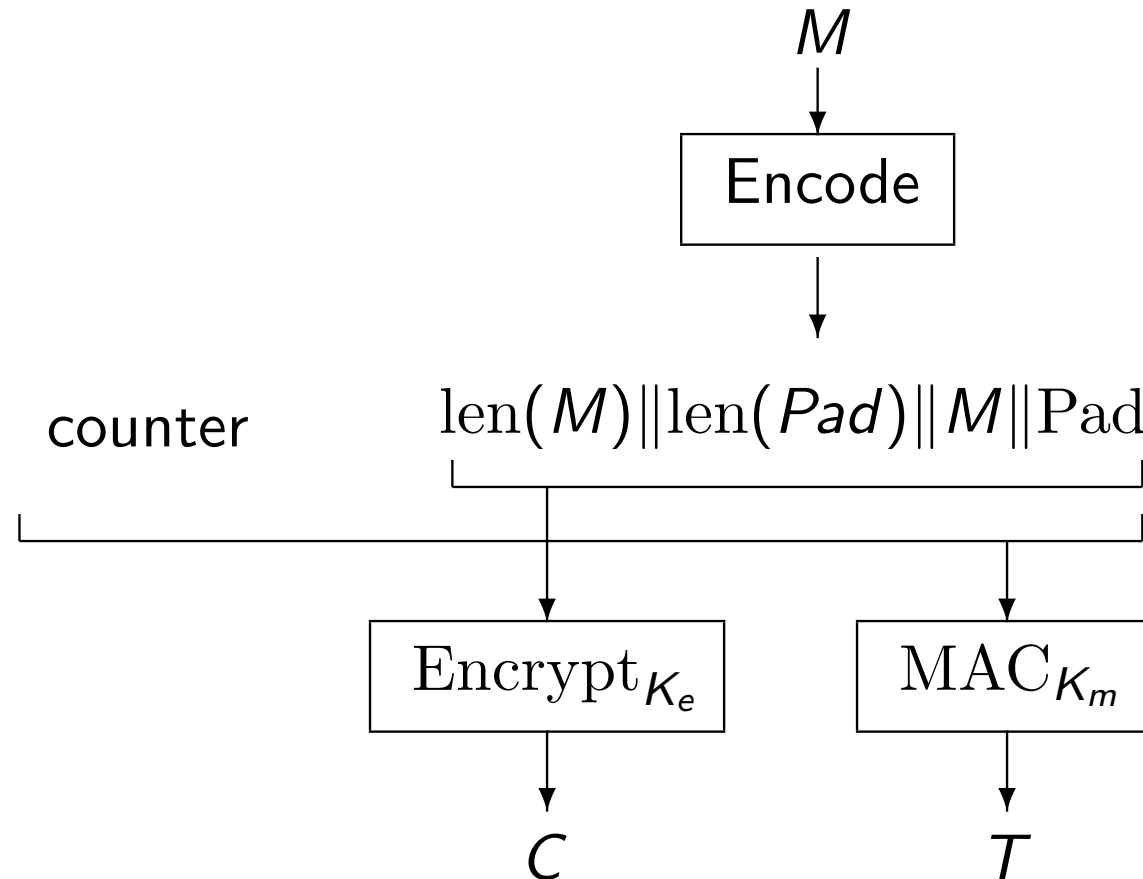
SSL 3.0 : POODLE CBC-\$  
(padding)

AE in	is based on	which in general is	and in this case is
SSH	E&M	insecure	secure
SSL	MtE	insecure	insecure
SSL + RFC 4344	MtE	insecure	secure
IPSec	EtM	secure	secure
WinZip	EtM	secure	insecure

Why?

- Encodings
- Specific “E” and “M” schemes
- For WinZip, disparity between usage and security model

# AE in SSH



SSH2 encryption uses inter-packet chaining which is insecure [D, BKN]. RFC 4344 [BKN] proposed fixes that render SSH provably IND-CPA + INT-CTXT secure. Fixes recommended by Secure Shell Working Group and included in OpenSSH since 2003. Fixes included in PuTTY since 2008.