#### Hash Functions

Adam O'Neill Based on http://cseweb.ucsd.edu/~mihir/cse207/

### Where we are

 We've seen a lower-level primitive (blockciphers) and a higher-level primitive (symmetric encryption)

#### Where we are

- We've seen a lower-level primitive (blockciphers) and a higher-level primitive (symmetric encryption)
- Now let's see another lower-level primitive, hash functions

### Hash functions

- MD: MD4, MD5, MD6
- SHA2: SHA1, SHA224, SHA256, SHA384, SHA512
- SHA3: SHA3-224, SHA3-256, SHA3-384, SHA3-512

Their primary purpose is collision-resistant data compression, but they have many other purposes and properties as well ... A hash function is often treated like a magic wand ...

#### Some uses:

- Certificates: How you know www.snapchat.com really is Snapchat
- Bitcoin
- Data authentication with HMAC: TLS, ...

## SHA1 is dead

BIZ & IT -

#### At death's door for years, widely used SHA1 function is now dead

Algorithm underpinning Internet security falls to first-known collision attack.

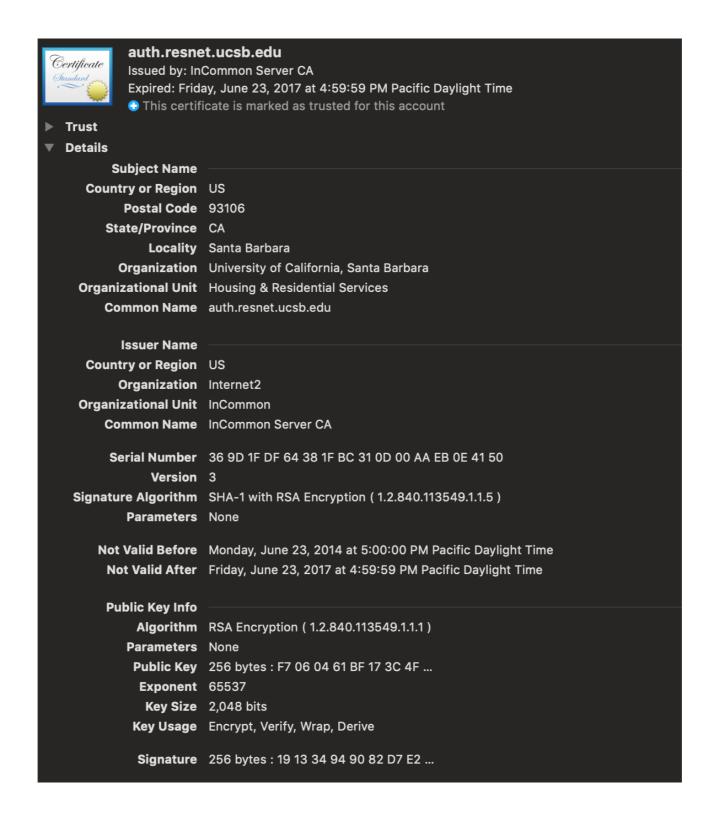
**DAN GOODIN - 2/23/2017, 5:01 AM** 





For more than six years, the SHA1 cryptographic hash function underpinning Internet security has been at death's door. Now it's officially dead, thanks to the submission of the first known instance of a fatal exploit known as a "collision."

### A SHA1 certificate



# No longer...

May 5, 2016

#### Microsoft will cease support for TLS certs signed by SHA1

**Greg Masters** Managing Editor

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Microsoft browsers will no longer display a lock when on HTTPS sites protected by SHA1 certs.

Microsoft announced it will soon cease support for TLS certificates signed by the SHA1 hashing algorithm, according to ArsTechnica.

After hinting in November that it might, the tech giant made it official last week. The end was expected following new research that revealed the popular cryptographic algorithm was susceptible to collision attacks in which miscreants attempt to find two inputs producing the same hash value. Should they succeed, they would be able to forge digital

signatures.

## Implication for Bitcoin?





The cryptography world has been buzzing with the news that researchers at Google and CWI Amsterdam have succeeded in successfully generating a 'hash collision' for two different documents using the SHA1 encryption algorithm, rendering the algorithm 'broken' according to cryptographic standards.

But what does this mean in plain language, and what are the implications for the bitcoin network?

#### **Hash collisions**

As laid out in a recent CoinDesk explainer, a hash function (of which SHA1 is an example) is used to take a piece of data of any length, process it, and return another piece of data – the 'hash digest' – with a fixed length.

One way that hash functions are used in computing is to check whether the contents of files are identical: as long as a hash function is secure, then two files which hash to the same value will always have the same contents.

However, a hash collision occurs when two different files hash to the same value.

Given the mathematical laws that govern hash functions, it is inevitable that hash collisions will occur for some values of input data (because the range of data you could put into the hash function is potentially infinite, but the output length is fixed).

For a secure hash function, the probability of this should be so small that, in practice, it is not possible to make a sufficient number of calculations to find it.

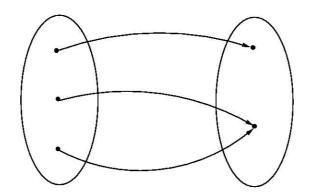
The significance of the Google/CWI team's results is in the fact that they were able to create a hash collision by finding a much more efficient method – 100,000 times more efficient in fact – than simply guessing every possible value of data.

### Collisions

A **collision** for a function  $h: D \to \{0,1\}^n$  is a pair  $x_1, x_2 \in D$  of points such that

- $h(x_1) = h(x_2)$ , and
- $x_1 \neq x_2$ .

If  $|D| > 2^n$  then the pigeonhole principle tells us that there must exist a collision for h.



We want that even though collisions exist, they are hard to find.

### The Formalism

The formalism considers a family H: Keys $(H) \times D \to R$  of functions, meaning for each  $K \in \text{Keys}(H)$  we have a map  $H_K : D \to R$  defined by  $H_K(x) = H(K, x)$ .

```
Game CR_H procedure Finalize(x_1, x_2)

procedure Initialize

K \leftarrow^{\$} Keys(H)

Return K If (x_1 = x_2) then return false

K \leftarrow^{\$} Keys(H)

Return (H_K(x_1) = H_K(x_2))
```

Let

$$\mathbf{Adv}_H^{\mathrm{cr}}(A) = \Pr\left[\mathrm{CR}_H^A \Rightarrow \mathsf{true}\right].$$

### The Formalism

```
Game CR_H procedure Finalize (x_1, x_2)

procedure Initialize If (x_1 = x_2) then return false If (x_1 \notin D \text{ or } x_2 \notin D) then return false Return K Return (H_K(x_1) = H_K(x_2))
```

The Return statement in **Initialize** means that the adversary A gets K as input. The key K here is not secret!

Adversary A takes K and tries to output a collision  $x_1, x_2$  for  $H_K$ .

A's output is the input to **Finalize**, and the game returns true if the collision is valid.

## Example

Let  $N = 2^{256}$  and define

$$H: \underbrace{\{1,\ldots,N\}}_{\text{Keys}} \times \underbrace{\{0,1,2,\ldots\}}_{D} \to \underbrace{\{0,1,\ldots,N-1\}}_{R}$$

by

$$H(K,x)=(x \bmod K).$$

**Q:** Is *H* collision resistant?

## Another example

```
Let E: \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n be a blockcipher.
Let H: \{0,1\}^k \times \{0,1\}^{2n} \to \{0,1\}^n be defined by
```

Alg 
$$H(K, x[1]x[2])$$
  
 $y \leftarrow E_K(E_K(x[1]) \oplus x[2])$ ; Return  $y$ 

Let's show that H is not collision-resistant by giving an efficient adversary A such that  $\mathbf{Adv}_{H}^{\mathrm{cr}}(A)=1$ .

## Another example

Let  $E: \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$  be a blockcipher.

Let  $H: \{0,1\}^k \times \{0,1\}^{2n} \to \{0,1\}^n$  be defined by

$$\frac{\textbf{Alg } H(K,x[1]x[2])}{y \leftarrow E_K(E_K(x[1]) \oplus x[2]); \text{ Return } y}$$

Let's show that H is not collision-resistant by giving an efficient adversary A such that  $Adv_H^{cr}(A) = 1$ .

**Idea:** Pick  $x_1 = x_1[1]x_1[2]$  and  $x_2 = x_2[1]x_2[2]$  so that

$$E_K(x_1[1]) \oplus x_1[2] = E_K(x_2[1]) \oplus x_2[2]$$

## Another example

```
Alg H(K, x[1]x[2])

y \leftarrow E_K(E_K(x[1]) \oplus x[2]); Return y
```

**Idea:** Pick 
$$x_1 = x_1[1]x_1[2]$$
 and  $x_2 = x_2[1]x_2[2]$  so that  $E_K(x_1[1]) \oplus x_1[2] = E_K(x_2[1]) \oplus x_2[2]$ 

#### adversary A(K) $x_1 \leftarrow 0^n 1^n$ ; $x_2[2] \leftarrow 0^n$ ; $x_2[1] \leftarrow (E_K^{-1})(E_K(x_1[1]) \oplus x_1[2] \oplus x_2[2])$

return  $x_1, x_2$ 

Then  $\mathbf{Adv}_{H}^{\mathrm{cr}}(A) = 1$  and A is efficient, so H is not CR.

Note how we used the fact that A knows K and the fact that E is a blockcipher!

## Keyless Hash Functions

We say that H: Keys  $\times D \to R$  is keyless if Keys  $= \{\varepsilon\}$  consists of just one key, the empty string.

In this case we write H(x) in place of  $H(\varepsilon, x)$  or  $H_{\varepsilon}(x)$ .

Practical hash functions like the MD, SHA2 and SHA3 series are keyless.

## **SHA256**

The hash function SHA256:  $\{0,1\}^{<2^{64}} \rightarrow \{0,1\}^{256}$  is **keyless**, with

- Inputs being strings X of any length strictly less than  $2^{64}$
- Outputs always having length 256.

```
Alg SHA256(X)  // |X| < 2^{64}
M \leftarrow \text{shapad}(X)  // |M| mod 512 = 0
M^{(1)}M^{(2)}\cdots M^{(n)} \leftarrow M  // Break M into 512 bit blocks
H_0^{(0)} \leftarrow 6a09e6677; H_1^{(0)} \leftarrow bb67ae85; \cdots; H_7^{(0)} \leftarrow 5be0cd19
H_1^{(0)} \leftarrow H_1^{(0)}H_2^{(0)}\cdots H_7^{(0)}  // |H_i^{(0)}| = 32, |H^{(0)}| = 256
For i = 1, \ldots, n \text{ do } H^{(i)} \leftarrow \text{sha256}(M^{(i)}) |H^{(i-1)})
Return H^{(n)}
```

sha256:  $\{0,1\}^{512+256} \to \{0,1\}^{256}$  is the **compression function**.

Willin Deliale CC5D

# Underlying blockcipher

```
Alg E^{sha256}(x, v) // x is a 512-bit key, v is a 256-bit input
x_0 \cdots x_{15} \leftarrow x // Break x into 32-bit words
For t = 0, \ldots, 15 do W_t \leftarrow x_t
For t = 16, \ldots, 63 do W_t \leftarrow \sigma_1(W_{t-2}) + W_{t-7} + \sigma_0(W_{t-15}) + W_{t-16}
v_0 \cdots v_7 \leftarrow v // Break v into 32-bit words
For j = 0, ..., 7 do S_i \leftarrow v_i // Initialize 256-bit state S
Fot t = 0, \ldots, 63 do // 64 rounds
     T_1 \leftarrow S_7 + \gamma_1(S_4) + Ch(S_4, S_5, S_6) + C_t + W_t
     T_2 \leftarrow \gamma_0(S_0) + Maj(S_0, S_1, S_2)
     S_7 \leftarrow S_6; S_6 \leftarrow S_5; S_5 \leftarrow S_4; S_4 \leftarrow S_3 + T_1
     S_3 \leftarrow S_2 \; ; \; S_2 \leftarrow S_1 \; ; \; S_1 \leftarrow S_0 \; ; \; S_0 \leftarrow T_1 + T_2
S \leftarrow S_0 \cdots S_7
Return S // 256-bit output
```

#### Internals

#### On the previous slide:

- $\sigma_0, \sigma_1, \gamma_0, \gamma_1, Ch, Maj$  are functions not detailed here.
- $C_1 = 428a2f98$ ,  $C_2 = 71374491$ , ...,  $C_{63} = c67178f2$  are constants, where  $C_i$  is the first 32 bits of the fractional part of the cube root of the i-th prime.

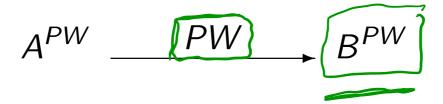
## Usage

Uses include hashing the data before signing in creation of certificates, data authentication with HMAC, key-derivation, Bitcoin, ...

These will have to wait, so we illustrate another use, the hashing of passwords.

#### Password authentication

- Client A has a password PW that is also stored by server B
- A authenticates itself by sending PW to B over a secure channel (TLS)



**Problem:** The password will be found by an attacker who compromises the server.

These types of server compromises are common and often in the news: Yahoo, Equifax, ...

## Hashed passwords

- Client A has a password PW and server stores  $\overline{PW} = H(PW)$ .
- A sends PW to B (over a secure channel) and B checks that  $H(PW) = \overline{PW}$



Server compromise results in attacker getting  $\overline{PW}$  which should not reveal PW as long as H is one-way, which is a consequence of collision-resistance.

But we will revisit this when we consider dictionary attacks!

This is how client authentication is done on the Internet, for example login to gmail.com.

password authenticated Icey exchange

## Birthday attack

Let  $H: \{0,1\}^k \times D \to \{0,1\}^n$  be a family of functions with  $|D| > 2^n$ . The q-trial birthday attack is the following adversary  $A_q$  for game  $CR_H$ :

#### adversary $A_q(K)$

for i = 1, ..., q do  $x_i \stackrel{\$}{\leftarrow} D$ ;  $y_i \leftarrow H_K(x_i)$  if  $\exists i, j \ (i \neq j \text{ and } y_i = y_j \text{ and } x_i \neq x_j)$  then return  $x_i, x_j$  else return  $\bot$ 

Interestingly, the analysis of this via the birthday problem is not trivial, but it shows that

$$\mathbf{Adv}_{H}^{\mathrm{cr}}(A_q) \geq 0.3 \cdot \frac{q(q-1)}{2^n} .$$

So a collision can usually be found in about  $q = \sqrt{2^n}$  trials.

#### Attack times

-	h	e	
		_	

Function	n	$T_B$
MD4	128	2 <sup>64</sup>
MD5	128	264
SHA1	160	280
SHA256	256	$2^{128}$
SHA512	512	2 <sup>256</sup>
SHA3-256	256	2 <sup>128</sup>
SHA3-512	512	2256

out

 $T_B$  is the number of trials to find collisions via a birthday attack.

Design of hash functions aims to make the birthday attack the best collision-finding attack, meaning it is desired that there be no attack succeeding in time much less than  $T_B$ .

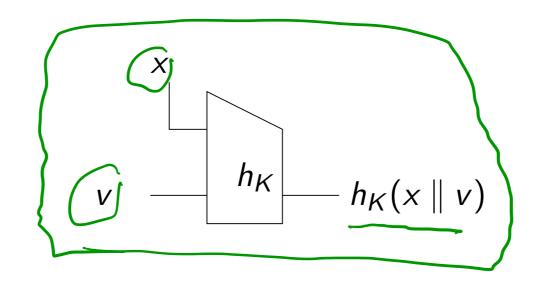
# Compression Functions

Darres-Meyer design principle

A compression function is a family  $h: \{0,1\}^k \times \{0,1\}^{b+n} \to \{0,1\}^n$  of hash functions whose inputs are of a fixed size b+n, where b is called the block size. "copacilow"

E.g. b = 512 and n = 160, in which case

$$h: \{0,1\}^k \times \{0,1\}^{672} \to \{0,1\}^{160}$$



#### Merkly Darrossog and

## **MD Transform**

Let  $h: \{0,1\}_{-}^{k} \times \{0,1\}_{-}^{b+n} \to \{0,1\}_{-}^{n}$  be a compression function with block length b. Let D be the set of all strings of at most  $2^{b}-1$  blocks.

The MD transform builds from h a family of functions

$$H: \{0,1\}^k \times D \to \{0,1\}^n$$

such that: If h is CR, then so is H.

The problem of hashing long inputs has been reduced to the problem of hashing fixed-length inputs.

There is no need to try to attack H. You won't find a weakness in it unless h has one. That is, H is guaranteed to be secure assuming h is secure.

For this reason MD is the design used in many hash functions, including the MD and SHA2 series. SHA3 uses a different paradigm.

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## MD Setup

assume input length is a multiple of the block length

Given: Compression function (b):  $\{0,1\}^k \times \{0,1\}^{b+n} \to \{0,1\}^n$ .

Build: Hash function  $\widehat{H}$ :  $\{0,1\}^k \times D \rightarrow \{0,1\}^n$ .

Since  $M \in D$ , its length  $\ell = |M|$  is a multiple of the block length b. We let  $||M||_b = |M|/b$  be the number of b-bit blocks in M, and parse as

$$M[1] \dots M[\ell] \leftarrow M$$
.  $\mathcal{L} \cup \mathcal{L} \subseteq \mathcal{L} \subseteq \mathcal{L}$ 

Let  $(\widehat{\ell \ell})$  denote the *b*-bit binary representation of  $\ell \in \{0, \dots, 2^b-1\}$ .

### The Transform

Given: Compression function  $h: \{0,1\}^k \times \{0,1\}^{b+n} \to \{0,1\}^n$ .

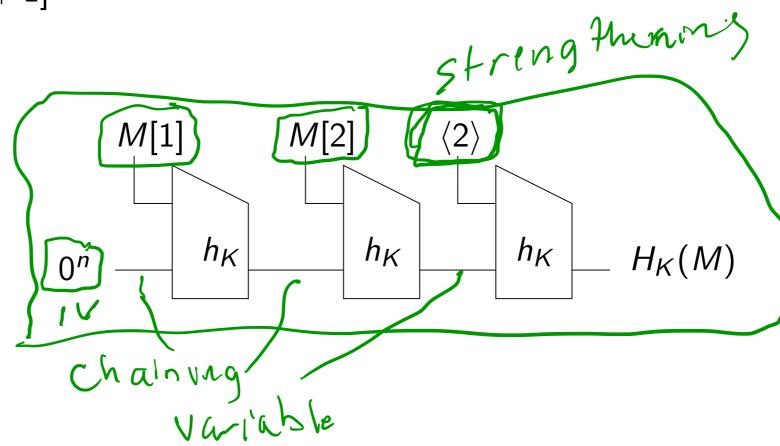
Build: Hash function  $H: \{0,1\}^k \times D \rightarrow \{0,1\}^n$ .

Algorithm  $H_K(M)$ 

$$m \leftarrow \|M\|_b$$
;  $M[m+1] \leftarrow \langle m \rangle$ ;  $V[0] \leftarrow 0^n$ 

For 
$$i = 1, ..., m + 1$$
 do  $v[i] \leftarrow h_K(M[i]||V[i - 1])$ 

Return V[m+1]



## MD preserves CR

**Theorem:** Let  $h: \{0,1\}^k \times \{0,1\}^{b+n} \to \{0,1\}^n$  be a family of functions and let  $H: \{0,1\}^k \times D \to \{0,1\}^n$  be obtained from h via the MD transform. Given a cr-adversary  $A_H$  we can build a cr-adversary  $A_h$  such that

and the running time of  $\widetilde{A_h}$  is that of  $A_H$  plus the time for computing h on the outputs of  $A_H$ .

Implication:  $\begin{array}{c} h \ \mathrm{CR} \\ & \Rightarrow \quad \mathsf{Adv}_h^{\mathrm{cr}}(A_h) \ \mathsf{small} \\ & \Rightarrow \quad \mathsf{Adv}_H^{\mathrm{cr}}(A_H) \ \mathsf{small} \\ & \Rightarrow \quad H \ \mathrm{CR} \end{array}$ 

#### Davies - Meyer

## Compression functions

Let  $E:\{0,1\}^b \times \{0,1\}^n \to \{0,1\}^n$  be a block cipher. Let us define keyless compression function  $h:\{0,1\}^{b+n} \to \{0,1\}^n$  by  $h(x||v) = E_x(v) \ .$ 

**Question:** Is *h* collision resistant?

We seek an adversary that outputs distinct  $x_1||v_1, x_2||v_2$  satisfying

$$E_{x_1}(v_1) = E_{x_2}(v_2)$$
.

con we design a CR compression function From a block cipher?

## Compression functions

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.

**Answer:** NO, h is NOT collision-resistant, because the following adversary A has  $Adv_h^{cr}(A) = 1$ :

#### adversary A

$$x_1 \leftarrow 0^b$$
;  $x_2 \leftarrow 1^b$ ;  $v_1 \leftarrow 0^n$ ;  $y \leftarrow E_{x_1}(v_1)$ ;  $v_2 \leftarrow E_{x_2}^{-1}(y)$   
Return  $x_1 ||v_1|, x_2 ||v_2|$ 

## Compression functions

Let  $E: \{0,1\}^b \times \{0,1\}^n \to \{0,1\}^n$  be a block cipher. Let us define keyless compression function  $h: \{0,1\}^{b+n} \to \{0,1\}^n$  by

$$h(x||v) = E_x(v) \oplus v$$
.

**Question:** Is *h* collision resistant?

Mihir Bellare UCSD 38

## Compression functions

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We seek an adversary that outputs distinct  $x_1 || v_1, x_2 || v_2$  satisfying

$$E_{x_1}(v_1) \oplus v_1 = E_{x_2}(v_2) \oplus v_2$$
.

**Answer:** Unclear how to solve this equation, even though we can pick all four variables.

## Davies-Meyer

Let  $E: \{0,1\}^b \times \{0,1\}^n \to \{0,1\}^n$  be a block cipher. Let us define keyless compression function  $h: \{0,1\}^{b+n} \to \{0,1\}^n$  by

$$h(x||v) = E_x(v) \oplus v$$
.

This is called the Davies-Meyer method and is used in the MD and SHA2 series of hash functions, modulo that the  $\oplus$  may be replaced by addition.

In particular the compression function sha256 of SHA256 is underlain in this way by the block cipher  $\mathsf{E}^\mathsf{sha256}:\{0,1\}^{512}\times\{0,1\}^{256}\to\{0,1\}^{256}$  that we saw earlier, with the  $\oplus$  being replaced by component-wise addition modulo  $2^{32}$ .

## Cryptanalytic attacks

So far we have looked at attacks that do not attempt to exploit the structure of h.

Can we get better attacks if we do exploit the structure?

Ideally not, but hash functions have fallen short!

## Cryptanalytic Attacks

When	Against	Time	Who
1993,1996	md5	$2^{16}$	[dBBo,Do]
2004	MD5	1 hour	[WaFeLaYu]
2005,2006	MD5	1 minute	[LeWadW,KI]
2005	SHA1	2 <sup>69</sup>	[WaYiYu]
2017	SHA1	$2^{63.1}$	[SBKAM]

Collisions found in compression function md5 of MD5 did not yield collisions for MD5, but collisions for MD5 are now easy.

https://shattered.io/.

2017: Google, Microsoft and Mozilla browsers stop accepting SHA1-based certificates.

The SHA256 and SHA512 hash functions are still viewed as secure, meaning the best known attack is the birthday attack.

#### SHA1

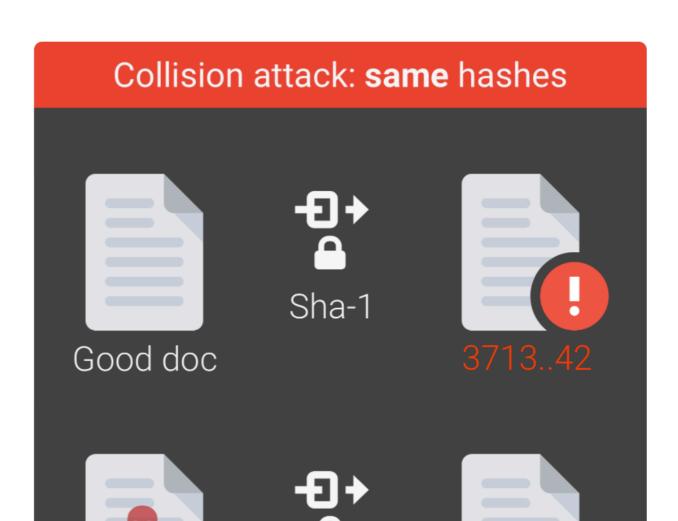


We have broken SHA-1 in practice.

This industry cryptographic hash function standard is used for digital signatures and file integrity verification, and protects a wide spectrum of digital assets, including credit card transactions, electronic documents, open-source software repositories and software updates.

It is now practically possible to craft two colliding PDF files and obtain a SHA-1 digital signature on the first PDF file which can also be abused as a valid signature on the second PDF file.

For example, by crafting the two colliding PDF files

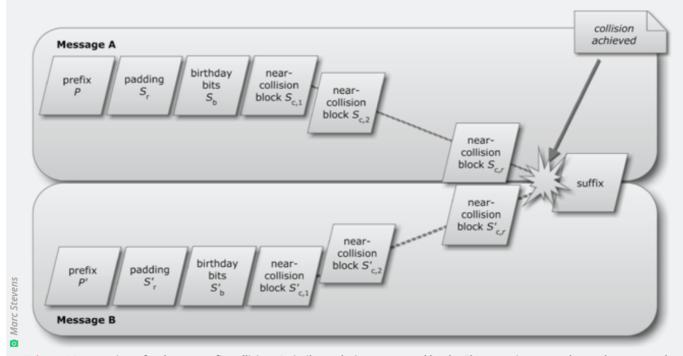


## Flame exploited MD5

#### Crypto breakthrough shows Flame was designed by world-class scientists

The spy malware achieved an attack unlike any cryptographers have seen before.

**DAN GOODIN - 6/7/2012, 11:20 AM** 



Enlarge / An overview of a chosen-prefix collision. A similar technique was used by the Flame espionage malware that targeted Iran. The scientific novelty of the malware underscored the sophistication of malware sponsored by wealthy nation states.



The Flame espionage malware that infected computers in Iran achieved mathematic breakthroughs that could only have been accomplished by world-class cryptographers, two of the world's foremost cryptography experts said.





"We have confirmed that Flame uses a yet unknown MD5 chosenprefix collision attack," Marc Stevens wrote in an e-mail posted to a cryptography discussion group earlier this week. "The collision attack itself is very interesting from a scientific viewpoint, and there are already some practical implications." Benne de Weger, a Stevens colleague and another expert in cryptographic collision

#### **Flame**

Revealed: Stuxnet "beta's" devious alternate attack on Iran nuke program

Massive espionage malware targeting governments undetected for 5 years

Iranian computers targeted by new malicious data wiper program

New in-the-wild malware

## The tightrope

Why don't cryptographers build secure hash functions?

Cryptographers seem perfectly capable of building secure hash functions.

The difficulty is that they strive for VERY HIGH SPEED.

SHA256 can run at 3.5 cycles/byte (eBACS: 2018 Intel Core i3-8121U, https://bench.cr.yp.to/results-hash.html) or 0.6 ns per byte, and hardware will make it even faster.

It is AMAZING that one gets ANY security at such low cost.

If you allow cryptographers a 10x slowdown, they can up rounds by 10x and designs seem almost impossible to break.

### SHA3

National Institute for Standards and Technology (NIST) held a world-wide competition to develop a new hash function standard.

#### Contest webpage:

http://csrc.nist.gov/groups/ST/hash/index.html

#### Requested parameters:

- Design: Family of functions with 224, 256, 384, 512 bit output sizes
- Security: CR, one-wayness, near-collision resistance, others...
- Efficiency: as fast or faster than SHA2-256

### SHA3

**Submissions:** 64

**Round 1:** 51

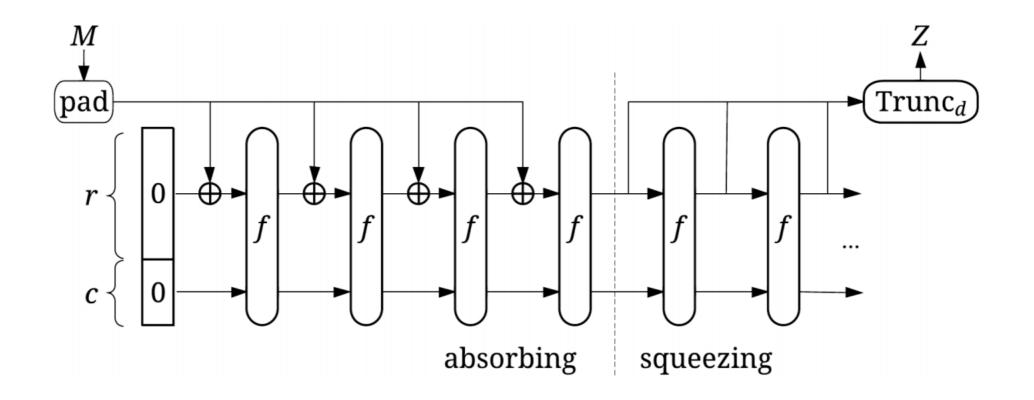
Round 2: 14: BLAKE, Blue Midnight Wish, CubeHash, ECHO, Fugue,

Grostl, Hamsi, JH, Keccak, Luffa, Shabal, SHAvite-3, SIMD, Skein.

Finalists: 5: BLAKE, Grostl, JH, Keccak, Skein.

SHA3: 1: Keccak

## SHA3: Sponge



 $f: \{0,1\}^{r+c} \to \{0,1\}^{r+c}$  is a (public, invertible!) permutation. d is the number of output bits, and c=2d.

SHA3 does not use the MD paradigm used by the MD and SHA2 series.

Shake(M, d)— Extendable-output function, returning any given number d of bits.

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