

Instructions Before attempting this homework, be sure to review the course's Academic Honesty policy on the course syllabus, and be certain to abide by it and the homework policies discussed in class. This homework is due on **Friday, February 8th at 11:59pm**, by which time you should have submitted your solutions as a PDF to **Gradescope**. Since we aren't using real paper here, please use at least **one page per solution** (in this case at least four pages: at least one each for Problem 1, Problem 2A, Problem 2B, and Problem 2C). Please also **label your solution pages correctly on Gradescope!** If you do not, you will be **deducted up to 10 points**, so make sure you leave yourself about five minutes to upload, especially if you are unfamiliar with the process.

Problem 1. (30 points.) In a recent interview with astrophysicist Neil Degrasse Tyson,¹ Edward Snowden suggested a reason why humans have not observed alien communication is that aliens' use of "good" encryption would make such communication look random and therefore indistinguishable from cosmic background radiation. (NB: He is talking about aliens communicating amongst themselves, not attempts to contact humans.) Critique this argument from a cryptographic standpoint.

Problem 2. (70 points.) Let $\mathbb{Z}_{10} = \{0, 1, \dots, 9\}$. Consider the symmetric-key encryption scheme $\text{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ with message-space $(\mathbb{Z}_{10})^4$ (that is, messages are four decimal digits) defined as follows. Key-generation algorithm \mathcal{K} outputs a uniformly random $\pi \in \text{Perm}(\mathbb{Z}_{10})$, where $\text{Perm}(\mathbb{Z}_{10})$ is the set of permutations on \mathbb{Z}_{10} , and encryption algorithm \mathcal{E} is defined by

Algorithm $\mathcal{E}_\pi(M)$:

Parse M as $M[1]M[2]M[3]M[4]$ where each $M[i] \in \mathbb{Z}_{10}$

For $i = 1$ to 4 do:

$P[i] \leftarrow M[i] + i \bmod 10$

$C[i] \leftarrow \pi(P[i])$

Return $C[1]C[2]C[3]C[4]$

(Part A -10 points.) Finish the description of SE . That is, specify a decryption algorithm \mathcal{D} such that $\text{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ is a *correct* symmetric-key encryption scheme with \mathcal{K}, \mathcal{E} as defined above.

(Part B - 30 points.) Is SE a substitution cipher? Why or why not?

(Part C - 30 points.) Is SE a Shannon-secure? Why or why not?

¹See <http://www.startalkradio.net/show/a-conversation-with-edward-snowden-part-1/>.