

# Lecture 5...

12<sup>th</sup> Sept 2017

$$\text{MDP } M = (S, A, P, R, d_0, \gamma)$$

$S$  = state,

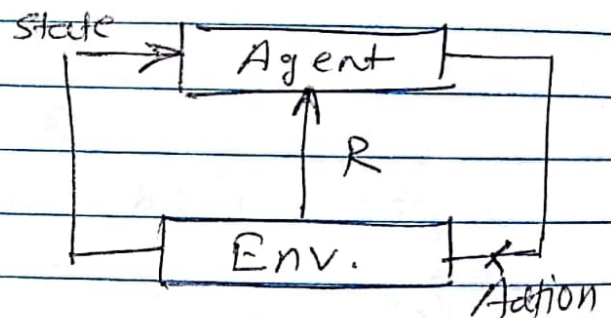
$A$  = action

$P$  = transition  $f^n$

$R$  = reward  $f^n$

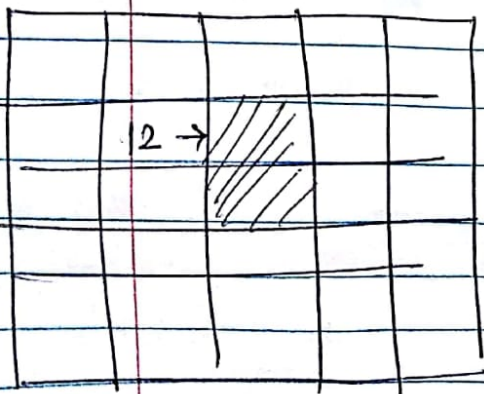
$d_0$  = initial distribut<sup>n</sup>

$\gamma$  = discounting par.



$$\pi(s, a) = P_r(A_t = a \mid S_t = s)$$

$$J(\pi) = \mathbb{E} \left[ \sum_{t=0}^{\infty} R_t \mid \pi \right]$$



$R(12, \text{attempt} - \text{right}, 12)$

$$= \mathbb{E} [R_t \mid S_t = 12, A_t = A-R, S_{t+1} = 12]$$

$$= \frac{-10 \times 0.8}{0.9}$$

Today's topic Environment

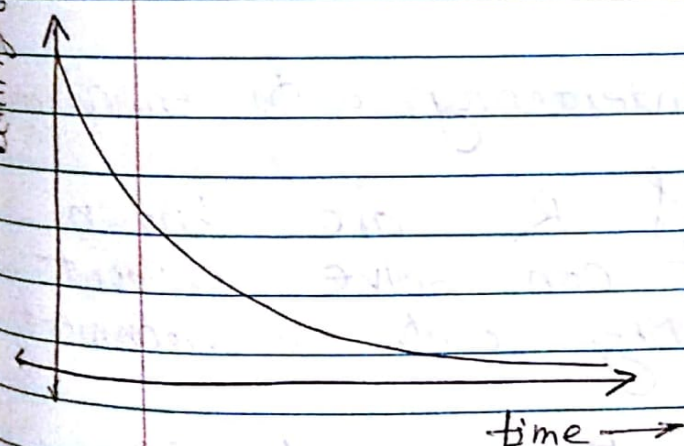
Reward discount ( $\gamma$ )

$$\gamma \in [0, 1]$$

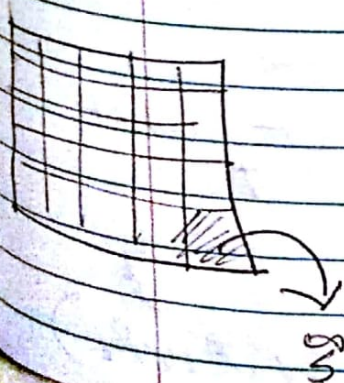
$$J(\pi) \triangleq \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t R_t \mid \pi \right]$$

- Including  $\gamma (< 1)$  ensures that  $J(\pi)$  is finite

$$0 \leq \sum_{t=0}^{\infty} \gamma^t R_t \leq \frac{1}{1-\gamma}$$



Terminal states : it is a state that always transition to a special state, called the 'Terminal absorbing state'



- Reward of transitioning into  $s_3$  is zero
- Once enter  $s_3$ , can't get out

Episode : when agent reaches  $\gamma$ , the current trial called an "episode", ends, and we start new one.

— set  $t = 0$   
— sample  $s_0 \sim d_0$

- Episodic MDP is one where, agent always reaches  $\gamma$

### RL Definition Revisited:

Learn from interacting with environment

- Planning:  $P$  &  $R$  are known, agent can solve without interacting with environment
- RL: Atleast  $P$  is unknown, most methods will not estimate  $P$ .

### Terminology :

- ① A history,  $H_t$ , is history of what has happened upto time  $t$

$$H_t \triangleq (s_0, A_0, R_0, s_1, A_1, R_1, \dots, s_t, A_t, R_t)$$

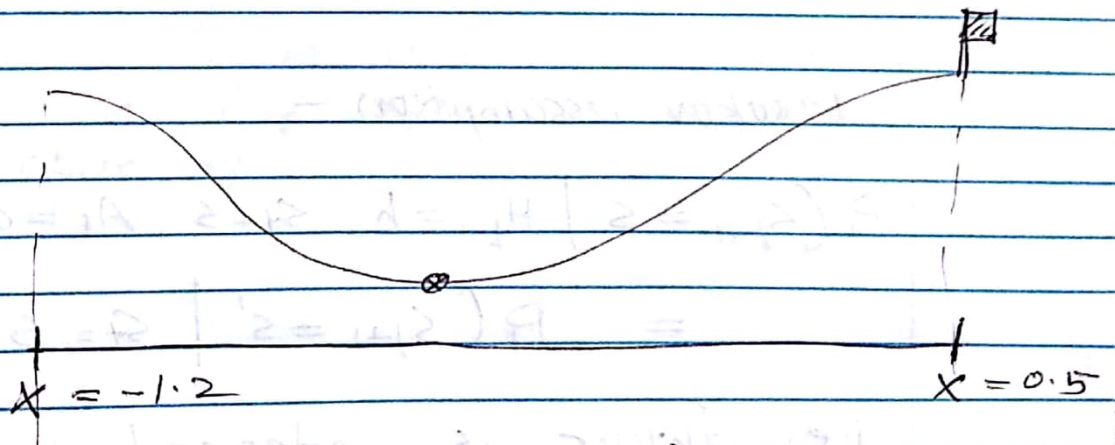
② Trajectory : is the history of an entire episode  $H_n$ .

③ Discounted Reward : of trajectory is the discounted sum of rewards

$$G \triangleq \sum_{t=0}^{\infty} \gamma^t \cdot R_t$$

$$J(\pi) = \mathbb{E}[G | \pi]$$

Mountain Car :



State :  $S = (x, v)$

Action :  $a \in \{ \text{Forward, neutral, reverse} \}$

Dynamics :  $v_{t+1} = v_t + 0.001a - 0.0025 \cos(3x)$

Dynamics:  $X_{t+1} = X_t + V_t$

if  $X_{t+1}$  is outside  $[-1.2, 0.5]$   
it is moved to this range &  
 $V_{t+1} \leftarrow 0$

terminal state:  $X_t = 0.5$

Reward:  $R_t = -1$  everywhere

initial state:

### Markov Problem Property

$$P(h, s, a, s') \triangleq \Pr(s_{t+1} = s' \mid H_{t+1} = h, \\ s_t = s, A_t = a)$$

Markov assumption -

$$\Pr(s_{t+1} = s' \mid H_{t+1} = h, s_t = s, A_t = a) \\ = \Pr(s_{t+1} = s' \mid s_t = s, A_t = a)$$

i.e. future is independent of  
past given the present

$$\Pr(R_t = r \mid H_{t+1}, s_t = s, A_t = a) = \\ \Pr(R_t = r \mid s_t = s, A_t = a, s_{t+1} = s')$$

Code :: Blocks  
Visual Studios

$$P(A_t = a | S_t = s, H_{t-1} = h) =$$

$$P(A_t = a | S_t = s)$$

- Mountain car problem:

$$\begin{array}{ll} \text{if } S = (x, v) & \checkmark \\ S = (x) & \times \end{array}$$

Markovianess is a property of the state representation

- One can always define a Markovian ~~is~~ state representation given a non-Markovian  $\rightarrow$  —.

Let  $S_t$  be a non-Markovian s.rep.  
Then  $(S_t, h_{t-1})$  is a Markovian state.rep.