COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

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University of Massachusetts Amherst. Fall 2021.

Lecture 9

Last Class:

- Finish up distinct elements counting. High-level overview of the HyperLogLog algorithm.
- Introduction of Jaccard similarity and the similarity search problem.

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This Class:

- Locality sensitive hashing and fast similarity search.
- MinHashing for Jaccard similarity search.

0, 1

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\#\text{ shared elements}}{\#\text{ total elements}}.$$

Want Fast Implementations For:

- **Near Neighbor Search:** Have a database of n sets/bit strings and given a set A, want to find if it has high Jaccard similarity to anything in the database. $\Omega(n)$ time with a linear scan.
- **All-pairs Similarity Search:** Have n different sets/bit strings and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Goal: Speed up Jaccard similarity search (near neighbor and all-pairs similarity search).

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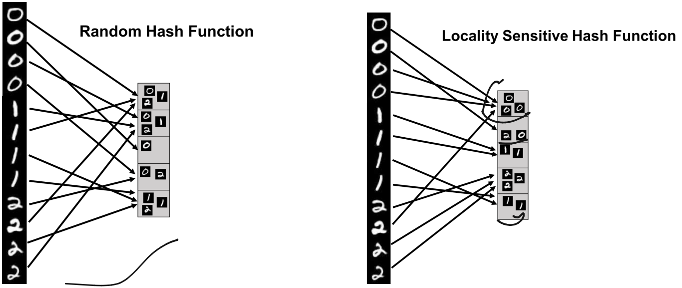
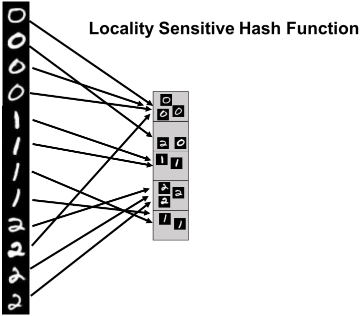


How does locality sensitive hashing (LSH) help with similarity search?



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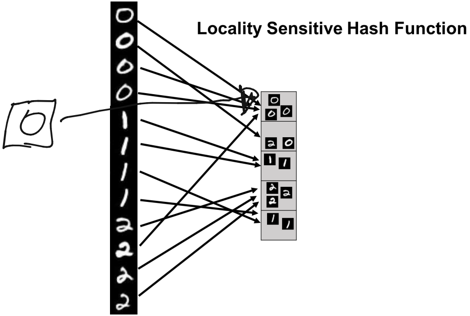

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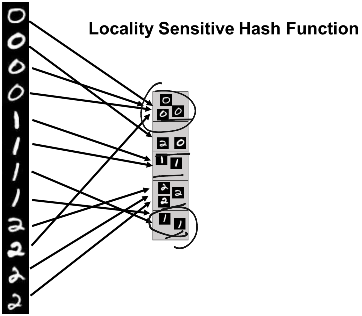

- **Near Neighbor Search:** Given item x, compute $\mathrm{h}(x)$. Only search for similar items in the $\mathrm{h}(x)$ bucket of the hash table.
- **All-pairs Similarity Search:** Scan through all buckets of the hash table and look for similar pairs within each bucket.

An Example: Locality sensitive hashing for Jaccard similarity.

MINHASHING

minHash(A) : sets → numbers

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MinHash(A): [Andrei Broder, 1997 at Altavista]

- Let $\mathbf{h} : U \rightarrow [0, 1]$ be a random hash function
- $\mathbf{s} := 1$
- For $x_1, \ldots, x_{|A|} \in A$
  - $\mathbf{s} := \min(\mathbf{s}, \mathbf{h}(x_k))$
- Return $\mathbf{s}$

the dog walked fast

.79 .37 .43 .64

mH(A) = .37

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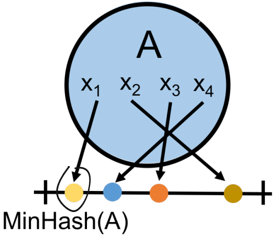

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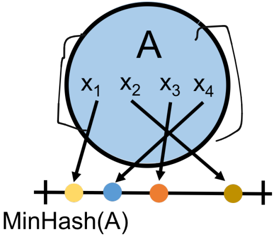

Identical to our distinct elements sketch!

For two sets A and B, what is $\Pr(MinHash(A) = MinHash(B))$?

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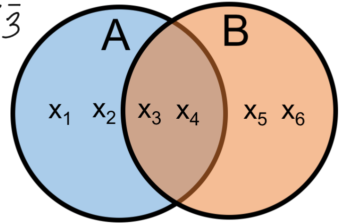
$J(A,B) = \frac{2}{6} = \frac{1}{3}$



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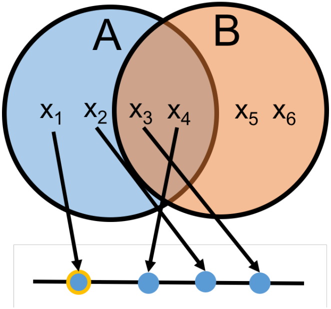

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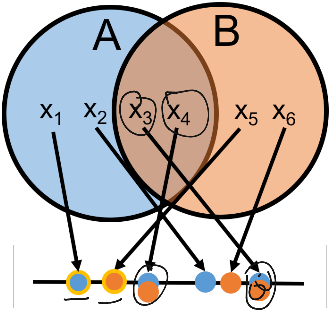

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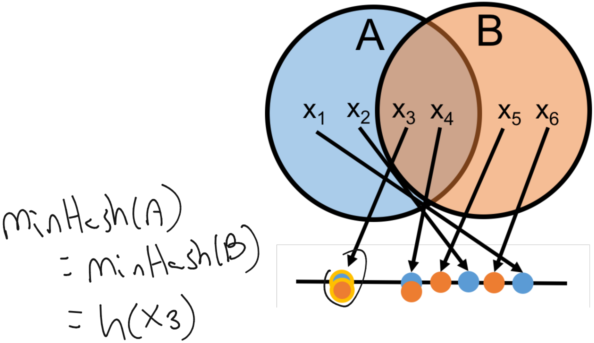

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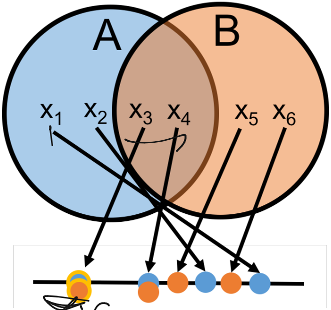

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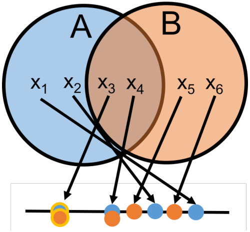

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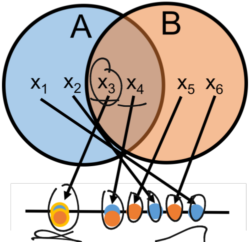

$\Pr(MinHash(A) = MinHash(B)) = ?$

$$\frac{|A \cap B|}{|A \cup B|}$$

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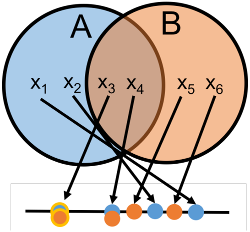

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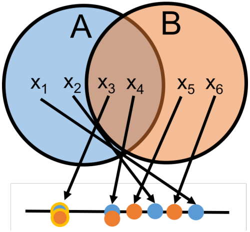

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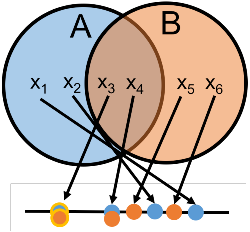

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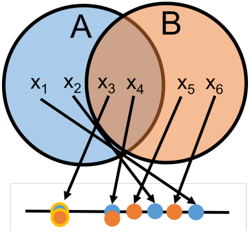

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Locality sensitive: the higher $J(A, B)$ is, the more likely $MinHash(A), MinHash(B)$ are to collide.

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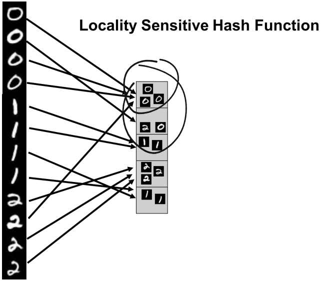

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  "1/2 = Pr(mH(x) = mH(y))

- For every document x in your database with $J(x, y) \geq 1/2$ what is the probability you will find x in bucket $\mathbf{g}(MinHash(y))$?

  $\geq 1/2$

REDUCING FALSE NEGATIVES

With a simple use of MinHash, we miss a match x with $J(x, y) = 1/2$ with probability $1/2$. How can we reduce this false negative rate?

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Potential for a lot of false positives! Slows down search time.

BALANCING HIT RATE AND QUERY TIME

We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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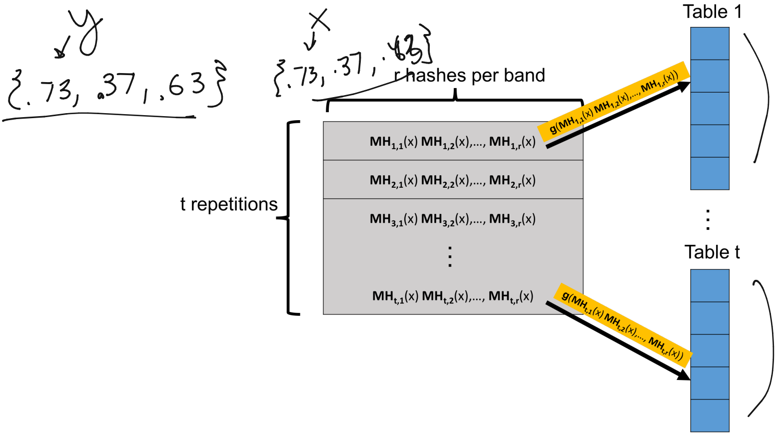

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

Consider searching for matches in t hash tables, using MinHash signatures of length r. For x and y with Jaccard similarity $J(x, y) = s$:

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BALANCING HIT RATE AND QUERY TIME

Consider searching for matches in t hash tables, using MinHash signatures of length r. For x and y with Jaccard similarity $J(x, y) = s$:

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Hit Probability: $1 - (1 - s^r)^t$.

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THE s-CURVE

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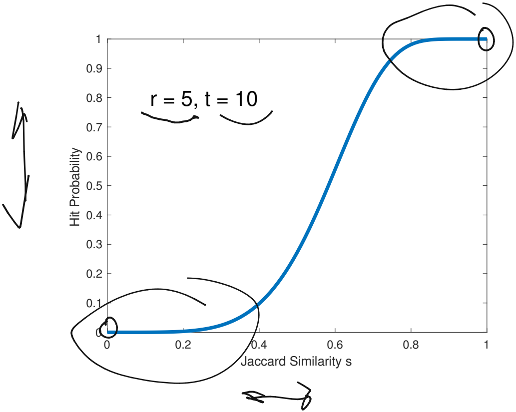

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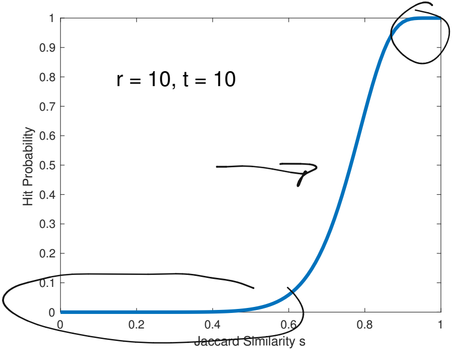

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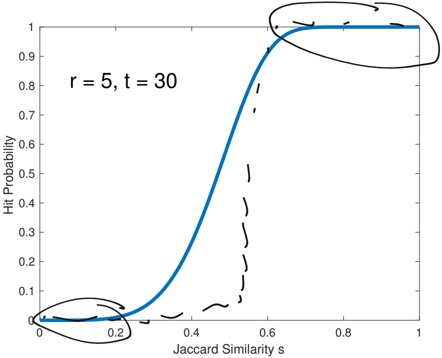

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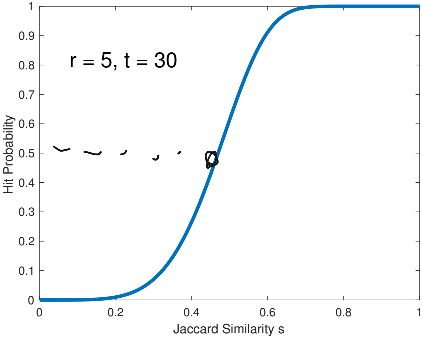

r and t are tuned depending on application. 'Threshold' when hit probability is 1/2 is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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Expected Number of Items Scanned: (proportional to query time)

$$\leq 10 + .98 * 10,000 + .007 * 9,989,990 \approx 80,000$$

s-CURVE EXAMPLE

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HASHING FOR DUPLICATE DETECTION

	Hash Table	Bloom Filters	MinHash Similarity Search	Distinct Elements
Goal	Check if x is a duplicate of any y in database and return y.	Check if x is a duplicate of y in database.	Check if x is a duplicate of any y in database and return y.	Count # of items, excluding duplicates.
Space	$O(n)$ items	$O(n)$ bits	$O(n \cdot t)$ items (when t tables used)	$O\left(\frac{\log \log n}{\epsilon^2}\right)$
Query Time	$O(1)$	$O(1)$	Potentially $o(n)$	NA
Approximate Duplicates?	✗	✗	✓	✗

All different variants of detecting duplicates/finding matches in large datasets. An important problem in many contexts!

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