COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

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University of Massachusetts Amherst. Fall 2021.

Lecture 6

LOGISTICS

I assign m requests to k servers

Server loads are $R_1, R_2 \ldots R_k$

$$\text{Var}(R_1 + R_2 + \ldots R_k) = 0 \neq \text{Var}(R_1) + \ldots \text{Var}(R_k)$$

- Problem Set 1 is due this Friday at 8pm in Gradescope.
- My office hours have moved to Thursday 5-6pm on Zoom.

Last Class:

- Exponential concentration bounds – Bernstein and Chernoff
- Connection to the central limit theorem

This Class:

- Bloom filters: random hashing to maintain a large set in small space.
- Possibly start on distinct items counting

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APPROXIMATELY MAINTAINING A SET

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hash tables

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Solution: Bloom filters (repeated random hashing). Will use much less space than a hash table.

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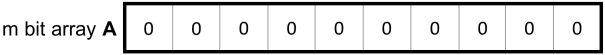

BLOOM FILTERS

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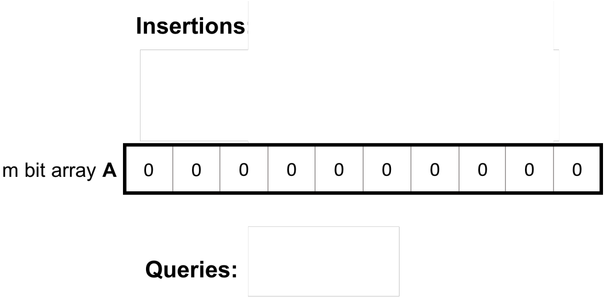

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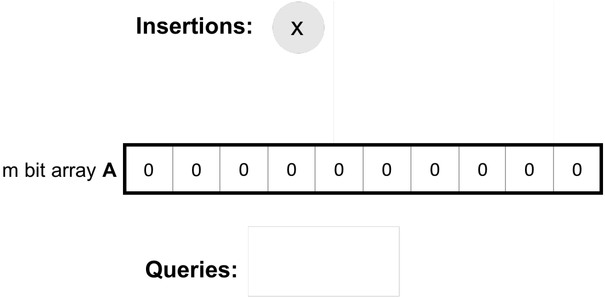

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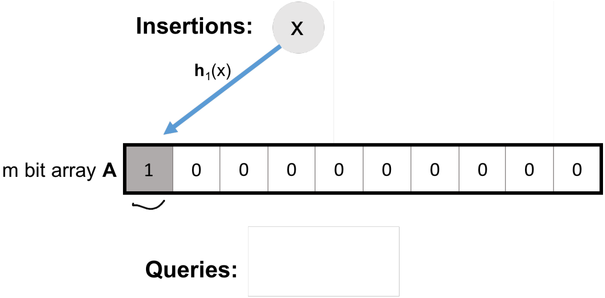

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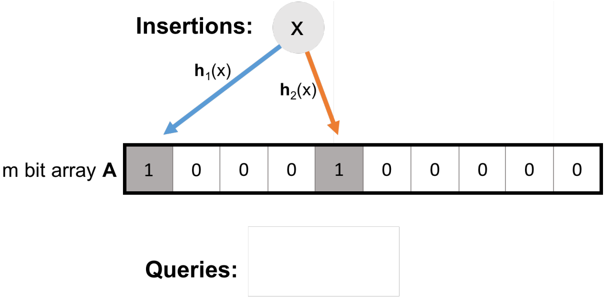

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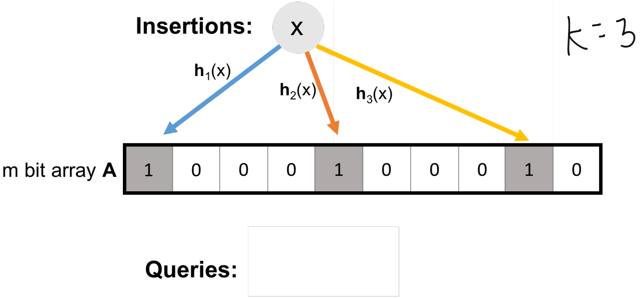

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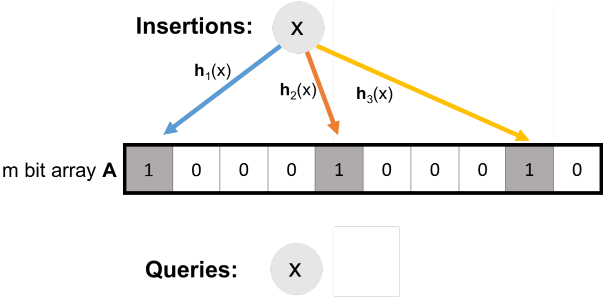

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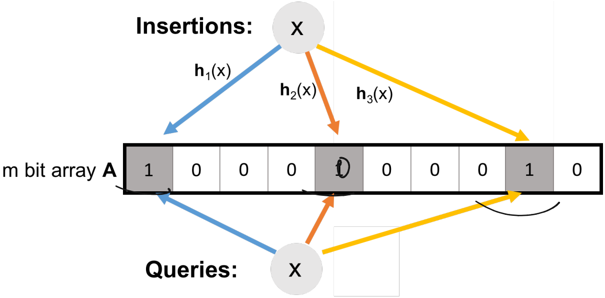

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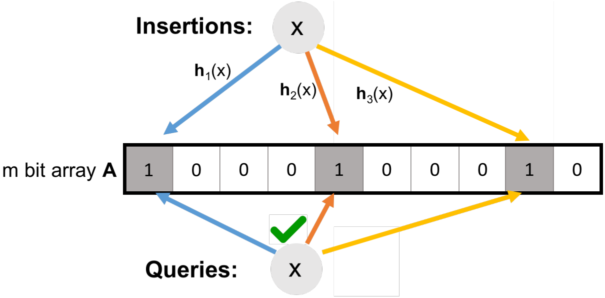

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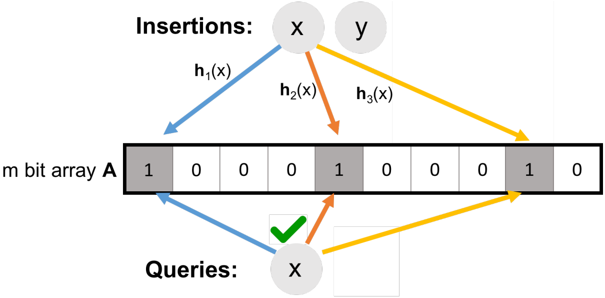

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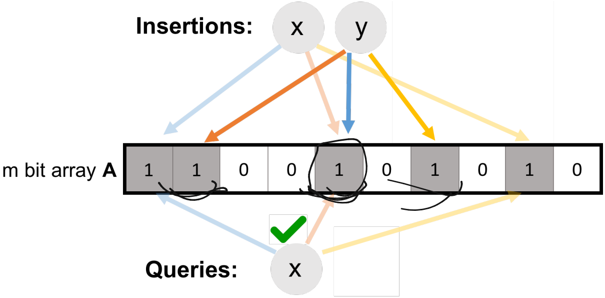

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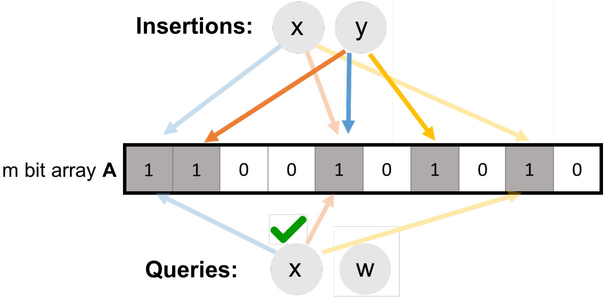

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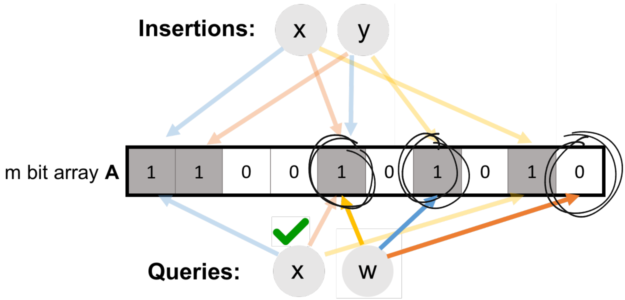

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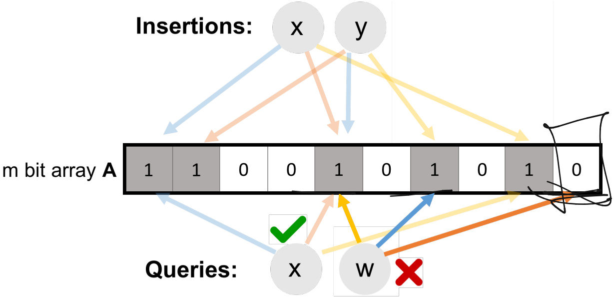

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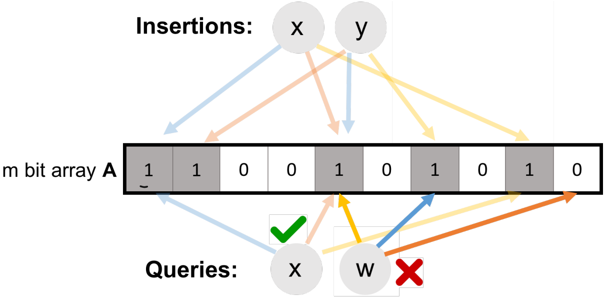
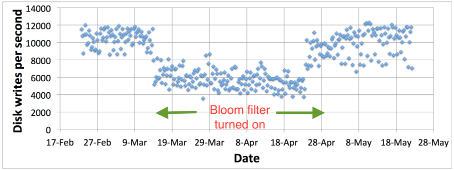


No false negatives. False positives more likely with more insertions.

Akamai (Boston-based company serving 15 – 30% of all web traffic) applies bloom filters to prevent caching of 'one-hit-wonders' – pages only visited once fill over 75% of cache.



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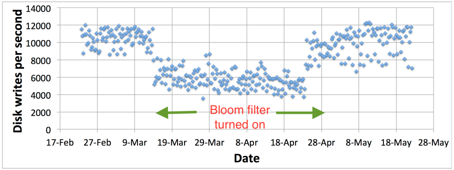

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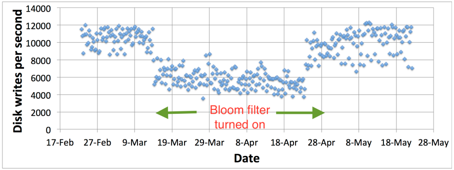

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- **False positive:** A new url (possible one-hit-wonder) is cached. If the bloom filter has a false positive rate of $\delta = .05$, the number of cached one-hit-wonders will be reduced by at least 95%.

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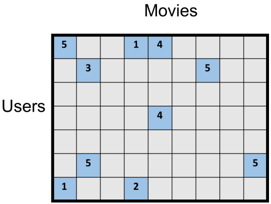

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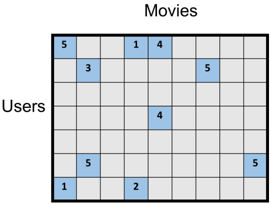

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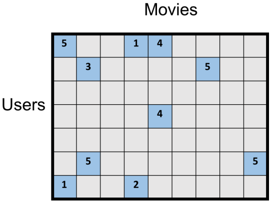

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- **False positive:** A read is made to a possibly empty cell. A $\delta = .05$ false positive rate gives a 95% reduction in these empty reads.

- Database Joins: Quickly eliminate most keys in one column that don't correspond to keys in another.
- Recommendation systems: Bloom filters are used to prevent showing users the same recommendations twice.
- Spam/Fraud Detection:
  - Bit.ly and Google Chrome use bloom filters to quickly check if a url maps to a flagged site and prevent a user from following it.
  - Can be used to detect repeat clicks on the same ad from a single IP-address, which may be the result of fraud.
- Digital Currency: Some Bitcoin clients use bloom filters to quickly pare down the full transaction log to transactions involving bitcoin addresses that are relevant to them (SPV: simplified payment verification).

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k·n

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inclusion exclusion

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k=1
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query(w)

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$$= \left(1-e^{-\frac{kn}{m}}\right)^k$$

Actually Incorrect!

n: total number items in filter, m: number of bits in filter, k: number of random hash functions, $\mathbf{h}_1, \ldots \mathbf{h}_k$: hash functions, A: bit array, δ: false positive rate.

How does the false positive rate δ depend on m, k, and the number of items inserted?

Step 1: What is the probability that after inserting n elements, the i^{th} bit of the array A is still 0?

$$\Pr(A[i] = 0) = \left(1 - \frac{1}{m}\right)^{kn} \approx e^{-\frac{kn}{m}}$$

Step 2: What is the probability that querying a new item w gives a false positive?

$$\begin{aligned}\Pr\left(A[\mathbf{h}_1(w)] = \ldots = A[\mathbf{h}_k(w)] = 1\right) \\ &= \Pr(A[\mathbf{h}_1(w)] = 1) \times \ldots \times \Pr(A[\mathbf{h}_k(w)] = 1) \\ &= \left(1 - e^{-\frac{kn}{m}}\right)^k\end{aligned}$$

Actually Incorrect! Dependent events.

n: total number items in filter, m: number of bits in filter, k: number of random hash functions, $\mathbf{h}_1, \ldots \mathbf{h}_k$: hash functions, A: bit array, δ: false positive rate.

Step 1: To avoid dependence issues, condition on the event that the A has t zeros in it after n insertions, for some $t \leq m$. For a non-inserted element w, after conditioning on this event we correctly have:

$$\Pr(A[\mathbf{h}_1(w)] = \ldots = A[\mathbf{h}_k(w)] = 1)$$
$$= \Pr(A[\mathbf{h}_1(w)] = 1) \times \ldots \times \Pr(A[\mathbf{h}_k(w)] = 1).$$

I.e., the events $A[\mathbf{h}_1(w)] = 1, \ldots, A[\mathbf{h}_k(w)] = 1$ are independent conditioned on the number of bits set in A. Why?

- Conditioned on this event, for any j, since $\mathbf{h}_j$ is a fully random hash function, $\Pr(A[\mathbf{h}_j(w)] = 1) = \frac{t}{m}$.
- Thus conditioned on this event, the false positive rate is $\left(1 - \frac{t}{m}\right)^k$.
- It remains to show that $\frac{t}{m} \approx e^{-\frac{kn}{m}}$ with high probability. We already have that $\mathbb{E}[\frac{t}{m}] = \frac{1}{m}\sum_{i=1}^{m} \Pr(A[i] = 0) \approx e^{-\frac{kn}{m}}$.

Need to show that the number of zeros t in A after n insertions is bounded by $O\left(e^{-\frac{kn}{m}}\right)$ with high probability.

Can apply Theorem 2 of: `http://cglab.ca/~morin/publications/ds/bloom-submitted.pdf`

False Positive Rate: with m bits of storage, k hash functions, and n items inserted $\delta \approx \left(1 - e^{\frac{-kn}{m}}\right)^k$.

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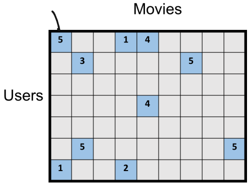

- We have 100 million users and $10,000$ movies. On average each user has rated only 10 movies so of these 10^{12} possible (user,movie) pairs, only $10 * 100,000,000 = 10^9 = n$ (user,movie) pairs have non-empty entries in our table.
- We allocate $m = 8n = 8 \times 10^9$ bits for a Bloom filter (1 GB).

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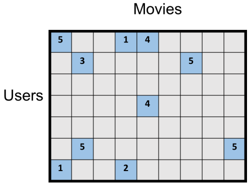

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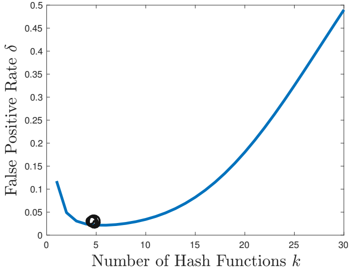

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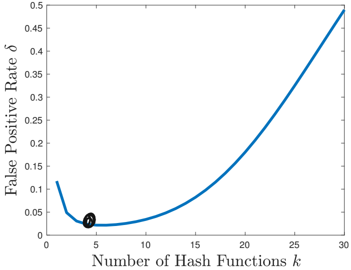

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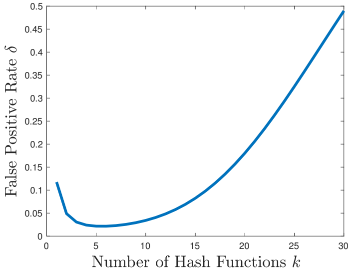

- Can differentiate to show optimal number of hashes is $k = \ln 2 \cdot \frac{m}{n}$.
- Balances between filling up the array with too many hashes and having enough hashes so that even when the array is pretty full, a new item is unlikely to have all its bits set (yield a false positive)

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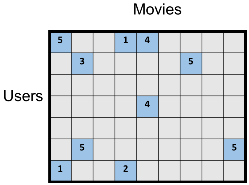

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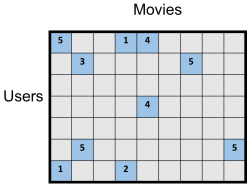

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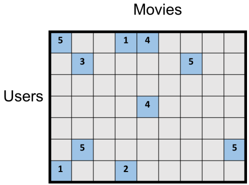

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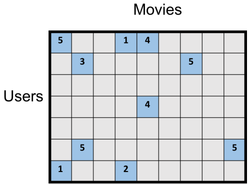

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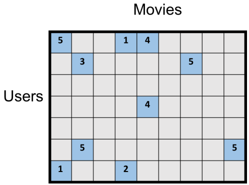

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An observation about Bloom filter space complexity:

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I.e., storing n items in a bloom filter requires $O(n)$ space. So what's the point? Truly $O(n)$ bits, rather than $O(n \cdot \text{item size})$.

Questions on Bloom Filters?