COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

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University of Massachusetts Amherst. Fall 2021.

Lecture 10

LOGISTICS

- Problem Set 2 is due next Friday at 11:59pm.
- The midterm is will in class on Tuesday 10/19. Midterm study material has been posted in the Schedule Tab and in Moodle.

Suppose a patient's temperature is 98 degrees. Every time their temperature is taken, the reading is <97 degrees with probability 1/3 and is >99 degrees with probability 1/3. Suppose their temperature is taken three times. What's the probability the median of these three readings is ≥97 and ≤99 degrees? Give your answer to two decimal places.

Question **1**

Correct

2.00 points out of 2.00

Flag question

Edit question

[ABC | |]

[AB | C |]

1/3, 1/3, 1/3

97, 98, 99

A, B, C

(1) $= \frac{3!}{27} = \frac{6}{27}$

(2) $= 1/3^3 = \frac{1}{27}$

(3) $= \frac{\binom{3}{1}}{27} = \frac{3}{27}$

(4) $= 3/27$

$$\frac{6+1+3+3}{27} = \frac{13}{27} = .48$$

Last Class:

- Locality sensitive hashing for near neighbor search.
- MinHash as a locality sensitive hash function for Jaccard similarity
- Balancing false positives and negatives with LSH signatures and repeated hash tables.

SUMMARY

Last Class:

- Locality sensitive hashing for near neighbor search.
- MinHash as a locality sensitive hash function for Jaccard similarity
- Balancing false positives and negatives with LSH signatures and repeated hash tables.

This Class:

- Finish up LSH: SimHash for cosine similarity.
- Frequent Items Estimation
- Count-min sketch algorithm

Next Few Classes:

- Random compression methods for high dimensional vectors. The Johnson-Lindenstrauss lemma.
- Connections to the weird geometry of high-dimensional space.

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- PCA, low-rank approximation, and the singular value decomposition.
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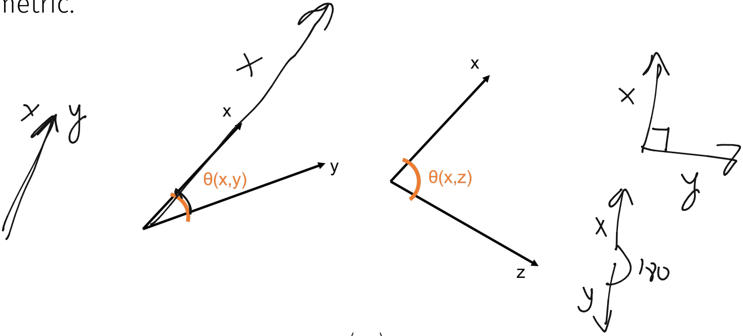
Will use a lot of linear algebra. May be helpful to refresh.

- Vector dot product, addition, Euclidean norm. Matrix vector multiplication.
- Linear independence, column span, orthogonal bases, rank.
- Orthogonal projection, eigendecomposition, linear systems.

SIMHASH FOR COSINE SIMILARITY

Repetition and s-curve tuning can be used for fast similarity search with any similarity metric, given a locality sensitive hash function for that metric.



Cosine Similarity: $\cos(\theta(x,y)) = \frac{\langle x,y\rangle}{\|x\|_2 \cdot \|y\|_2}$.

- $\cos(\theta(x,y)) = 1$ when $\theta(x,y) = 0°$ and $\cos(\theta(x,y)) = 0$ when $\theta(x,y) = 90°$, and $\cos(\theta(x,y)) = -1$ when $\theta(x,y) = 180°$.

SimHash: LSH for cosine similarity.

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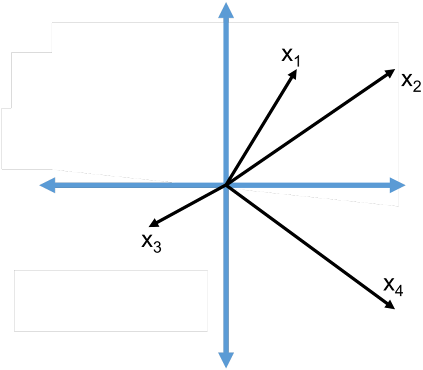
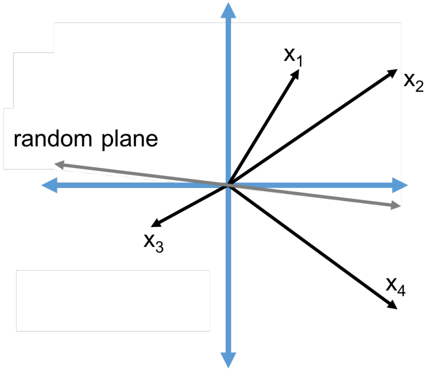


SimHash: LSH for cosine similarity.

$h(x_i) \to$ scalar



SIMHASH FOR COSINE SIMILARITY

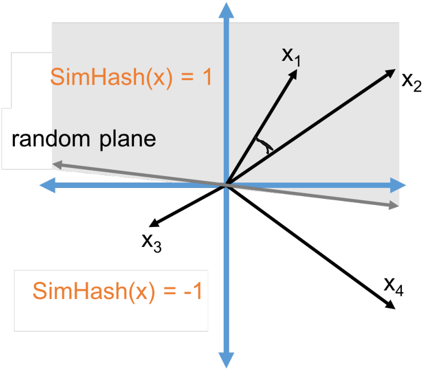
SimHash: LSH for cosine similarity.

$h(x_1) = 1$

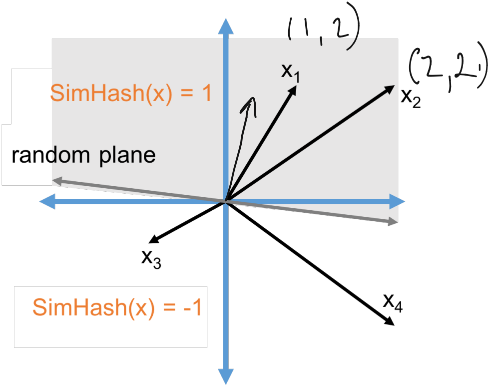
$h(x_2) = 1$

$h(x_3) = -1$

$h(x_4) = -1$



SimHash: LSH for cosine similarity.



$SimHash(x) = \text{sign}(\langle x, t\rangle)$ for a random vector t.

What is $\Pr[SimHash(x) = SimHash(y)]$?

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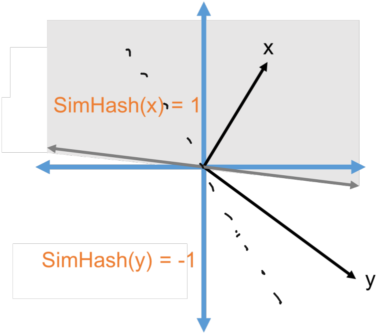
$SimHash(x) \neq SimHash(y)$ when the plane separates x from y.



What is $\Pr[SimHash(x) = SimHash(y)]$?

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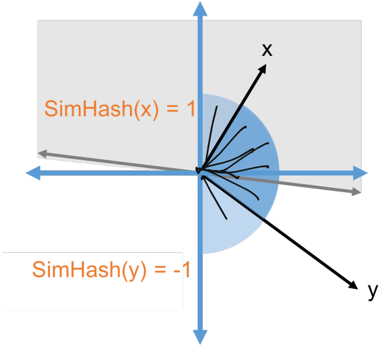
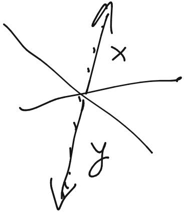
cos(θ(x,y)) = -1
P(SH(x) = SH(y))
= 0





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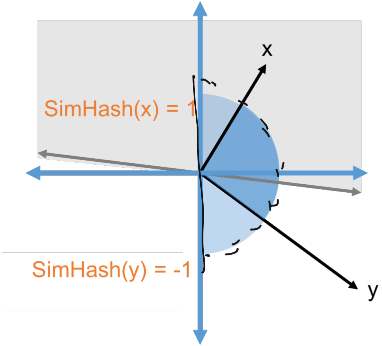

- $\Pr[SimHash(x) \neq SimHash(y)] = \frac{\theta(x,y)}{\pi}$

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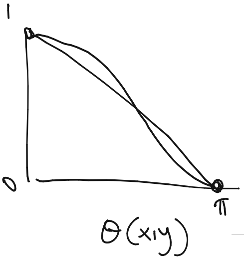

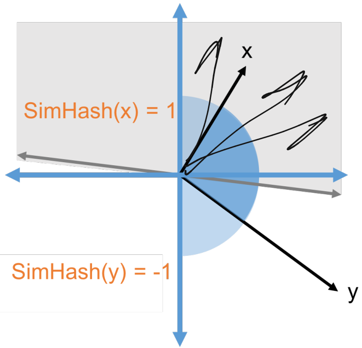

- $\Pr[SimHash(x) \neq SimHash(y)] = \frac{\theta(x,y)}{\pi}$
- $\Pr[SimHash(x) = SimHash(y)] = 1 - \frac{\theta(x,y)}{\pi} \approx \frac{\cos(\theta(x,y))+1}{2}$

k-Frequent Items (Heavy-Hitters) Problem: Consider a stream of n items $x_1, \ldots, x_n$ (with possible duplicates). Return any item at appears at least $\frac{n}{k}$ times.

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$k = 3 \quad \frac{n}{k} = \frac{9}{3} = 3$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
5	12	3	3	4	5	5	10	3

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k=n

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- What is the maximum number of items that can be returned?

a) n b) k c) n/k d) $\log n$

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- What is the maximum number of items that can be returned?
  a) n b) k c) n/k d) $\log n$
- Trivial with $O(n)$ space – store the count for each item and return ~~the~~ any one that appears $\geq n/k$ times.
- Can we do it with less space? I.e., without storing all n items?

Applications of Frequent Items:

- Finding top/viral items (i.e., products on Amazon, videos watched on Youtube, Google searches, etc.)
- Finding very frequent IP addresses sending requests (to detect DoS attacks/network anomalies).
- ‘Iceberg queries’ for all items in a database with frequency above some threshold.

Generally want very fast detection, without having to scan through database/logs. I.e., want to maintain a running list of frequent items that appear in a stream.

Association rule learning: A very common task in data mining is to identify common associations between different events.

- Identified via frequent itemset counting. Find all sets of t items that appear many times in the same basket.
- Frequency of an itemset is known as its support.
- A single basket includes many different itemsets, and with many different baskets an efficient approach is critical. E.g., baskets are Twitter users and itemsets are subsets of who they follow.

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MMDS

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FREQUENT ITEMSET MINING

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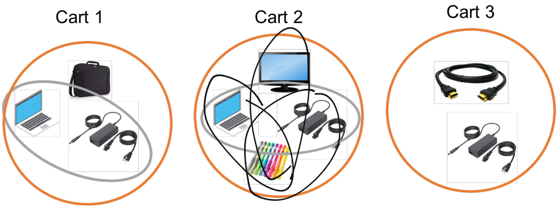

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Issue: No algorithm using $o(n)$ space can output just the items with frequency $\geq n/k$. Hard to tell between an item with frequency n/k (should be output) and $n/k - 1$ (should not be output).

x_1	x_2	x_3	x_4	x_5	x_6	...	$x_{n-n/k+1}$	...	x_n
3	12	9	27	4	101		3		3

n/k-1 occurrences

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n/k-1 occurrences

(ϵ, k)-Frequent Items Problem: Consider a stream of n items $x_1, \ldots, x_n$. Return a set F of items, including all items that appear at least $\frac{n}{k}$ times and only items that appear at least $(1-\epsilon) \cdot \frac{n}{k}$ times.

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- An example of relaxing to a 'promise problem': for items with frequencies in $[(1-\epsilon) \cdot \frac{n}{k}, \frac{n}{k}]$ no output guarantee.

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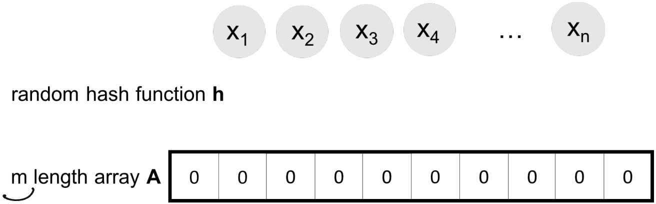

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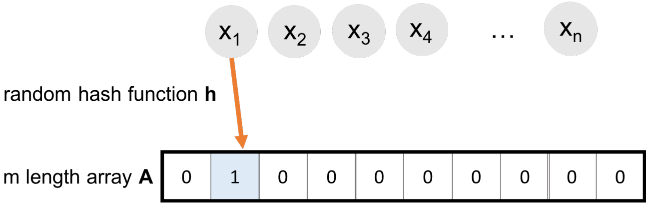

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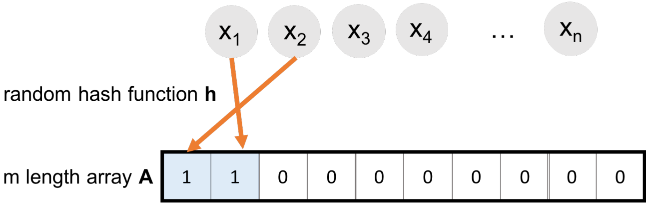

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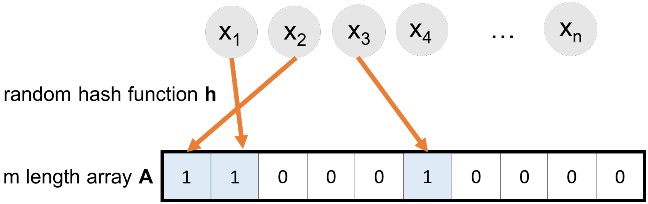

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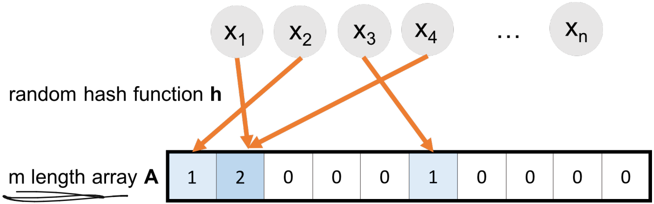

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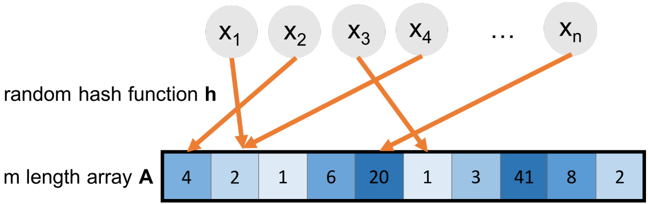

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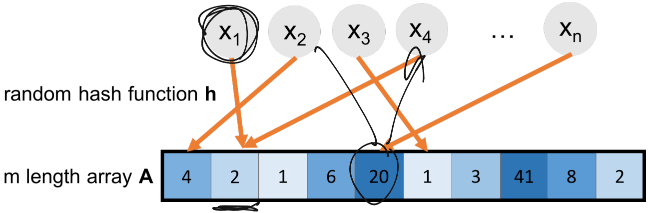
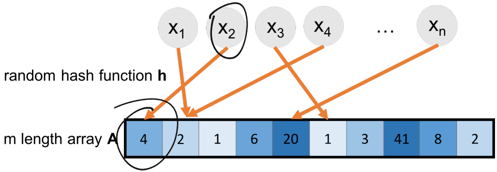


Will use $A[\mathbf{h}(x)]$ to estimate $f(x)$, the frequency of x in the stream. I.e., $|\{x_i : x_i = x\}|$.

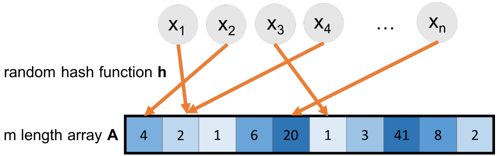


Use $A[\mathbf{h}(x)]$ to estimate $f(x)$.

Claim 1: We always have $A[\mathbf{h}(x)] \geq f(x)$. Why?

$f(x)$: frequency of x in the stream (i.e., number of items equal to x). $\mathbf{h}$: random hash function. m: size of Count-min sketch array.

COUNT-MIN SKETCH ACCURACY



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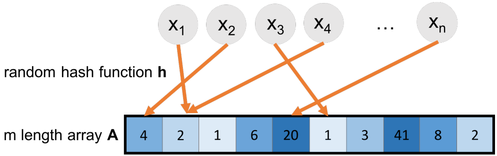

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- $A[\mathbf{h}(x)]$ counts the number of occurrences of any y with $\mathbf{h}(y) = \mathbf{h}(x)$, including x itself.
- $A[\mathbf{h}(x)] = f(x) + \sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)$.

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COUNT-MIN SKETCH ACCURACY

$$A[\mathbf{h}(x)] = f(x) + \underbrace{\sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)}_{\text{error in frequency estimate}} .$$

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Expected Error:

$$\mathbb{E}\left[\sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)\right] =$$

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Expected Error:

$$\mathbb{E}\left[\sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)\right] = \sum_{y \neq x} \Pr(\mathbf{h}(y) = \mathbf{h}(x)) \cdot f(y) = \sum_{y \neq x} \frac{1}{m} \cdot f(y)$$

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$$= \sum_{y \neq x} \frac{1}{m} \cdot f(y) = \frac{1}{m} \cdot (n - f(x)) \leq \frac{n}{m}$$

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What is a bound on probability that the error is $\geq \frac{2n}{m}$?

$$\frac{\mathbb{E}(\text{error})}{2n/m} = \frac{n/m}{2n/m} = \frac{1}{2}$$

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Markov's inequality: $\Pr\left[\sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y) \geq \frac{2n}{m}\right] \leq \frac{1}{2}$.

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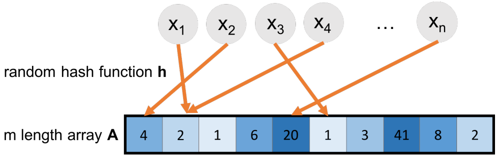
What property of **h** is required to show this bound? a) fully random b) pairwise independent c) 2-universal d) locality sensitive

$f(x)$: frequency of x in the stream (i.e., number of items equal to x). **h**: random hash function. m: size of Count-min sketch array.



Claim: For any x, with probability at least 1/2,

$$f(x) \leq A[\mathbf{h}(x)] \leq f(x) + \frac{2n}{m}.$$

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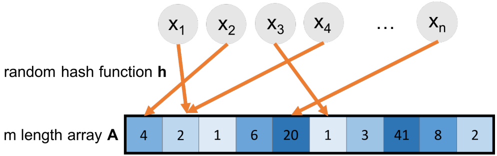
COUNT-MIN SKETCH ACCURACY

$O\left(\frac{k}{\varepsilon}\right)$

$\frac{n}{k}$ $\quad (1-\varepsilon)\frac{n}{k}$



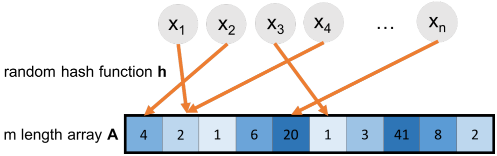
Claim: For any x, with probability at least 1/2,

$$f(x) \le A[\mathbf{h}(x)] \le f(x) + \frac{2n}{m}.$$

$$\frac{2n}{m} = \frac{2n}{\frac{2k}{\varepsilon}} = \boxed{\frac{\varepsilon n}{k}}$$

To solve the (ϵ, k)-Frequent elements problem, set $m = \frac{2k}{\epsilon}$.

$f(x)$: frequency of x in the stream (i.e., number of items equal to x). $\mathbf{h}$: random hash function. m: size of Count-min sketch array.



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How can we improve the success probability?

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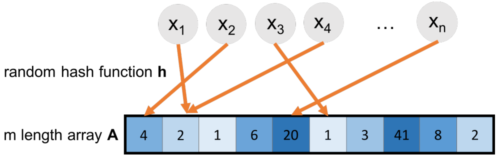
COUNT-MIN SKETCH ACCURACY



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How can we improve the success probability? Repetition.

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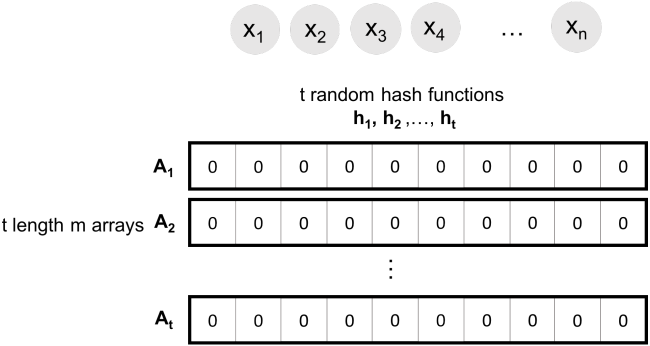
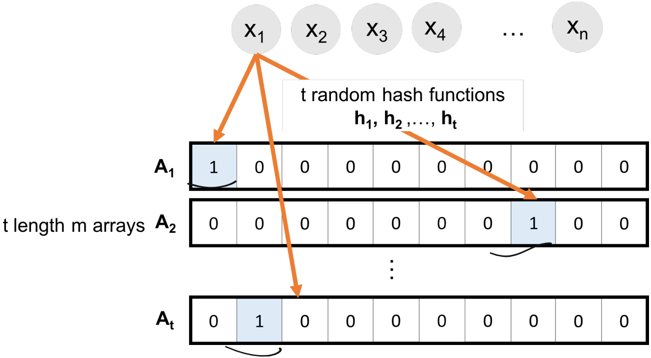
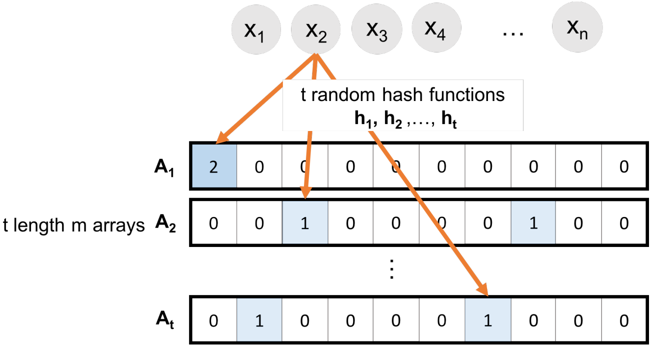
COUNT-MIN SKETCH ACCURACY





x_1
x_2
x_3
x_4
…
x_n
t random hash functions
$h_1, h_2, \ldots, h_t$
A_1
1 0 0 0 0 0 0 0 0 0
t length m arrays
A_2
0 0 0 0 0 0 0 1 0 0
A_t
0 1 0 0 0 0 0 0 0 0


x1
x2
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t random hash functions
h1, h2 ,…, ht
A1
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A2
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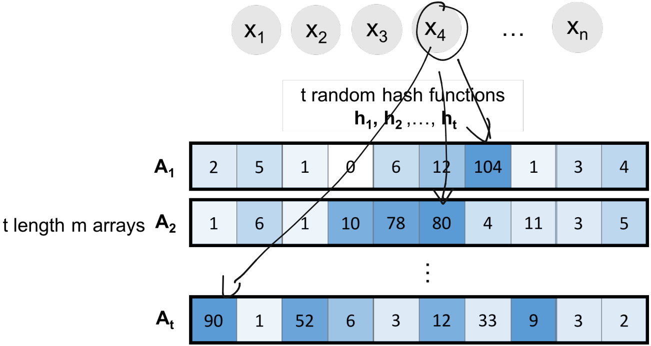
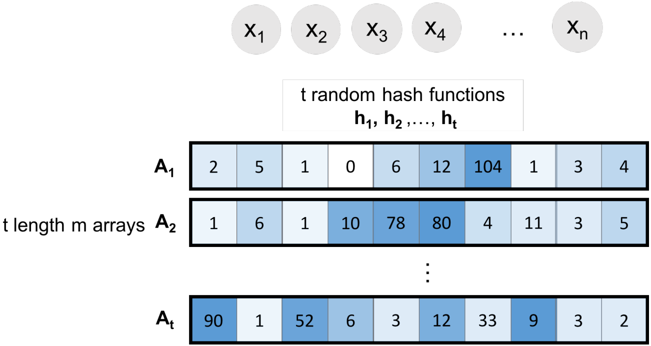
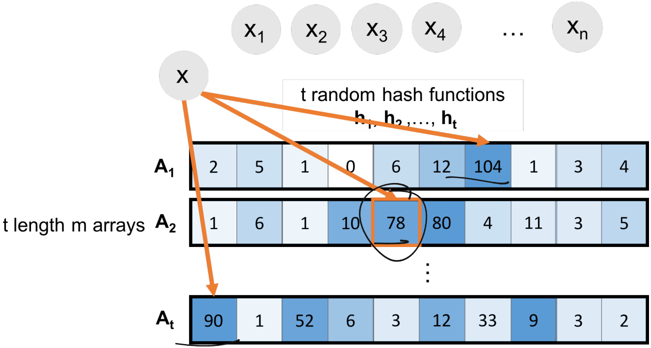

x1
x2
x3
x4
...
xn
t random hash functions
h1, h2 ,..., ht
A1
2 5 1 0 6 12 104 1 3 4
t length m arrays
A2
1 6 1 10 78 80 4 11 3 5
At
90 1 52 6 3 12 33 9 3 2

COUNT-MIN SKETCH ACCURACY



Estimate $f(x)$ with $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$. (count-min sketch)



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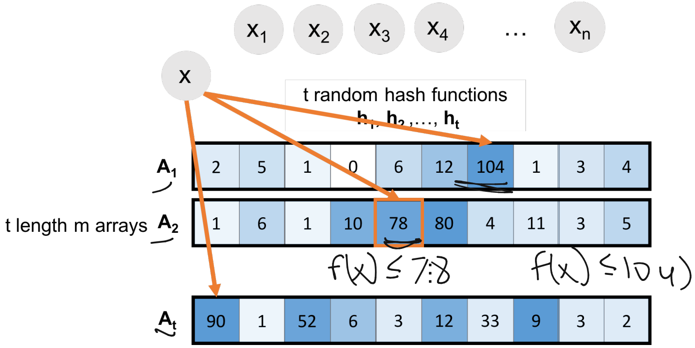
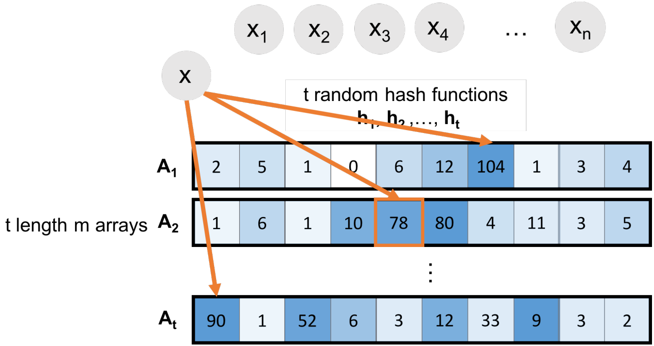
$\hat{f}(x) = 78$



Estimate $f(x)$ with $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$. (count-min sketch)

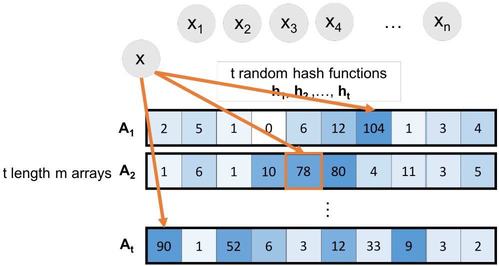
Why min instead of mean or median?

$m = \frac{2k}{\varepsilon}$

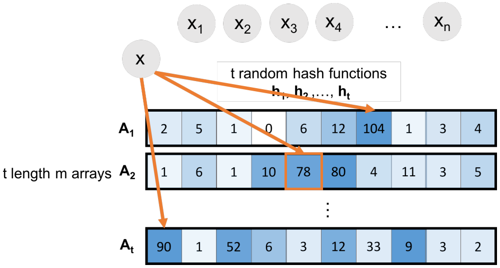


Estimate $f(x)$ with $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$. (count-min sketch)

Why min instead of mean or median? The minimum estimate is always the most accurate since they are all overestimates of the true frequency!



Estimate $f(x)$ by $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$



Estimate $f(x)$ by $\tilde{f}(x) = \min_{i\in[t]} A_i[\mathbf{h}_i(x)]$

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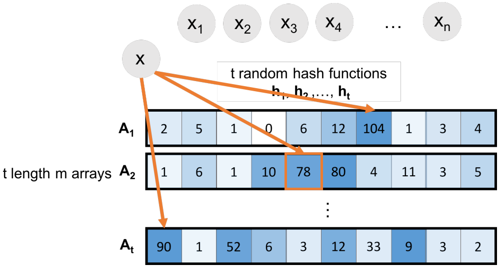

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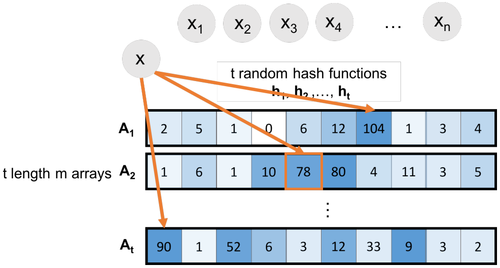

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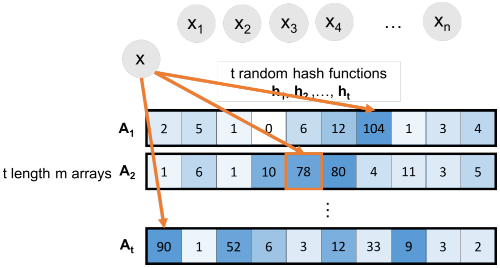

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$1 - \frac{1}{2^{\log(1/\delta)}} = 1 - \delta$

- What is $\Pr[f(x) \leq \tilde{f}(x) \leq f(x) + \frac{\epsilon n}{k}]$? $1 - 1/2^t$.
- To get a good estimate with probability $\geq 1 - \delta$, set $t = \log(1/\delta)$.

Upshot: Count-min sketch lets us estimate the frequency of every item in a stream up to error $\frac{\epsilon n}{k}$ with probability $\geq 1 - \delta$ in $O\left(\log(1/\delta) \cdot k/\epsilon\right)$ space.

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$O(t \cdot m)$

- Accurate enough to solve the (ϵ, k)-Frequent elements problem – distinquish between items with frequency $\frac{n}{k}$ and those with frequency $(1 - \epsilon)\frac{n}{k}$.

COUNT-MIN SKETCH

Upshot: Count-min sketch lets us estimate the frequency of every item in a stream up to error $\frac{\epsilon n}{k}$ with probability $\geq 1 - \delta$ in $O(\log(1/\delta) \cdot k/\epsilon)$ space.

- Accurate enough to solve the (ϵ, k)-Frequent elements problem – distinquish between items with frequency $\frac{n}{k}$ and those with frequency $(1 - \epsilon)\frac{n}{k}$.
- How should we set δ if we want a good estimate for all items at once, with 99% probability?

$\frac{.01}{n} \cdot n$ $\delta = \frac{.01}{n}$

$O(t \cdot m) = O(\log(n) \cdot k/\epsilon)$

Count-min sketch gives an accurate frequency estimate for every item in the stream. But how do we identify the frequent items without having to store/look up the estimated frequency for all elements in the stream?

Count-min sketch gives an accurate frequency estimate for every item in the stream. But how do we identify the frequent items without having to store/look up the estimated frequency for all elements in the stream?

One approach:

- When a new item comes in at step i, check if its estimated frequency is $\geq i/k$ and store it if so.
- At step i remove any stored items whose estimated frequency drops below i/k.
- Store at most $O(k)$ items at once and have all items with frequency $\geq n/k$ stored at the end of the stream.

Questions on Frequent Elements?