COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

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University of Massachusetts Amherst. Fall 2020.
Lecture 8

- We uploaded Problem Set 2 last night. It will be due Monday 9/28 at 8pm.
- Start early. Give yourself time to mull over the problems.
- Some reminders from your friendly 514 grading staff:
  - You need to **mark your group-mates as part of the submission in Gradescope**. Just having their name on written on the front page is not enough.
  - Tag the location of each individual subquestion, not just the first page of the full question.
  - If you write in pencil please be sure to write darkly. It can be very hard to read once scanned.
- Quiz 4 is due Monday at 8pm.

Last Class:

- Boosting the success probability of distinct elements estimation with the median trick.
- Sketched the idea of practical distinct elements algorithms: LogLog and HyperLogLog.
- Started on fast similarity search. MinHashing to estimate the Jaccard similarity between two sets.

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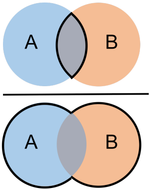
This Class:

- MinHash and locality sensitive hashing (LSH).
- Application of LSH to fast similarity search.

Jaccard Similarity: $J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\text{\# shared elements}}{\text{\# total elements}}$.



Two Common Use Cases:

- **Near Neighbor Search:** Have a database of n sets/bit strings and given a set A, want to find if it has high similarity to anything in the database. Naively $\Omega(n)$ time.
- **All-pairs Similarity Search:** Have n different sets/bit strings. Want to find all pairs with high similarity. Naively $\Omega(n^2)$ time.

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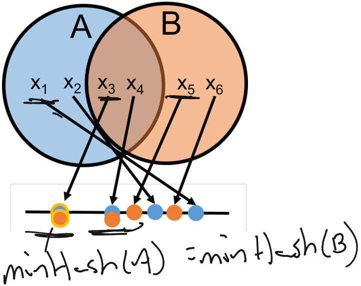

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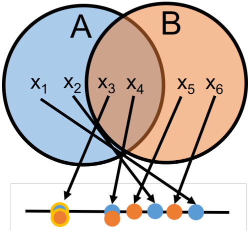

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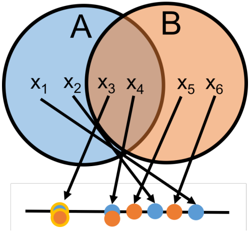

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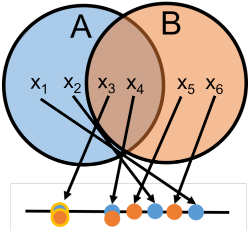

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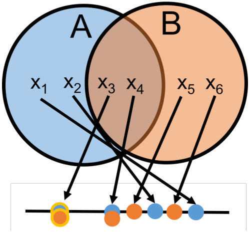

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collision probability

$$= \frac{|A \cap B|}{|A \cup B|} = J(A, B).$$

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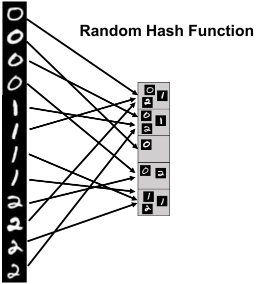

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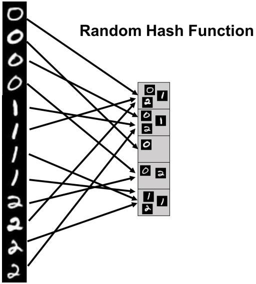
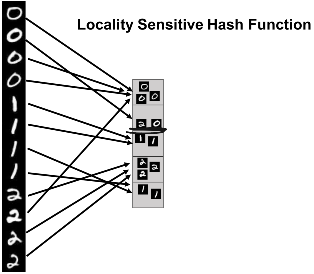
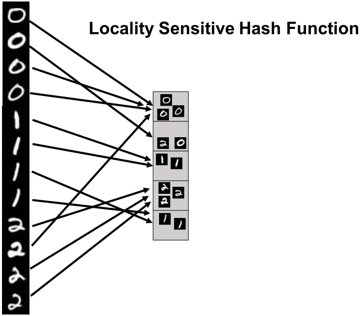




How does locality sensitive hashing (LSH) help with similarity search?



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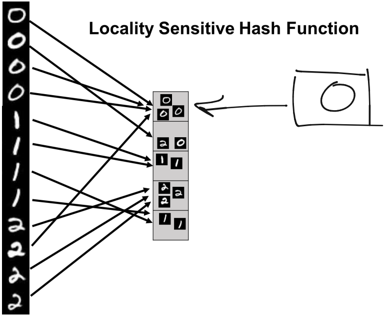

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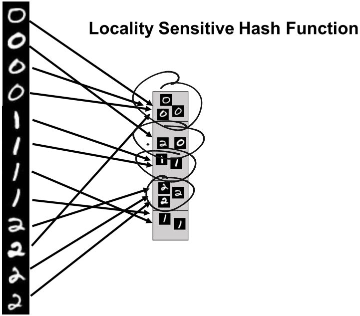

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LSH FOR SIMILARITY SEARCH

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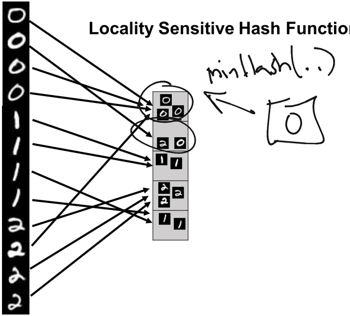

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- We will use $\mathbf{h}(x) = \mathbf{g}(MinHash(x))$ where $\mathbf{g} : [0, 1] \to [n]$ is a random hash function. Why?

Goal: Given a document y, identify all documents x in a database with Jaccard similarity (of their shingle sets) $J(x, y) \geq 1/2$.

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- For every document x in your database with $J(x, y) \geq 1/2$ what is the probability you will find x in bucket $\mathbf{g}(MinHash(y))$? 1/2

REDUCING FALSE NEGATIVES

With a simple use of MinHash, we miss a match x with $J(x, y) = 1/2$ with probability $1/2$. How can we reduce this false negative rate?

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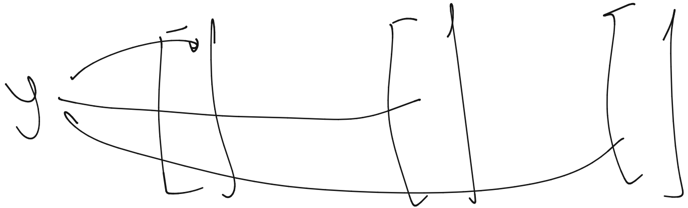

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false negatives: query (y), $J(x,y) \geq 1/2$ I don't
x in the same bucket as y

MinHash : sets → [m]

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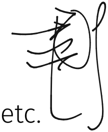

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Potential for a lot of false positives! Slows down search time.

BALANCING HIT RATE AND QUERY TIME

We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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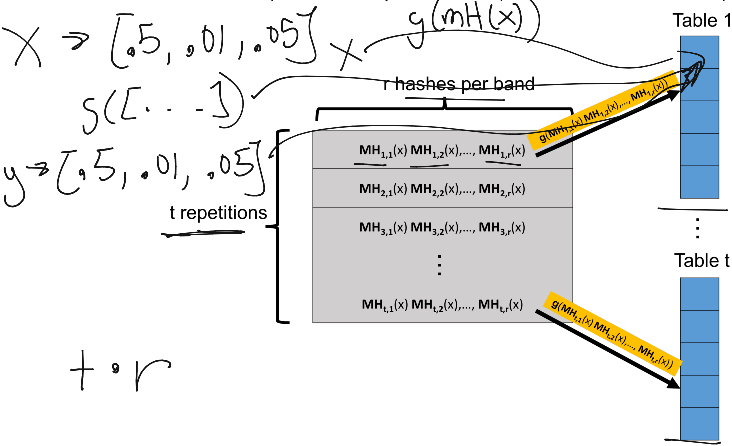

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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[.5, .6, .1] [.5, .6, .1]

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- Probability that x and y don't match in *all repetitions*:

t tables/
repetitions

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BALANCING HIT RATE AND QUERY TIME

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$$\text{Hit Probability: } 1 - (1 - s^r)^t.$$

t = # tables
r = signature length
s = similarity

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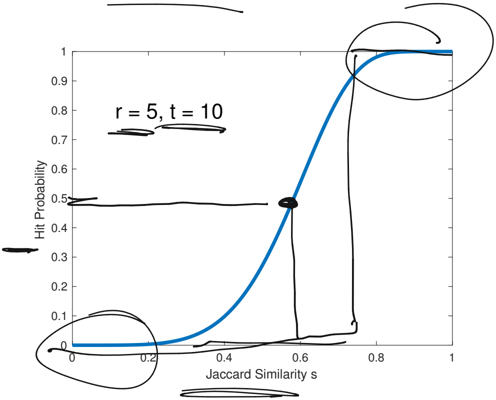

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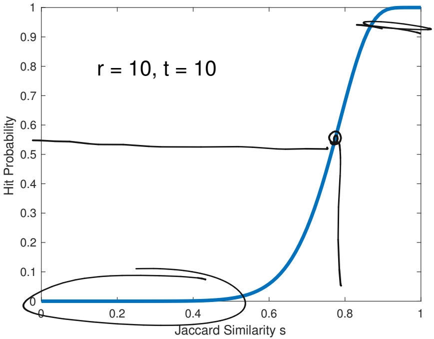

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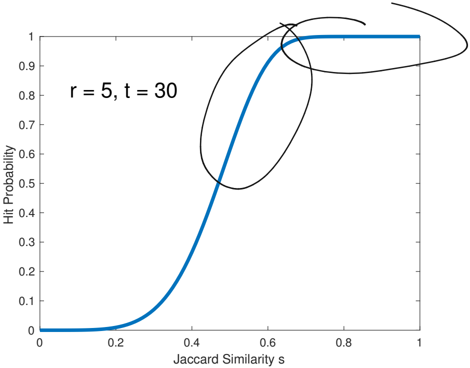

THE s-CURVE

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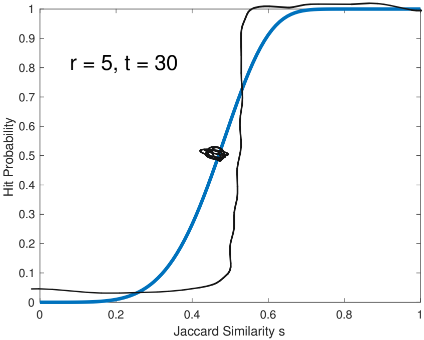
r=50
t=50



r and t are tuned depending on application. 'Threshold' when hit probability is 1/2 is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

s-curve example

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	Hash Table	Bloom Filters	MinHash Similarity Search	Distinct Elements
Goal	Check if x is a duplicate of any y in database and return y.	Check if x is a duplicate of y in database.	new Check if x is a duplicate of any y in database and return y.	Count # of items, excluding duplicates.
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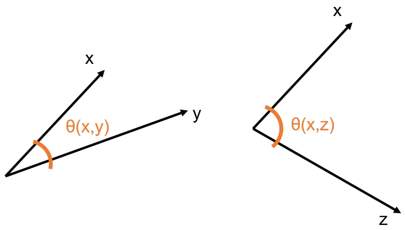

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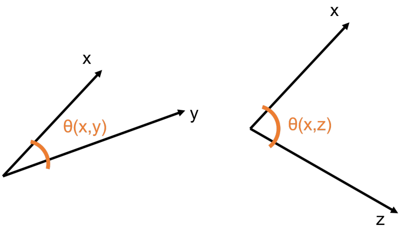

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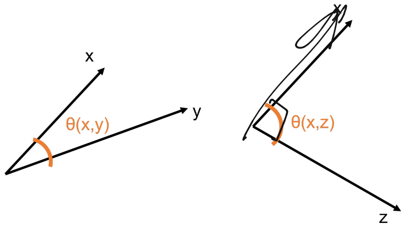

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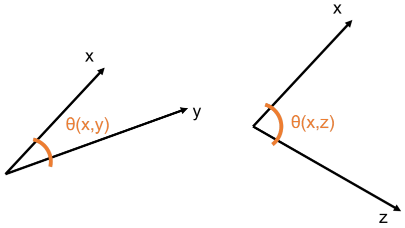

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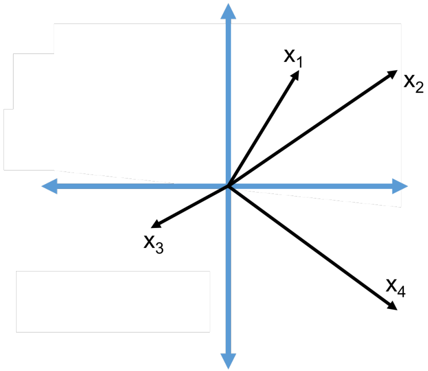

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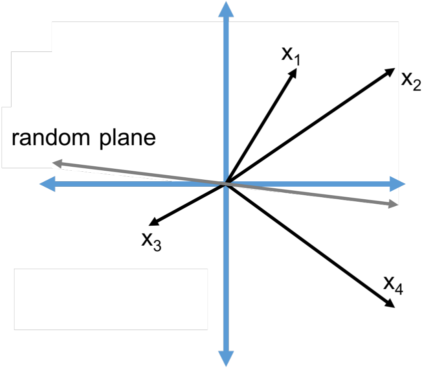

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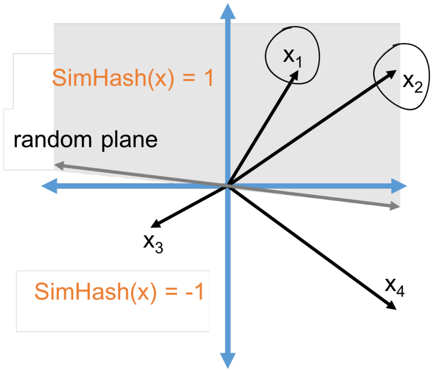

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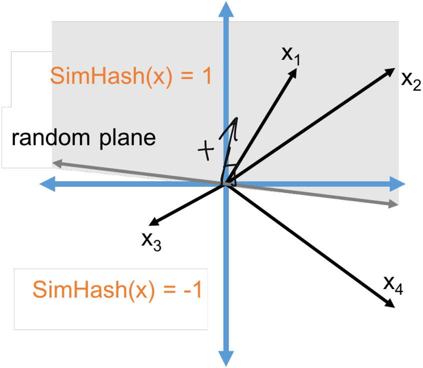

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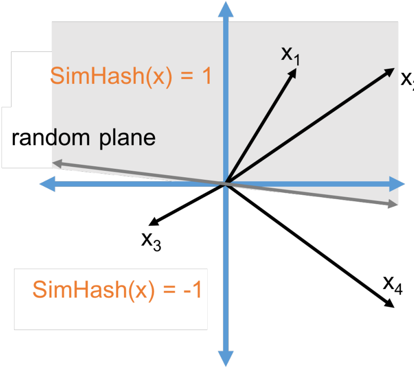

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SIMHASH FOR COSINE SIMILARITY

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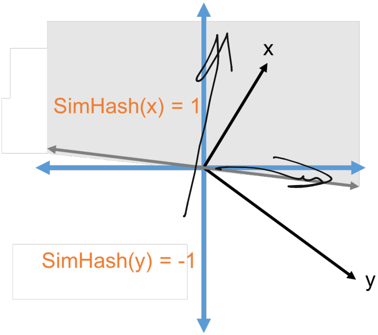

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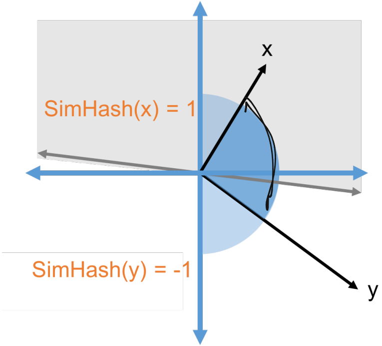

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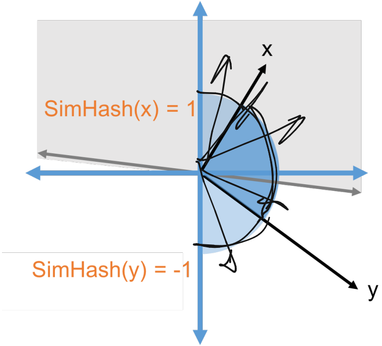

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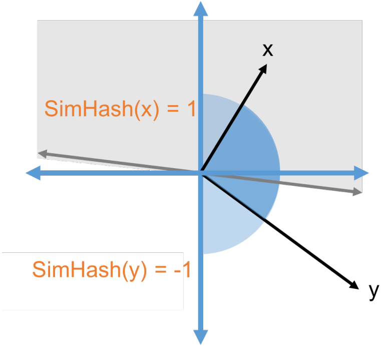

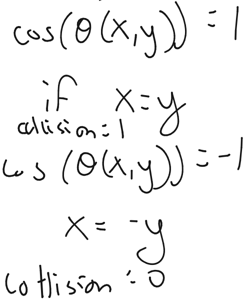

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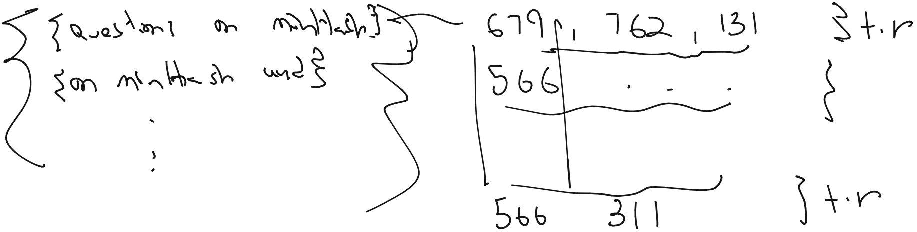
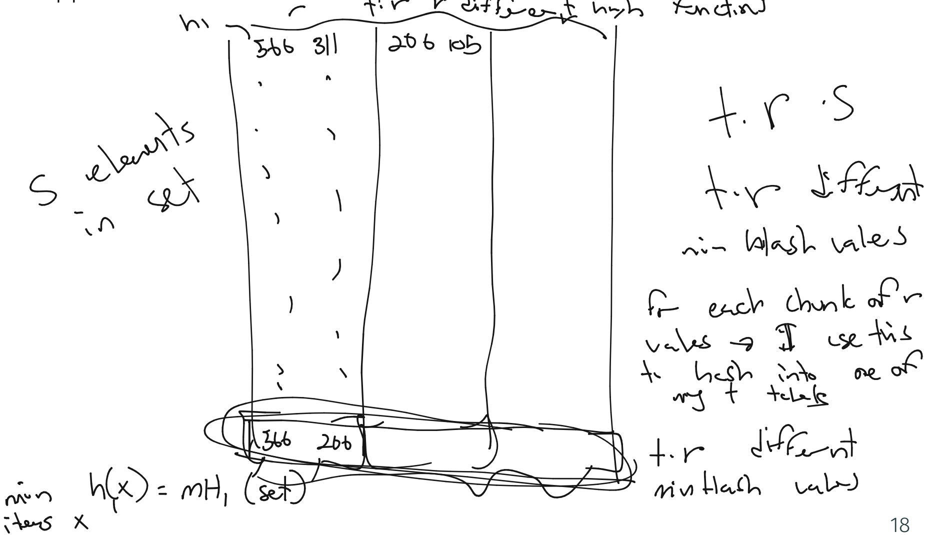


Questions on MinHash and Locality Sensitive Hashing?

- minhash is a locality sensitive hash function for Jaccard
- if we use it to hash into a hash table we can find similar items
- tune # tables, + signature length via S curve, to balance false positives + false negatives

Many applications outside traditional similarity search. E.g., approximate neural net computation (Anshumali Shrivastava).



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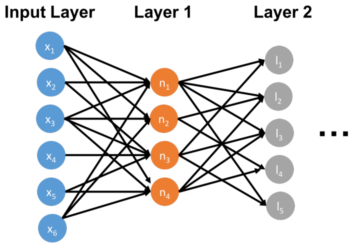

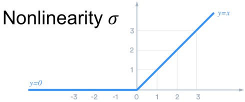

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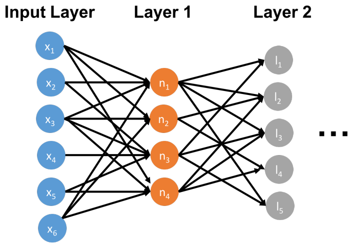

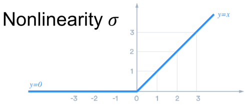

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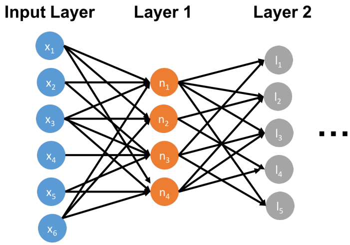

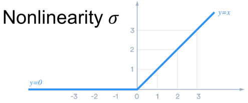

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HASHING FOR NEURAL NETWORKS

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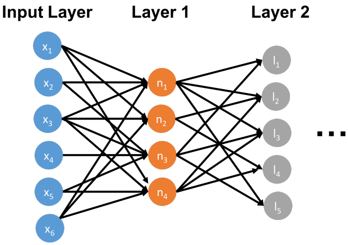

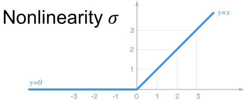

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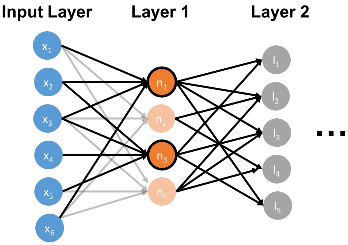

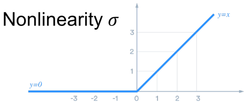

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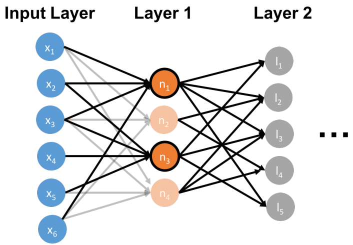
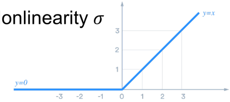


Nonlinearity σ



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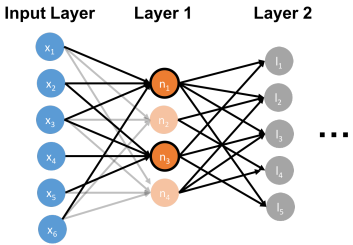

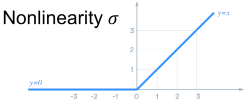

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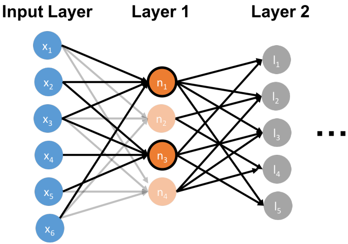

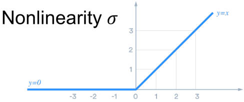

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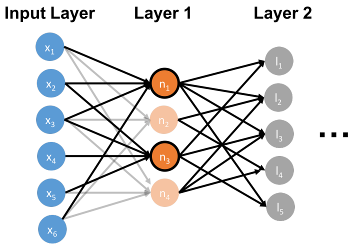
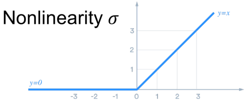
HASHING FOR NEURAL NETWORKS





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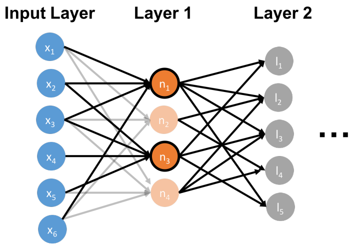

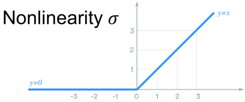

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- Store each weight vector w_i (corresponding to each node) in a set of hash tables and check inputs x for similarity to these stored vectors.