AN OPTIMIZATION-BASED FRAMEWORK FOR AUTOMATED MARKET MAKING

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RECAP: PREDICTION MARKETS
MERGE BELIEFS

Consensus estimate
BELIEFS, DISTRIBUTIONS, PRICES
“PROBABILITY DOES NOT EXIST”

- The above phrase is what Bruno de Finetti wanted “printed in capital letters in the preface” to his Theory of Probability.

- de Finetti: a probability P should be interpreted as the odds of a bet one would offer when our opponent can take any side of this bet.

- The laws of probability can derived via simple “no-arbitrage conditions” of these odds.
TWO PROBLEMS FOR AUTOMATED MARKET MAKERS

• As contracts are purchased, how shall we set prices?

• How to handle combinatorial outcome spaces, i.e. when $N$ is large?
  
  • Tournament outcome: $N = n!$

  • Multi-candidate election: $N = \binom{n}{k}$
NAIVE APPROACH: ONE CONTRACT PER OUTCOME

• A natural strategy would just be to sell one contract for each of the large set of outcomes
BETTER APPROACH: A SMALL “MENU” OF CONTRACTS

• Consider a multi-candidate election, where outcome is a set of k winners from n candidates,

• Market maker sells n contracts, one for each i, of the form:

  \[\text{pays off } \$1 \text{ when } i \text{ is among } k \text{ winners}\]

• That is, we allow bets on only a subset of “relevant” dimensions
## THE PAYOFF MATRIX

### SMALL

<table>
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<tr>
<th></th>
<th>Cand. 1</th>
<th>Cand. 2</th>
<th>Cand. 3</th>
<th>Cand. 4</th>
<th>Cand. 5</th>
<th>Cand. 6</th>
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<td>Outcome N</td>
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</tbody>
</table>
THE PRICE SPACE

• For simple markets, prices lie in the simplex:

• For “complex” markets, what constraints must we impose on the prices?

• Price vector must lie in ConvexHull(outcomes)!
PRICING VIA REGULARIZATION: SIMPLE MARKETS

LMSR:

\[ p(i) = \frac{\exp(\eta q(i))}{\sum_j \exp(\eta q(j))} \]

Alternative:

\[ p := \arg \max_{p' \in K} \ q \cdot p' - \frac{R(p')}{\eta} \]

Neg. Entropy

Simplex
PRICING VIA REGULARIZATION: COMPLEX MARKETS

\[ p := \arg \max_{p' \in K} q \cdot p' - \frac{R(p')}{\eta} \]

Convex Hull of Outcome Space

Some Curved Regularization
RESULTS

• We have an efficient way to set prices in a prediction market with a combinatorial outcome space

• The “liquidity” (i.e. price stability) depends on the curvature properties of $R$ -- more curved $\Rightarrow$ more stability

• The worst-case loss of the market maker is no more than

$$\frac{\max_K R - \min_K R}{\eta}$$
QUESTIONS?

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