From J&M chapter 7 -- Jason Eisner's ice cream / weather HMM example.

**Model**

![Hidden Markov Model Diagram]

*Figure 7.3* A hidden Markov model for relating numbers of ice creams eaten by Jason (the observations) to the weather (H or C, the hidden variables).

**Viterbi algorithm**

![Viterbi Trellis Diagram]

*Figure 7.4* The Viterbi trellis for computing the best path through the hidden state space for the ice-cream eating events. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of \( V_{t}(j) \) for two states at two time steps. The computation in each cell follows Eq. 7.19: \( V_{t}(j) = \max_{1 \leq k \leq N} V_{t-1}(k) a_{ij} b_{j}(o_{t}) \). The resulting probability expressed in each cell is Eq. 7.18: \( V_{t}(j) = P(q_{0}, q_{1}, \ldots, q_{t-1}, o_{1}, o_{2}, \ldots, o_{t}; q_{t} = j|\lambda) \).

**Declaratively:**

\[
V_{t}[k] = \max_{y_{1} \ldots y_{t-1}} P(y_{t} = k, y_{1} \ldots y_{t-1}, w_{1} \ldots w_{t})
\]

**Algorithm**, for each \( t=1..N \),

\[
V_{t}[k] := \max_{j=1..K} \left( V_{t-1}[j] \cdot P_{\text{trans}}(j \rightarrow k) \cdot P_{\text{emit}}(w_{t}|k) \right)
\]
Forward algorithm

Figure 7.7 The forward trellis for computing the total observation likelihood for the ice-cream events 3 1 3. Hidden states are in circles, observations in squares. White (unfilled) circles indicate illegal transitions. The figure shows the computation of $\alpha_t(j)$ for two states at two time steps. The computation in each cell follows Eq. 7.14: $\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$. The resulting probability expressed in each cell is Eq. 7.13: $\alpha_t(j) = P(o_1, o_2 \ldots o_t, q_t = j \mid \lambda)$.

Declaratively:

$$\alpha_t[k] = \sum_{y_1 \ldots y_t} P(y_t = k, w_1 \ldots w_t, y_1 \ldots y_{t-1})$$

Algorithm, for each $t=1..N$,

$$\alpha_t[k] := \sum_{j=1..K} \left( \alpha_{t-1}[j] P_{trans}(j \rightarrow k) P_{emit}(w_t|k) \right)$$

Forward