Role of incentives in networks

CS 653, Fall 2010
Selfish behavior in networks

- Game theory in networks an area in itself
- Today: Three case studies
  - BitTorrent
  - Selfish routing
    - Overlay routing
    - Interdomain routing
BitTorrent
BitTyrant: Optimizing return-on-investment

For each peer $p$, maintain estimates of expected download performance $d_p$ and upload required for reciprocation $u_p$.

Initialize $u_p$ and $d_p$ assuming the bandwidth distribution in Figure 2.

$d_p$ is initially the expected equal split capacity of $p$.

$u_p$ is initially the rate just above the step in the reciprocation probability.

Each round, rank order peers by the ratio $d_p/u_p$ and unchoke those of top rank until the upload capacity is reached.

\[
\begin{align*}
\frac{d_0}{u_0}, \frac{d_1}{u_1}, \frac{d_2}{u_2}, \frac{d_3}{u_3}, \frac{d_4}{u_4}, \ldots
\end{align*}
\]

choose $k \mid \sum_{i=0}^{k} u_i \leq cap$

At the end of each round for each unched peer:

If peer $p$ does not unchoke us: $u_p \leftarrow (1 + \delta) u_p$

If peer $p$ unchokes us: $d_p \leftarrow$ observed rate.

If peer $p$ has unchoked us for the last $r$ rounds:

\[
\begin{align*}
u_{rn} \leftarrow (1 - \gamma) u_{rn}
\end{align*}
\]
BitTorrent is an auction [LLSB08]

Algorithm 1 BitTorrent’s current auction.

1. Run an auction for $i$’s bandwidth; accept bandwidth $b_j(t-1)$ from interested peer $j$ as $j$’s bid.

2. Send $1/s$ fraction of $i$’s total outgoing bandwidth to each of the highest $(s - 1)$ interesting bidders from the previous round, as well as one other interested peer at random.

Algorithm 4 Proportional share auction clearing.

1. Run an auction for $i$’s bandwidth; accept bandwidth $b_j^i(t)$ from peer $j$ as $j$’s bid.

2. Let $B_i$ represent $i$’s total available upload bandwidth. Send to peer $j$ his proportional share:

   $$b_j^i(t) = B_i \cdot \frac{b_j^i(t-1)}{\sum_k b_k^i(t-1)}. \quad (1)$$
BitTyrant as a Sybil attack

Algorithm 2 A Sybil attack on the BitTorrent.

1. Find the maximum $k$ such that $k \cdot (c_{s-k} + \epsilon) \leq U_p$.

2. Create $k$ Sybils, $\{\sigma_1, \ldots, \sigma_k\}$, and with $\sigma_i$ bid $c_{s-k} + \epsilon$.

c_1 \geq c_2 \geq \ldots \geq c_{(s-1)}$ are the rates required to make it to the $i$’th slot in the auction.
BitTorrent

- Does the BitTorrent game reach a desirable equilibrium state?
Nash equilibrium

- A state where no player has an incentive to unilaterally change their strategy
- Classic example: Prisoner’s dilemma

Example PD payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>Prisoner B Stays Silent</th>
<th>Prisoner B Betrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prisoner A Stays Silent</td>
<td>Each serves 6 months</td>
<td>Prisoner A: 10 years Prisoner B: goes free</td>
</tr>
<tr>
<td>Prisoner A Betrays</td>
<td>Prisoner A: goes free</td>
<td>Each serves 5 years</td>
</tr>
<tr>
<td>Cooperate</td>
<td>3, 3</td>
<td>0, 5</td>
</tr>
<tr>
<td>Defect</td>
<td>5, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
Does BT reach a NE?

- BitTorrent has not been shown to reach NE under a reasonable game model
  - Example from 4.2 in [QS04]
- What about PropShare?
  - Best-response not prop-share, so not guaranteed to reach NE

6.1 Best response to prop-share

We can capture peer $i$’s best response to all other nodes playing proportional share as follows:

$$\begin{align*}
\text{maximize} & \quad \sum_j B_j \cdot \frac{b_j^i(t)}{\sum_k b_k^j(t)} \\
\text{s.t.} & \quad \sum_j b_j^i \leq B_i \quad \text{and} \quad \forall k : b_k^i \geq 0
\end{align*}$$
Selfish routing
Selfish routing

- If users choose routes (e.g., overlay routing), is there an efficiency loss?
Braess’s paradox

![Diagram of Braess’s paradox showing the network before and after adding a link.](image)

**Q:** What is the price of selfish routing here?
Price of selfish routing [RT02]

Assuming one unit of flow, what is the "price of anarchy" here?

- With linear cost functions, this is about as bad as it gets.
Selfish routing in Internet-like environments [QYZS06]

- Underlay: OSPF or MPLS
- Overlay
  - Independent source routing
  - Cooperative routing in each overlay
- Link latency functions: M/M/1 and others
- Summary: Selfish routing
  - Achieves near-optimal average latency while overloading some links
  - Interacts poorly with traffic engineering
Selfish interdomain routing [MWA07]

- Example of price of anarchy

Figure 1: Unilaterally controlled routing leads to longer paths and higher resource consumption inside ISPs. The solid lines depict early-exit routes. The dashed line depicts a route that is better overall as well as for the left and right ISPs.