# CMPSCI 240: "Reasoning Under Uncertainty"

Lecture 21

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# Reminders

- Check the course website: `http://www.cs.umass.edu/~wallach/courses/s12/cmpsci240/`
- Sixth homework is due on Friday

# Grades

1. Add discussion section scores, divide by 14, multiply by 10
2. Add homework scores, divide by 250, multiply by 30
3. Divide midterm score by 100, multiply by 30
4. Add 1–3 to obtain your score (max. possible is 70)
5. If your score is 40 or less, you're in danger of getting a D

# Markov Chains

# Reasoning About Complex Scenarios

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- Markov chains give us a way to exploit independence assumptions when we are reasoning about temporal scenarios
- e.g., the sequence of daily prices of a stock, the sequence of websites a web surfer visits (Google PageRank), the number of packets in a network buffer over time, the sequence of words in an "English-looking" document, ...

# Examples of Markov Chains

- e.g., my router can be either online or offline. If it is online it will be online the next day with probability 0.8. If it is offline it will remain offline the next day with probability 0.4.

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- e.g., consider a line at the Apple store. Every minute someone is served with probability $1/2$. Every minute someone joins the line with probability 1 if the line has length 0, with probability $2/3$ if the line has length 1, with probability $1/3$ if the line has length 2, and with probability 0 if the line has length 3.

# States and Transitions

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- The system changes state according to transition probabilities
- States and transitions between them can be represented using a transition probability graph or state transition diagram

# Examples of Markov Chains

- e.g., my router can be either online or offline. If it is online it will be online the next day with probability 0.8. If it is offline it will remain offline the next day with probability 0.4. Draw the transition probability graph for this system.

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$$p_{ij} = P(X_{t+1} = j \mid X_t = i)$$

- Transition probabilities out of any state must sum to one:

$$\sum_j p_{ij} = \sum_j P(X_{t+1} = j \mid X_t = i) = 1$$

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$$P(X_{t+1}=j \mid X_t=i, X_{t-1}=h, \ldots) = P(X_{t+1}=j \mid X_t=i)$$

- If we know the state at time $t$ and the transition probabilities then there is nothing else we need to specify about the system in order to characterize/predict its future behavior

# Markov Property and State Space

- Can still use a Markov chain model even if there is a dependence on the states at several previous points in time

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- Can still use a Markov chain model even if there is a dependence on the states at several previous points in time
- Do this by defining additional states which encode the relevant information from previous points in time

# Examples of Markov Chains

- e.g., my router can be either online or offline. If it is online it will remain online the next day with probability 0.8. If it is offline it will remain offline the next day with probability 0.4. If it is offline for 4 days straight, I can get it (temporarily) repaired, resetting it to online. What are the states and transition probabilities? Draw the transition probability graph.

# Transition Probabilities

# Probability of a Path

- Can compute the probability of any sequence of states using the multiplication rule and the Markov property:

$$P(X_1 = b, X_2 = c, X_3 = d, X_4 = e \mid X_0 = a) = p_{ab}\, p_{bc}\, p_{cd}\, p_{de}$$

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- e.g., if my router is initially online, what is the probability that my router will then be offline for 4 days straight?

- e.g., what if I am only 50% certain it is initially online?

# $n$-Step Transition Probabilities

- If we know the initial probability of each state $P(X_0 = i)$ and the transition probabilities, then we can compute the $n$-step transition probability $P(X_n = j)$ using the recursive formula:

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- This is just the law of total probability!

# Examples of $n$-Step Transition Probabilities

- e.g., my router can be either online or offline. If it is online it will remain online the next day with probability 0.8. If it is offline it will remain offline the next day with probability 0.4. If it is initially online, what is the probability that it will be online after 5 days? What is the probability it will be offline?

[Got to here in class...]

# Examples of Markov Chains

- e.g., consider a line at the Apple store. Every minute, the server serves a customer with probability 1/2. Every minute someone joins the line with probability 1 if the line has length 0; with probability 2/3 if the line has length 1; with probability 1/3 if the line has length 2; with probability 0 if the line has length 3. What are the states and transition probabilities? Draw the transition probability graph.

# For Next Time

- Check the course website: `http://www.cs.umass.edu/~wallach/courses/s12/cmpsci240/`
- Sixth homework is due on Friday