CMPSCI 240: "Reasoning Under Uncertainty"

Lecture 12

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Reminders

- Check the course website: `http://www.cs.umass.edu/~wallach/courses/s12/cmpsci240/`
- Fourth homework is due TOMORROW at 11:59pm

Recap

Last Time: Transmitting Information

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- Error probability: overall probability of there being an undetected error when using some encoding scheme

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- Error detecting codes, e.g., parity check codes
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- Fundamental trade-off: want encoding schemes that minimize both the error probability and the information rate

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- 2 bit flips can be detected using a global parity bit $\implies 8/4$

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- If so, flipping this bit accounts for the parity violation

Correlation and Covariance

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- When $\text{cov}(X, Y) = 0$, X and Y are uncorrelated
- If X and Y are independent, what is $\text{cov}(X, Y)$?
- If $\text{cov}(X, Y) = 0$, are necessarily X and Y independent?

Examples of Independence and Correlation

- e.g., suppose the pair of random variables (X, Y) take on values $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ each with probability $1/4$. What is the joint PMF of X and Y? What is the marginal PMF of X? What is the marginal PMF of Y? What are $\mathbb{E}[X]$ and $\mathbb{E}[Y]$? What is the covariance of X and Y?

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- What is $\rho(X, Y)$ if X and Y are independent?

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- $|\rho|$ is a measure of how true this is

Examples of the Correlation Coefficient

- e.g., consider n independent coin flips, where p is the probability of heads. Let X and Y be the number of heads and tails. What is $X + Y$? What about $\mathbb{E}[X + Y]$? What does this tell us about the relationship between $X - \mathbb{E}[X]$ and $Y - \mathbb{E}[Y]$? What is $\text{cov}(X, Y)$? What is $\rho(X, Y)$?

Variance of the Sum of Random Variables

- In general, for any $X_1, X_2, \ldots, X_n$

$$\text{var}(X_1 + \ldots + X_n) = \sum_{i=1}^{n} \text{var}(X_i) + \sum_{\{(i,j) \mid i \neq j\}} \text{cov}(X_i, X_j)$$

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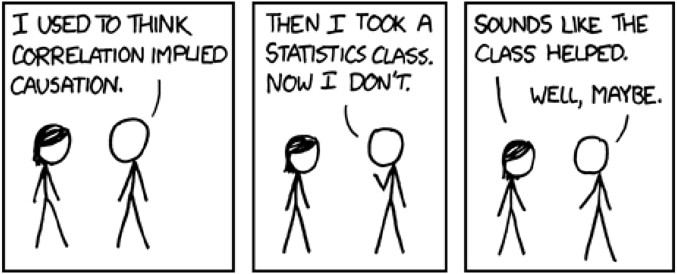
- What if $X_1, X_2, \ldots, X_n$ are independent?

Causation


I USED TO THINK CORRELATION IMPLIED CAUSATION.
THEN I TOOK A STATISTICS CLASS. NOW I DON'T.
SOUNDS LIKE THE CLASS HELPED.
WELL, MAYBE.

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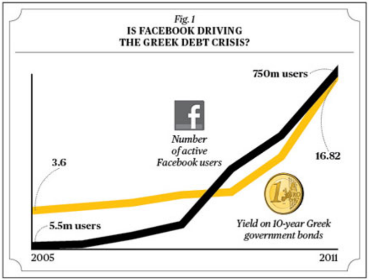
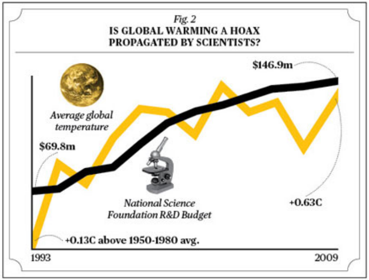
- "Relationship" is a coincidence or very complex/indirect...

Correlation vs. Coincidence





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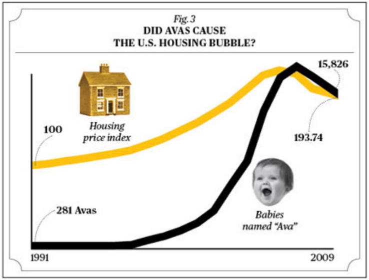

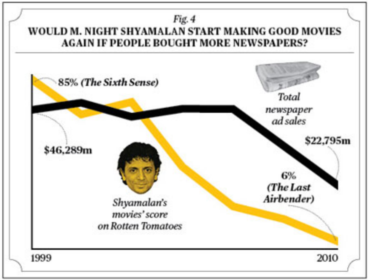

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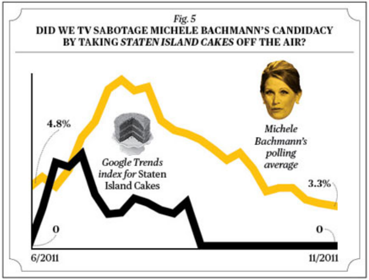

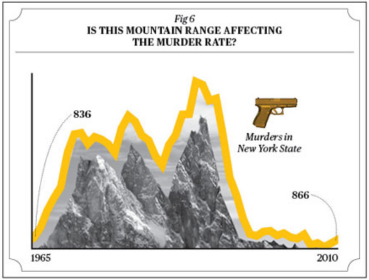

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