CMPSCI 240
Reasoning Under Uncertainty
Homework 1

Prof. Hanna Wallach

Assigned: February 3, 2012
Due: February 10, 2012

Question 1: (5 points) There are two events A and B, both with nonzero probabilities. If the occurrence of B makes the occurrence of A more likely (that is, if $\mathbb{P}[A \mid B] > \mathbb{P}[A]$), is it ALWAYS true that the occurrence of A also makes the occurrence of B more likely? Provide a proof or counterexample to support your answer. (A proof should be rigorous, and a counterexample should include well-specifed probabilities for any events of interest.)

Question 2: (5 points) We are told that events A and B are independent. In addition, events A and C are independent. Is it ALWAYS true that A is independent of B $\cup$ C? Provide either a proof or counterexample.

Question 3: (10 points) The disc containing your favorite MP3 has been scratched very badly. The disc was mixed up with three other scratched discs that were lying around. It is equally likely that any of the four discs holds your favorite MP3. A friend offers to take a look, claiming to recover songs from any disc with a 90% probability (assuming the song is there).

(a) Given that he searches on disc 1 but cannot recover your MP3, what is the probability that your MP3 is on disc i for $i = 1, 2, 3, 4$?

(b) Your friend has searched both disc 1 and disc 2 and has not yet found

the MP3. What is the probability it is on disc i for $i = 1, 2, 3, 4$? (HINT: Given that the MP3 is on disc *i*, the event that your friend finds it on disc 1 is independent of the event that your friend finds it on disc 2.)

Question 4: (10 points) A parking lot consists of a single row containing n parking spaces for some reasonably large n (say, $n \geq 5$). Phil arrives when all spaces are free. Sandeep is the next person to arrive. Each person makes an equally likely choice among all available spaces at the time of arrival.

(a) Describe a natural sample space for this experiment mathematically. (Recall: a sample space is defined as a set of all possible outcomes.)

(b) Find $\mathbb{P}[A]$, the probability the spaces selected by Phil and Sandeep are at most 2 spaces apart (i.e., at most 1 empty space between them).

Question 5: (10 points) A company has a data center with a total of 1,000 computers. The computers are grouped into clusters of 25 computers each. A cluster fails if 2 or more of its computers fail. Given that 3 of the 1,000 computers fail on particular day, what is the probability that at least 1 cluster fails? Assume that all computers are equally likely to have failed.

Question 6: (10 points) You have three hypotheses concerning the weather this evening: dry, rainy, and snowy. At this time of year, the weather in the evening is dry on 40% of days, rainy on 20% of days, and snowy on 40% days. At noon you notice clouds in the sky. Clouds appear at noon on

- 6% of the days that will have dry evenings,
- 2% of the days that will have rainy evenings,
- 4% of the days that will have snowy evenings.

Find the probability of each type of weather given that you saw clouds.