

CMPSCI 105: Lecture #4 Gates and Truth Tables

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First, an aside: Binary Addition

- Since numbers can be converted between bases, addition in one base gives the same answer as addition in another base,
- Adding in binary is simpler than adding in decimal,
- So we build computers to add in binary rather than in decimal!

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Binary Addition Table

+	0	1
0	0	1
1	1	10

Zero, with a Carry

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Example

. ← Carries
 0110010101000101
+0110011100010110
 1100110001011011

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Now back to Logic...

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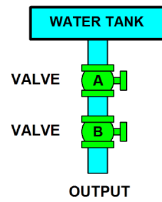
When is the sentence True?

- **"I am going to the store and going to the beach"**
- Clause A: "going to the store"
- Clause B: "going to the beach"
- Either may independently be True or False.
- When is the overall sentence True?
- Only when both are True.

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When does Water flow?

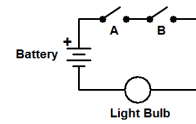
- When both valves are open



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When does the light come on?

- When both switches are closed



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Commonalities

- In all three cases the underlying logic is the same:
 - Both clause A and clause B must be True for the sentence to be True.
 - Both Valve A and Valve B must to open for water to flow.
 - Both Switch A and Switch B must be closed for the light bulb to come on.
- The only thing in common is **AND**.

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Truth Tables

- A **Truth Table** is a tabular way of describing all possible behaviors for a binary system.
- We could use T and F to indicate True and False, but instead we will define 1 for True and 0 for False.
- The number of rows in a Truth Table is 2 raised to the power of the number of inputs.

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Truth Table for AND (two inputs)

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

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Can we have more than Two inputs?

- **YES.**
- “I am going to the store and going to the beach and going to the movies.”
- The pipe contains three valves in series, all must be open for water to flow.
- The circuit contain three switches in series, all must be closed for the light bulb to come on.

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Truth Table for AND (three inputs)

A	B	C	AND
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

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What About...

- If we can describe the behavior of AND...
- ...can we also describe OR?
- Yes, but there's a catch:
- The OR we use in English is not the same as the OR we use in computer logic!

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The OR in English

- "I am going to the store OR going to the beach."
- If I don't do either, I've lied,
- if I do one or the other, I've told the truth,
- but *if I do both I've lied!*
- In English the inference is that if I say I'll do one thing or the other, I won't do both!
- This is called an **Exclusive OR** (aka **XOR**).

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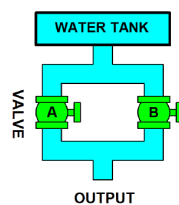
The OR we use in Logic

- If one input or the other is true, the output is true,
- If both inputs are true, the output is still true,
- This is called an **Inclusive OR**.

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When does Water flow?

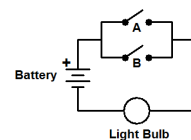
- When either or both valves are open



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When does the light come on?

- When either or both switches are closed



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Truth Table(s) for OR

A	B	XOR	A	B	OR
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	1	1	1

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General Rules for AND and OR

- AND: The output is 1 if all inputs are 1, the output is 0 if any input is 0.
- OR: The output is 1 if any input is 1, the output is 0 if all inputs are 0.
- XOR: The output is 1 if the inputs differ, the output is 0 if the inputs are the same.

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NOT

- "I am NOT going to the store."

A	NOT
0	1
1	0

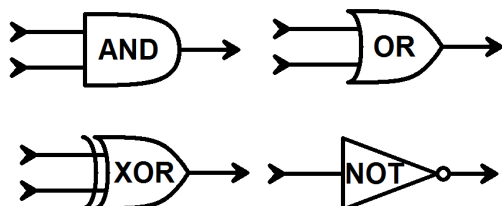
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Gates

- A Gate is:
 - A mathematical abstraction for...
 - ...a physical device that...
 - ...implements the function of a Truth Table.

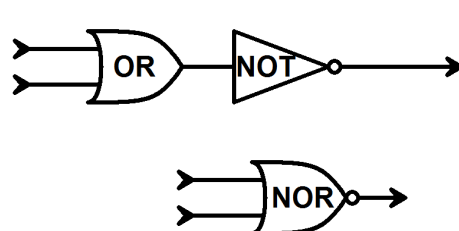
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AND, OR, and XOR gates



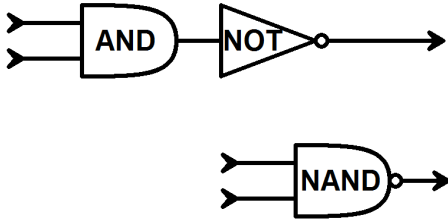
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NOT-OR = NOR



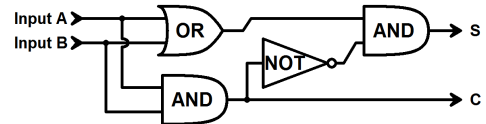
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NOT-AND = NAND



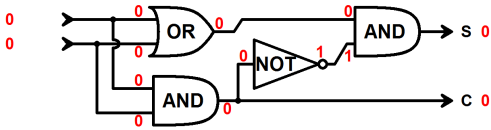
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What Does it Do?



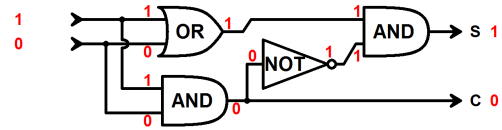
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Inputs 0,0



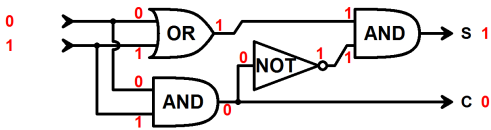
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Inputs 1,0



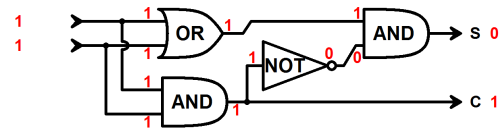
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Inputs 0,1



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Inputs 1,1



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What is its Function?

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

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A Binary Adder!

A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

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