A. (2 points) What is the spin rate of gear $G_2$ relative to the spin rate of $G_1$ (ignore differences in direction)?

\[ \frac{36}{144} = \frac{1}{4} \]

B. (3 points) What is the resistance of $R_1$ so that the voltage output of the resistor divider relative to its input is the same as the spin rate of $G_2$ relative to $G_1$?

6000Ω (2000Ω for $R_2$ is $\frac{1}{4}$ of the total, so the total is 8000Ω)

C. (5 points) Short Answer – How are these two systems similar? How are they different? Explain your answer in just a sentence or two.

Both are analog divide-by-four machines. One uses mechanical rotations, the other uses electrical voltages.
<2> 15 Points – Which of the following are digital devices and which are analog?

A. (3 points) A slide rule  
   Analog

B. (3 points) A vacuum tube amplifier circuit  
   Analog

C. (3 points) A relay adder  
   Digital

D. (3 points) A radio’s volume control that you turn by hand  
   Analog

E. (3 points) A NOR gate  
   Digital

<3> 15 Points – Consider the problems below, some using four-digit decimal arithmetic (base 10, with the left-most digit the sign digit), and the others using eight-bit binary arithmetic (base 2, with the left-most bit the sign bit). Solve each problem, and then tell me if it exhibits unsigned overflow, signed overflow, both, or neither?

A. (3 points) Decimal: 1408 + 4975 = 6383 (signed overflow)

B. (3 points) Decimal: 1408 + 2069 = 3477 (neither)

C. (3 points) Decimal: 1408 + 9904 = 11312 (unsigned overflow)

D. (3 points) Binary: 00010011 + 00101111 = 01000010 (neither)

E. (3 points) Binary: 11100011 + 11101010 = 111001101 (unsigned)

<4> 15 Points – Show the decimal (base 10) value of the eight-bit binary number 10011000 interpreted in each of the following ways:

A. (3 Points) Interpreted as Unsigned  
   10011000 = 152

B. (3 Points) Interpreted as Sign & Magnitude  
   10011000 = -24

C. (3 Points) Interpreted as One’s Complement  
   10011000 = -103

D. (3 Points) Interpreted as Two’s Complement  
   10011000 = -104

E. (3 Points) Interpreted as BCD  
   10011000 = 98
<5> 10 Points – The following circuit is of a full-adder constructed from half-adders. The table to the right shows the outputs for all possible input behaviors, but several of the outputs in the table are wrong. Circle the ones that are wrong. (Errors shown in red.)

<table>
<thead>
<tr>
<th>Input</th>
<th>Outputs</th>
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<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>B</td>
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<td>0</td>
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</tbody>
</table>

<6> 15 Points – Convert the decimal number **41.5625** into True Binary and Binary Scientific Notation. Show your work for partial credit.

True Binary (10 points):

**101001.1001**

Binary Scientific (5 points):

**1.010011001 × 2^5**

\[
41 ÷ 2 = 20 \text{ R } 1 \text{ (least sig. bit)} \quad .5625 \times 2 = 1.125 \text{ (most sig. bit)}
\]
\[
20 ÷ 2 = 10 \text{ R } 0 \quad .125 \times 2 = 0.25
\]
\[
10 ÷ 2 = 5 \text{ R } 0 \quad .25 \times 2 = 0.5
\]
\[
5 ÷ 2 = 2 \text{ R } 1 \quad .5 \times 2 = 1.0 \text{ (least sig. bit)}
\]
\[
2 ÷ 2 = 1 \text{ R } 0 \quad 2 \text{ ÷ 2 = 0 \ R } 1 \text{ (most sig. bit)}
\]

<7> 10 Points – Consider the binary number **1011.101** (without converting it to decimal):

A. What is the *binary* representation of this number *multiplied by two*?

**10111.01**

B. What is the *binary* representation of this number *divided by two*?

**101.1101**
10 Points – SHORT ESSAY – Pick one of the following questions about representations, and write your answers on the back of this page. Do both for +5 points extra-credit. Please do not write more than four to five sentences in total for either question. In your chosen question, think about why the representation is the way that it is, what alternatives might exist, and what the advantages and disadvantages of each representation may be. These representations were chosen for a reason over all the alternatives – why?

Option A.

Most fire hydrants in the United States have an access bolt with five sides, instead of four or six like most traditional bolts. Why?

Most bolts are either six-sided or four-sided to make wrenches able to grab the bolt head securely, and be able to move from position to position easily. More than six sides makes the shape of the bolt head asymptotically approach a circle, hard to grab securely. Three sided bolts are easy to grab, but would make the corners too sharp. Five sides is a compromise, which works as well as either four or six.

However, no consumer bolts are five-sided, so five-sided wrenches are uncommon. Firefighters do have such wrenches as part of their standard tool-kit.

The combination makes fire hydrants able to be deployed publically, with little danger of non-firefighters being able to open them.

Option B.

The Inuit people of Greenland use carved wooden maps, instead of maps drawn on paper. Why?

Paper is difficult to obtain, paper is not very sturdy when wet, and paper maps have to be unfolded in order to be studied. In contrast, wooden maps are rugged, and can be used and studied by feel in cold weather without removing them from inside mittens. They also provide a tactile sense of 3D topology not possible with flat paper.