CMPSCI 145 MIDTERM #2

SPRING 2014

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Solution Key

<1> 15 Points – Answer 15 of the following problems (1 point each). Answer more than 15 for extra credit. Incorrect or blank answers will be ignored.

137	What is the decimal value of the 8-bit ***unsigned*** binary number **`10001001`** ?
-9	What is the decimal value of the 8-bit ***sign and magnitude*** binary number **`10001001`** ?
-119	What is the decimal value of the 8-bit ***two's complement signed*** binary number **`10001001`** ?
W232	What is the SOUNDEX code for the name **`WASHINGTON`**?
J250	What is the SOUNDEX code for the name **`JACKSON`**?
L000	What is the SOUNDEX code for the name **`LEE`**?
0	Circle #1 is at <0,0> with radius 8. Circle #2 is at <14,0> with radius 5. How many points of intersection are there?
1	Circle #1 is at <0,0> with radius 8. Circle #2 is at <13,0> with radius 5. How many points of intersection are there?
2	Circle #1 is at <0,0> with radius 8. Circle #2 is at <12,0> with radius 5. How many points of intersection are there?
<1,1,-3>	A 3D ray has equations $x(t)=4t-1$, $y(t)=-4t+3$, $z(t)=2t-4$. What are the coordinates of the point at $t=½$?
<6,6>	What point in 2D is halfway between <0,5> and <12,7>?
Yes	Yes or No: Can I mathematically "draw" a line (i.e., compute intermediate points) between two points in 12 dimensions?
Sphere or Triangle	What geometric object is "most suitable" for ray-tracing? (Two allowed answers.)
(A+B)×C	What algebraic expression is equivalent to the RPN expression **`PUSH A`**, **`PUSH B`**, **`ADD`**, **`PUSH C`**, **`MULTIPLY`**?
O(N)	What is the running time (O(1), $O(\log_2(N))$, O(N), $O(N\times\log_2(N))$, $O(N^2)$, $O(N^3)$, etc.) of a ***linear search***?
$O(\text{Log}_2(N))$	What is the running time (O(1), $O(\log_2(N))$, O(N), $O(N\times\log_2(N))$, $O(N^2)$, $O(N^3)$, etc.) of a ***binary search***?
$O(N^2)$	What is the running time (O(1), $O(\log_2(N))$, O(N), $O(N\times\log_2(N))$, $O(N^2)$, $O(N^3)$, etc.) of a ***bubble sort***?
$O(N\times\log_2(N))$	What is the running time (O(1), $O(\log_2(N))$, O(N), $O(N\times\log_2(N))$, $O(N^2)$, $O(N^3)$, etc.) of a ***merge sort***?
O(1)	What is the running time (O(1), $O(\log_2(N))$, O(N), $O(N\times\log_2(N))$, $O(N^2)$, $O(N^3)$, etc.) of ***hashing***?
False	True or False: For a binary search to work correctly, the data in the list may be in any arbitrary order.

<2> 20 Points – (5 points each) Here is a 8-bit binary number interpreted as ***unsigned 10-bit fixed-point***, with five bits to the left and five bits to the right of the decimal point:

`10101.10100`

A. What is the decimal (base 10) value of this number?

21 5/8 or **21.625**

B. What is the ***smallest binary number*** that this representation can hold? (Show the result in 10-bit binary, including the decimal point.)

`00000.00000` (zero) or `00000.00001` (non-zero)

C. What is the decimal (base 10) value of the answer in part B?

0 (zero) or 1/32 = 0.03125 (non-zero)

D. What is the ***largest binary number*** that this representation can hold? (Show the result in 10-bit binary, including the decimal point.)

`11111.11111`

<3> 20 Points – (5 points each) Here is a decimal number and its true binary equivalent to be converted into ***three-quarter precision***, which is a 24-bit floating-point format that follows a 1-7-16 pattern (1 bit for sign, 7 bits for the biased exponent, 16 bits for the mantissa):

(Decimal) **`-55.375`** = (True Binary) **`-110111.011`**

A. What is the ***binary scientific notation*** value for this number?

-1.10111011×2^{5} (fraction 10111011 to be stored in mantissa)

B. What is the value in decimal of the ***bias*** for three-quarter precision?

$2^{7-1}-1 = 2^{6}-1 = 64-1 =$ **63** (added to true exponent to get biased exp.)

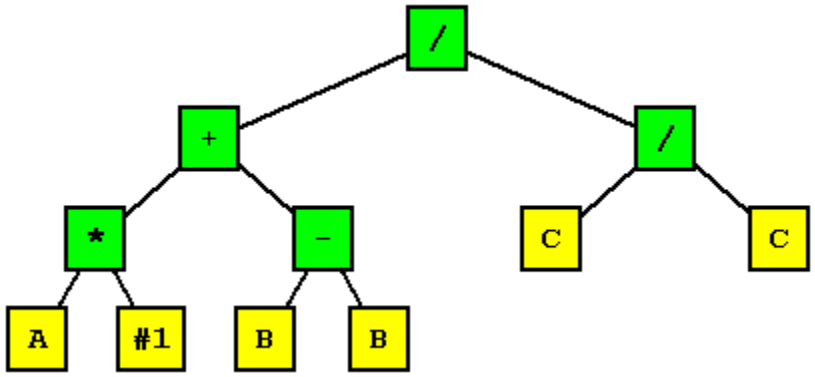
C. What is the value in decimal of the ***biased exponent*** for the number in this problem?

63 (bias) + 5 (true exponent) = **68** (stored as binary 1000100 in exponent field)

D. What is the binary representation for this number in ***three-quarter precision***? (Write your answer in the boxes below. The exponent field is shaded.)

1	1	0	0	0	1	0	0	1	0	1	1	1	0	1	1	0	0	0	0	0	0	0	0

<4> 10 Points – The image below shows the tree representation for an arithmetic expression (**A**, **B**, and **C** are algebraic variables, **#1** is just the numeric constant 1, ***** means multiplication, and **/** means division):



1. (5 points) Write out the complete algebraic equation for the current tree representation ***as shown***, including parentheses where necessary.

((A × 1) + (B – B)) / (C / C)

2. (5 points) Figure out how to ***optimize*** the tree to <u>most</u> efficiently compute the exact same value as what is shown here, and then <u>draw that resulting tree</u>.

A * 1 → A,
B – B → 0,
A + 0 → A,
C / C → 1,
A / 1 → A

<5> 15 Points – In the following Lagrange interpolation for a parabola, fill in the blanks with numbers so that the curve passes through P_0 when t = 5, through P_1 when t = 10, and through P_2 when t = 15.

$$f(t)=\frac{(t-\underline{10})(t-\underline{15})}{(\underline{5}-\underline{10})(\underline{5}-\underline{15})}P_0+\frac{(t-\underline{5})(t-\underline{15})}{(\underline{10}-\underline{5})(\underline{10}-\underline{15})}P_1+\frac{(t-\underline{5})(t-\underline{10})}{(\underline{15}-\underline{5})(\underline{15}-\underline{10})}P_2$$

<6> 15 Points – Here is a linear list of names. The "front" of the list is at the left. In each case, show the result of the list after a search, where the order of the names in the list is updated according to various ***self-organizing list*** techniques.

(5 points) Tom Bob Anne Mary Fred Sam Carol

Search for Mary, Swap with Front:

Mary Bob Anne Tom Fred Sam Carol

(5 points) Tom Bob Anne Mary Fred Sam Carol

Search for Mary, Move to Front:

Mary Tom Bob Anne Fred Sam Carol

(5 points) Tom Bob Anne Mary Fred Sam Carol

Search for Mary, Promote One Slot:

Tom Bob Mary Anne Fred Sam Carol

<7> 5 Points – Short Answer – If someone asked you what a particular binary number "meant," how would you answer them in light of what we have learned in this class?

It depends entirely on the *context* in which the number was generated. There is nothing inherent in any binary number which tells us which interpretation to use – although we can discount some interpretations (for example, `1100` cannot be BCD).