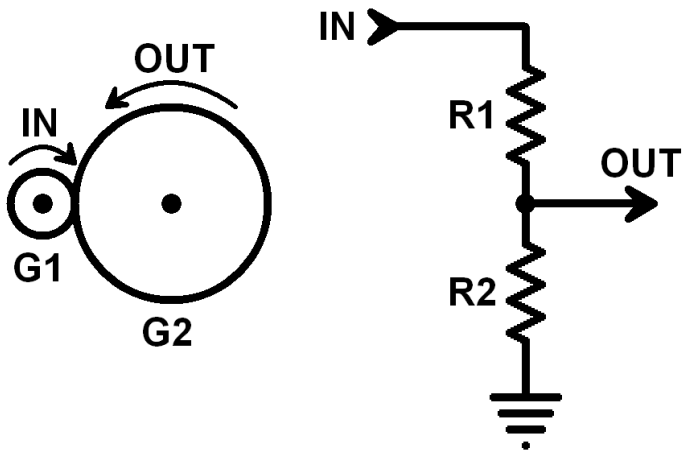
CMPSCI 145 MIDTERM #1

SOLUTION KEY

SPRING 2014

February 28, 2014

<1> 10 Points – Examine the following diagram of two systems, one involving gears and the other involving resistors. Resistor **R1** has a resistance of 5000 Ohms, and **R2** has a resistance of 1000 Ohms. Gear **G1** has 36 teeth.



A. (2 points) What is the voltage of the output of the resistor divider, relative to any arbitrary input voltage?

The output voltage is $\frac{1}{6}$ of the input voltage.

The total resistance is 5000 + 1000 = 6000 Ohms. The output voltage is measured across the 1000 Ohm resistor, which is $\frac{1}{6}$ of the total resistance, so it must be $\frac{1}{6}$ of the input voltage. Mathematically, if the input voltage is E volts, then by Ohm's Law ($I = \frac{E}{R}$) the current through both resistors is ($\frac{E}{6000}$), and by Ohm's Law again ($E = I \times R$) the voltage across R2 is thus ($\frac{E}{6000}$) $\times 1000 = \frac{1}{6}E$.

B. (3 points) How many teeth does gear **G2** have so that the spin rate of **G2** relative to **G1** (ignoring differences in direction) is the same as the relationship of the output voltage to the input voltage in the resistor divider?

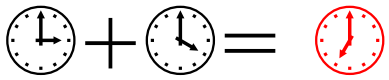
216 teeth.

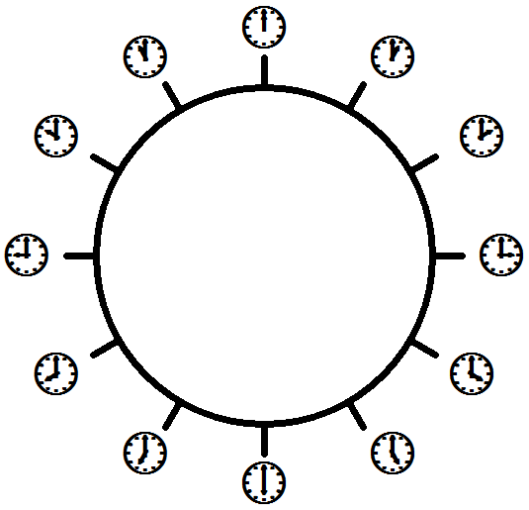
From the diagram, G1 will spin faster than G2. Because of our answer to part A, we will want G2 to spin at $\frac{1}{6}$ the rate of G1. Thus, for every revolution of G1 we want G2 to go around $\frac{1}{6}$ of a revolution: G2 has 6 times the number of teeth as G1. G1 has 36 teeth, so therefore G2 has 6×36 = 216 teeth.

C. (5 points) Short Answer – How are these two systems similar? How are they different? Explain your answer in just a sentence or two.

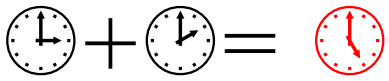
Both are **analog systems** that divide their input values by six. Within the limits of the respective systems, there are an **infinite number of legal input values** (spin rates, voltages), and the **output values are always a proportion of that**. They use **different technologies** to achieve the result, one mechanical and one electrical, so they can only be applied to the systems for which they were designed.

<2> 10 Points – The image to the right shows a standard 12-hour clock. Compute the following sums, and tell me if there is an unsigned overflow, a signed overflow, both or neither:





3+4=7, **signed overflow only** - in the clock base arithmetic system, 7 o'clock is the same as -5 o'clock, and going into or past 6 o'clock is a signed overflow. The sum did not pass 12, so there is no unsigned overflow.



3+2=5, **no overflow** - did not pass either 6 (signed overflow) or 12 (unsigned overflow).

<3> 15 Points – Consider the sums below, some using ***four-digit decimal*** arithmetic (base 10, with the left-most digit the sign digit), and the others using ***eight-bit binary*** arithmetic (base 2, with the left-most bit the sign bit). If the results are longer than four digits or longer than eight bits, that means a carry was also generated. For each sum tell me if it exhibits ***unsigned overflow***, ***signed overflow***, ***both***, or ***neither***?

A. (3 points) Decimal: 3408 + 5925 = 9333 **neither**

B. (3 points) Decimal: 3408 + 1769 = 5177 **signed**

C. (3 points) Decimal: 3408 + 8104 = 11512 **unsigned**

D. (3 points) Binary: 01011011 + 00101111 = 10001010 **signed**

E. (3 points) Binary: 11110011 + 11100010 = 111010101 **unsigned**

<4> 15 Points – Show the ***decimal*** (base 10) value of the eight-bit binary number 10001001 interpreted in each of the following ways:

A. (3 Points) Interpreted as Unsigned 10001001 = **137**

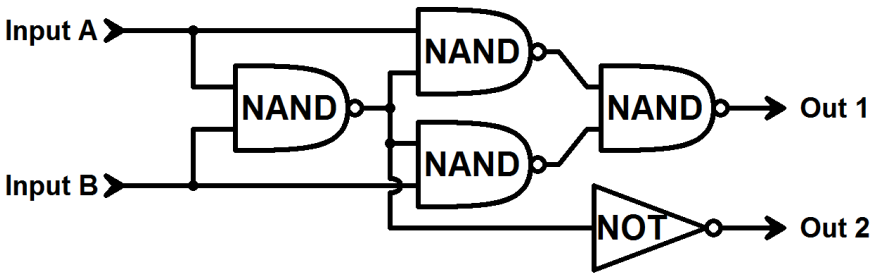
B. (3 Points) Interpreted as Sign & Magnitude 10001001 = **-9**

C. (3 Points) Interpreted as One's Complement 10001001 = **-118**

D. (3 Points) Interpreted as Two's Complement 10001001 = **-119**

E. (3 Points) Interpreted as BCD 10001001 = **89**

<5> 15 Points – Trace the following circuit and show its outputs (fill in the table) for all possible input behaviors.



Input		Outputs	
A	B	Out 2	Out 1
0	0	**0**	**0**
0	1	**0**	**1**
1	0	**0**	**1**
1	1	**1**	**0**

5 Points EXTRA CREDIT – What is the functional ***purpose*** of this circuit?

It is another version of the **half-adder**, for adding two bits together.

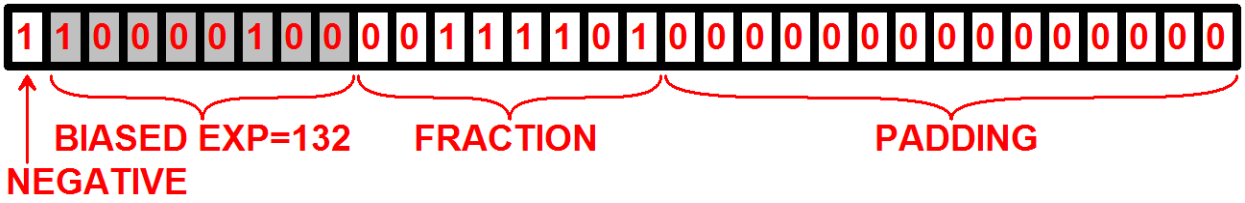
<6> 15 Points – Convert the decimal number **-39.625** into True Binary, Binary Scientific, and then into Single Precision Floating Point. Single precision is 32 bits total, with 1 bit for sign, 8 bits for biased exponent, and 23 bits for the mantissa. Show your work for partial credit.

True Binary (4 points): **-100111.101**

Binary Scientific (4 points): **-1.00111101**×2^{5}

True Exponent + Bias, in Decimal (2 Points): 5(exp) + 127(bias) = **132**

Single Precision (5 points, fill in *every* box with either a 0 or a 1):



<7> 10 Points – Consider the binary number 1001010.1101 (without converting it to decimal):

A. What is the binary representation of this number ***multiplied by two***?

Shifted Left: 10010101.101

B. What is the binary representation of this number ***divided by two***?

Shifted Right: 100101.01101

<8> 10 Points – SHORT ESSAY – Pick one of the following questions about representations, and write your answers on the back of this page. Do both for +5 points extra credit. Please do not write more than four to five sentences in total for either question. In your chosen question, think about why the representation is the way that it is, what alternatives might exist, and what the advantages and disadvantages of each representation may be. These representations were chosen for a reason over all the alternatives – why?

A. Most fire hydrants in the United States have an access bolt with five sides, instead of four or six like most traditional bolts. Why?

B. The Inuit people of Greenland use carved wooden maps, instead of maps drawn on paper. Why?

(A) Most bolts are either six-sided or four-sided to make wrenches able to grab the bolt head securely, and be able to move from position to position easily. More than six sides makes the shape of the bolt head asymptotically approach a circle, hard to grab securely. Three sided bolts are easy to grab, but would make the corners too sharp. Five sides is a compromise, which works as well as either four or six.

However, no consumer bolts are five-sided, so five-sided wrenches are uncommon. Firefighters do have such wrenches as part of their standard tool-kit.

The combination makes fire hydrants able to be deployed publically, with little danger of non-firefighters being able to open them.

(B) Paper is difficult to obtain, paper is not very sturdy when wet, and paper maps have to be unfolded in order to be studied. In contrast, wooden maps are rugged, and can be used and studied by feel in cold weather without removing them from inside mittens. They also provide a tactile sense of 3D topology not possible with flat paper.