CMPSCI 145 MIDTERM #1

SOLUTION KEY

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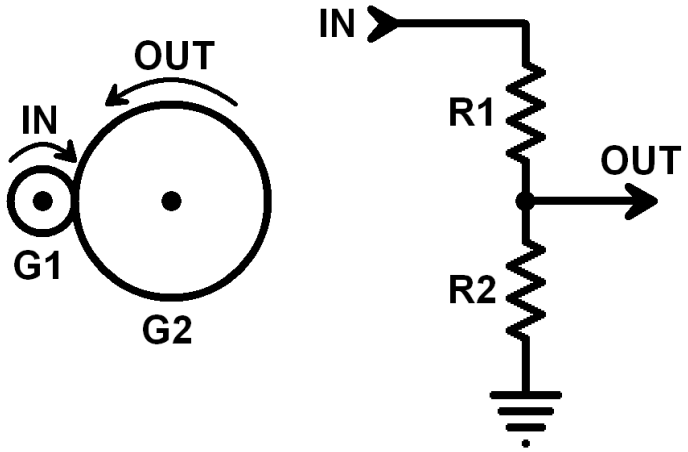
<1> 10 Points – Examine the following diagram of two systems, one involving gears and the other involving resistors. Resistor **R1** has a resistance of 2000 Ohms, and **R2** has a resistance of 1000 Ohms. Gear **G2** has 72 teeth.



A. (2 points) What is the voltage of the output of the resistor divider, relative to any arbitrary input voltage?

The output voltage is ⅓ of the input voltage.

The total resistance is 2000 + 1000 = 3000 Ohms. The output voltage is measured across the 1000 Ohm resistor, which is ⅓ of the total resistance, so it must be ⅓ of the input voltage. Mathematically, if the input voltage is E volts, then by Ohm's Law ($I = {}^{E}\!/_{R}$) the current through both resistors is ($^{E}\!/_{3000}$), and by Ohm's Law again ($E = I \times R$) the voltage across R2 is thus ($^{E}\!/_{3000}$) $\times 1000 = \frac{1}{3}E$.

B. (3 points) How many teeth does gear **G1** have so that the spin rate of **G1** relative to **G2** (ignoring differences in direction) is the same as the relationship of the output voltage to the input voltage in the resistor divider?

24 teeth.

From the diagram, G1 will spin faster than G2. Because of our answer to part A, we will want G2 to spin at ⅓ the rate of G1. Thus, for every revolution of G1 we want G2 to go around ⅓ of a revolution. Therefore, G1 must have ⅓ of the number of teeth of G2. G2 has 72 teeth, so therefore G1 has ⅓×72 = 24 teeth. If the problem was interpreted backwards, without looking at the diagram, a case could be made for G1 having 3 times the number of teeth of G2, or 216.

C. (5 points) Short Answer – How are these two systems similar? How are they different? Explain your answer in just a sentence or two.

Both are **analog systems** that divide their input values by three. Within the limits of the respective systems, there are an **infinite number of legal input values** (spin rates, voltages), and the **output values are always a proportion of that**. They use **different technologies** to achieve the result, one mechanical and one electrical, so they can only be applied to the systems for which they were designed.

<2> 10 Points – In the Op-Amp adder on page 81, R1=R2=R3=1000Ω, R*f*=2000Ω, V1=2.1 volts, V2=3.2 volts, and V3=1.6 volts. What is the output voltage? Show your work for partial credit.

–13.8 volts.

The equation on page 81 is $Vout = -Rf\left(\left({}^{V1}/_{R1}\right) + \left({}^{V2}/_{R2}\right) + \left({}^{V3}/_{R3}\right)\right)$. Since the three input resistors are all the same value *R*, this equation can be rewritten as and simplified to $Vout = -Rf\left({}^{(V1+V2+V3)}/_{R}\right)$, and then to $Vout = -\left({}^{Rf}/_{R}\right)\times(V1+V2+V3)$, or equivalently $Vout = -2(V1+V2+V3)$. Substituting the actual numbers in gives the computation: $Vout = -2(2.1+3.2+1.6) = -2(6.9) = -13.8$ volts.

<3> 15 Points – Consider the sums below, some using ***four-digit decimal*** arithmetic (base 10, with the left-most digit the sign digit), and the others using ***eight-bit binary*** arithmetic (base 2, with the left-most bit the sign bit). If the results are longer than four digits or longer than eight bits, that means a carry was also generated. For each sum tell me if it exhibits ***unsigned overflow***, ***signed overflow***, ***both***, or ***neither***?

A. (3 points) Decimal: 2478 + 1460 = ~~1738~~ 3938 **neither**

B. (3 points) Decimal: 2478 + 5995 = 8473 **neither**

C. (3 points) Decimal: 2478 + 7910 = 10388 **unsigned overflow**

D. (3 points) Binary: 01010011 + 00101110 = 10000001 **signed overflow**

E. (3 points) Binary: 11110001 + 11100010 = 111010011 **unsigned overflow**

In this problem I've done the sums for you, and even though I had the first decimal sum incorrect, your answer would not have been affected as a result. In all cases, a carry out indicates an unsigned overflow (C and E). Signed overflow only happens in addition when adding two positives with a negative result, or two negatives with a positive result. This happens in D, where two positives (leading bit = 0) have a negative result (leading bit = 1). Note that signed overflow *cannot* happen when adding a positive to a negative (B and C), even if there is a carry out (C). Also note that E is fine under a signed interpretation, even with a carry out, as you are adding a negative to a negative, with a negative result.

<4> 15 Points – Show the ***decimal*** (base 10) value of the eight-bit binary number 10000111 interpreted in each of the following ways:

A. (3 Points) Interpreted as Unsigned 10000111 = **135**

B. (3 Points) Interpreted as Sign & Magnitude 10000111 = **-7**

C. (3 Points) Interpreted as One's Complement 10000111 = **-120**

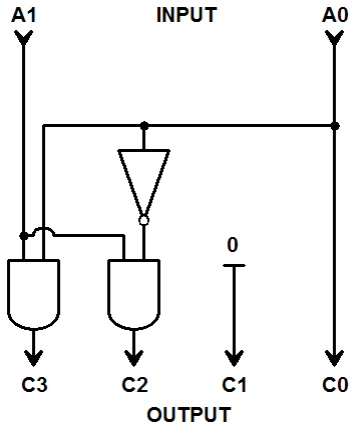
D. (3 Points) Interpreted as Two's Complement 10000111 = **-121**

E. (3 Points) Interpreted as BCD 10000111 = **87**

All the numbers that start with a 1-bit are negative in the signed representations (B, C, and D), so for those we have to figure out what they are the negative of (i.e., compute their absolute value, convert to decimal, and stick a minus sign in front). For sign-and-magnitude it is just the remaining bits. For one's complement we must flip the bits and interpret that bit pattern. For two's complement we flip the bits and add 1, and then interpret the result.

In BCD, we simply convert each 4-bit group to decimal separately: 1000 = 8, 0111 = 7.

<5> 15 Points – Trace the following circuit and show its outputs (fill in the table) for all possible input behaviors.



Input		Output			
A1	A0	C3	C2	C1	C0
0	0	0	0	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	1	0	0	1

5 Points EXTRA CREDIT – What is the functional ***purpose*** of this circuit?

The 4-bit result is the **square** of the 2-bit input. In decimal, the unsigned binary inputs are the values 0, 1, 2, 3, and the outputs are 0, 1, 4, 9, the squares of the inputs.

<6> 15 Points – Convert the decimal number **-42.125** into True Binary, Binary Scientific, and then into "Three Quarter Precision" Floating Point. Three-quarter precision is 24 bits total, following a 1-7-16 pattern (sign, 7 bits for biased exponent, 16 bits for the mantissa). You'll need to determine the appropriate bias value for 7 bits. Show your work for partial credit.

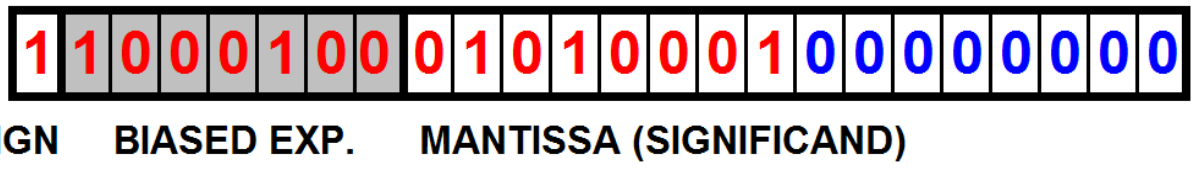
True Binary (4 points): **-101010.001**

Binary Scientific (4 points): **-1.01010001** $\times 2^{5}$

Bias for ¾ Precision (2 points): $\mathbf{2^{7-1}-1 = 2^{6}-1 = 64-1 = 63}$

(Biased Exponent = 63 + 5 = 68, or 1000100)

Three-Quarter Precision (5 points):



Zero-Bits in blue are padding bits needed to fill out the storage after the mantissa was inserted.

<7> 10 Points – Consider the binary number 1101011.0101 (without converting it to decimal):

A. What is the binary representation of this number ***multiplied by two***?

11010110.101 (shift left)

B. What is the binary representation of this number ***divided by two***?

110101.10101 (shift right)

<8> 10 Points – SHORT ESSAY – Pick one of the following questions about representations, and write your answers on the back of this page. Do both for +5 points extra credit. Please do not write more than four to five sentences in total for either question. In your chosen question, think about why the representation is the way that it is, what alternatives might exist, and what the advantages and disadvantages of each representation may be. These representations were chosen for a reason over all the alternatives – why?

A. Most fire hydrants in the United States have an access bolt with five sides, instead of four or six like most traditional bolts. Why?

Most bolts are either six-sided or four-sided to make wrenches able to grab the bolt head securely, and be able to move from position to position easily. More than six sides makes the shape of the bolt head asymptotically approach a circle, hard to grab securely. Three sided bolts are easy to grab, but would make the corners too sharp. Five sides is a compromise, which works as well as either four or six.

However, no consumer bolts are five-sided, so five-sided wrenches are uncommon. Firefighters do have such wrenches as part of their standard tool-kit.

The combination makes fire hydrants able to be deployed publically, with little danger of non-firefighters being able to open them.

B. The Inuit people of Greenland use carved wooden maps, instead of maps drawn on paper. Why?

Paper is difficult to obtain, paper is not very sturdy when wet, and paper maps have to be unfolded in order to be studied. In contrast, wooden maps are rugged, and can be used and studied by feel in cold weather without removing them from inside mittens. They also provide a tactile sense of 3D topology not possible with flat paper.