A Minimum Variance Method for Problems in Radio Antenna Placement

M. V. Panduranga Rao¹, Amrit Lal Ahuja¹, Srinivasan Iyengar², Kavita Iyer², Ranu Khade², Sachin Lodha¹, Dinesh Mehta³, Balasz Nagy³

¹Tata Research Development and Design Centre, Pune, India
²College of Engineering Pune, Pune, India
³Colorado School of Mines, Golden, USA

Abstract. Aperture synthesis radio telescopes generate images of celestial bodies from data obtained from several radio antennas. Placement of these antennas has always been a source of interesting problems. Often, several potentially contradictory objectives like good image quality and low infra-structural cost have to be satisfied simultaneously.

In this paper, we propose a general Minimum Variance method that focuses on obtaining good images in the presence of limiting situations. We show its versatility and goodness in three different situations: (a) Placing the antennas on the ground to get a target Gaussian UV distribution (b) Staggering the construction of a telescope in the event of staggered budgets and (c) Whenever available, using the mobility of antennas to obtain a high degree of fault tolerance.

1. Introduction

An important problem in interferometric radio-astronomy is to find the antenna placement on the ground that generates a required UV-distribution. A lot of interesting approaches have been suggested in the past for this problem and its variants (cf. Boone 2001 and 2002, Karastergiou et al 2006 and Su et al, 2004).

In this paper we look at three important problems in radio antenna placement that arise chiefly from the standpoint of astronomic merit:

- In general, a Gaussian distribution of UV points in the radial direction and a uniform distribution in the azimuthal direction is acceptable for Gaussian beams (see Boone 2002). A simple calculation shows that a uniformly random XY placement of antennas yields a radially tapering UV distribution. However, it may not be sufficiently close to Gaussian. How do we rectify this?

- Construction of large telescopes are likely to be staggered because of financial and logistical reasons. As such, can we come up with an antenna placement schedule such that good quality observations can be made all the time?

- The third problem is motivated by the setting of mobile antennas in Karastergiou et al, 2006. Can we utilize mobility of antennas among several pads to salvage image quality in the event of failure of some antennas?
We propose a single Minimum Variance Method (MVM) that tackles the above seemingly different problems. Informally, this technique involves choosing $N$ out of $M$ (where $M \geq N$) possible locations for placing the antennas. To achieve this, we start by placing an antenna on each of the $M$ locations. Then, we iteratively remove $M-N$ antennas, one at a time, such that we stay “close” to the target UV distribution. Our solution takes $O((M-N) \cdot M^3)$ time, which is an improvement over brute force solutions. A brute force solution would involve comparing distributions that result from all $\binom{M}{M-N}$ choices for antenna removal.

We report our experiments on a random placement scheme and on the placement data of 120 antennas/stations of the proposed Australian Square Kilometer Array (SKA).

In Section 2., we present the MVM. Section 3. shows how the technique applies to the three problems stated above, along with the experimental results. Some relevant literature is provided in the references section.\(^1\)

2. The Minimum Variance Method

The basic idea of the MVM is as follows. First, we divide the UV-plane into $p$ regions. Division is necessary because we wish a distributed removal of points so that some portion of the UV plane is not denuded of UV points. The desired UV distribution dictates the number of UV points each region should hold. We differ a comment of the shape and number of regions to the end of this section. We note, however, that the MVM can accommodate any scheme for division of the UV plane in principle.

The MVM starts with $M > N$ antennas and removes $M-N$ antennas, one at a time, in an iterative fashion. Suppose there are $p^{[r]}$ regions and $M^{[r]}$ antennas at the start of the $r^{th}$ iteration. Then, $M^{[r]} - 1$ UV points will be dropped as a result of the removal of one antenna during the $r^{th}$ iteration.

Depending on the required UV distribution, we would like the removal of $R^{[r]}_i$ UV points from the $i^{th}$ region in the $r^{th}$ iteration. However, in general, the number of UV points dropped from the region $i$ due to removal of antenna $j$ would be different, say $w_i(j)$. We capture the discrepancy between the actual and ideal UV points dropped in every region by removing the antenna $j$ with

$$Var(j) = \sum_{i=1}^{p^{[r]}} (w_i(j) - R^{[r]}_i)^2.$$ 

We now drop the antenna that has the least $Var(.)$ value. In short, the logic for deletion of the $N$ antennas is this:

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while there are more than $N$ antennas remaining do
   for each remaining antenna $j$
      Calculate $Var(j)$
   end for
   Remove the antenna that has the least $Var(.)$.
end while
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\(^1\)In this version, we omit a detailed literature survey, details of various results, results of our approach to the third problem, and relevant XY and XZ figures due to paucity of space. For these, please see the full version of the paper (arXiv:0901.4901).
This antenna removal logic can be tweaked for different problems. In general, this procedure would take \(O((M - N)M(M^2 + p)) = O((M - N) \cdot M^3)\) steps.

But for a rotation for changing coordinates, a UV point is defined as \(u = \frac{X_1 - X_2}{\lambda}\) and \(v = \frac{Y_1 - Y_2}{\lambda}\) for every pair \((X_1, Y_1)\) and \((X_2, Y_2)\) of XY coordinates. Therefore it is sufficient to work with the distribution that the XY coordinate differences may follow. In all our experiments, without loss of generality, we take \(\lambda = 1\).

There are at least two natural choices for the shape of the regions into which the UV plane is to be divided. One is the circle due to its radial symmetry. The second is the ellipse because of the fact that the UV tracks are elliptical in general, and that in several earth rotation aperture synthesis telescopes, a prolonged coverage is desirable. The number of regions can either remain the same (i.e., \(p\)) or vary through the execution of the MVM. Letting the number of regions remain constant makes all intermediate configurations mimic the final, while varying it will make the configuration in an iteration mimic that in the previous iteration.

3. Applications

3.1. Problem 1: UV to XY

The UV distribution that is desired in most cases is a Gaussian along the radial and uniform along the azimuthal direction. It is easy to show that a random distribution of antennas on the ground, that is, the XY-plane, gives a tapering distribution in the UV-plane. In other words, a random XY distribution induces more UV points corresponding to short XY distance pairs. With this as a starting point, we apply the Minimum Variance Method to arrive at the target Gaussian.

**Given:** (i) A geography, that is, dimensions of the land on which to place antennas. (ii) The number of antennas that are to be installed and (iii) The amplitude \(a_f\) of the desired Gaussian UV plane.

**Goal:** To find a placement of \(N\) antennas that yields a Gaussian UV distribution centered at the origin having a standard deviation \(\sigma = B/4\), where \(B\) is the largest inter UV point distance, and amplitude \(a_f\).

**Approach:** We start with an overkill of antennas. Instead of the required \(N\), we start with \(M \approx 2N\) antennas and place them randomly on the XY-plane. Our random choice results in a distribution of points on the UV-plane that deviates from the desired Gaussian.

We proceed by drawing a number of concentric circles on the UV plane with radii in arithmetic progression of common difference \(d\). The radii and the number of circles are determined by \(a_f\), \(\sigma\), \(N\) and \(B\). It is then possible to plot a density histogram with the number of points in the \(i^{th}\) annulus at the \(y\)-coordinate, and determine the area under the plot. Let the removal of an antenna \(j\) at the \(r^{th}\) iteration result in a fall of \(w_i(j)\) in the \(i^{th}\) \(y\)-coordinate.

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\(^2\)We work with \(\sigma = B/4\), since the area under the gaussian distribution curve before the 4\(\sigma\) coordinate closely approximates the total area.
This might differ from the ideal drop in the $y$ coordinate of a Gaussian with a diminished peak and S.D. $\sigma$. This discrepancy is to be minimized. If $q_i^{[r-1]}$ be the $y$-coordinate corresponding the $i^{th}$ at the end of the $r-1^{th}$ iteration, $Var(j)$ turns out to be $\sum_{i=1}^{p}(w_i(j) - (q_i^{[r-1]} - \frac{d(M-r)(M-r-1)/2}{\sigma\sqrt{\pi}} \cdot e^{-\frac{d(i-1/2)^2}{2\sigma^2}}))^2$.

For the experiment, we set a goal of generating a Gaussian distribution on the UV plane using $N = 28$ antennas with $a_f = 70$. We start with an initial set of $M = 48$ antennas that are generated randomly. The geography that we use, and the randomly generated antenna positions yields $B \approx 11180$ wavelengths. Then, $d \approx 6470$ wavelengths.

Figure 1(a) shows the UV-distribution generated by these antennas. The Density Histogram of the distribution is shown in Figure 1(c). Notice that the histogram deviates from the desired Gaussian at several places. Figure 1(d) is the cropped histogram arrived at finally. Observe that this histogram is closer to Gaussian than the initial one. Figure 1(b) shows the final UV distribution.

Figure 1. UV to XY

3.2. Problem 2: Staggered Construction

A telescope of $M$ antennas has been proposed with all the sites identified. However, the telescope construction has to be staggered and it has to be built in phases since the funding becomes available in small chunks over a time period. Suppose that we are given a budget of $N$ antennas for Phase 1. Which $N$ of the total $M$ antennas should we construct in Phase 1?

Ideally one would like to start making quality observations after completion of Phase 1 itself which means that the UV-distribution that is generated by
Phase 1 antennas should be a good approximation of the final UV-distribution that we would get after placing all \( N \) antennas.

In this case, we wish to remove the same number of UV points from each region during every iteration. Thus, \( R_i^{[r]} = \frac{M_i^{[r]} - 1}{p_i^{[r]}} \), and \( \text{Var}(j) = \sum_{i=1}^{p_i^{[r]}} (w_i(j) - \frac{M_i^{[r]} - 1}{p_i^{[r]}})^2 \).

We ran the experiment on the scaled down version of the Australian SKA for \( M = 120 \) antenna locations. Figures 2(a) and 2(b) show the corresponding UV-planes. The hour angle used is 0° and the declination −30°.

![Figure 2](image1.png)

(a) The UV plane–initial and 40 removed antennas
(b) The UV plane–initial and 80 removed antennas

Figure 2. Results for the proposed Australian SKA configuration.

### 3.3. Problem 3: Mobile Antennas

**Statement:** Suppose there are \( L \) antenna pads, but only \( M \leq L \) mobile antennas. Consider a demand for the \( M \) antennas. However, only \( N \) out of these \( M \) can be allocated (say, because of maintenance reasons, or being used for other experiments). Our problem is to choose \( N \) out of the \( M \) antenna pads that best approximate the desired UV pattern generated by the \( M \) antennas. Having done that, the antennas can be shifted to the recommended \( N \) pads.

The parameters set for the previous problem in Subsection 3.2. carry through exactly. We use \( M = 24 \) and \( N = 19 \) for the experiment. Using the MVM, one can recommend which 19 pads to be used.

**Conclusions:** We proposed a Minimum Variance Method for the radio antenna placement problem and demonstrated its goodness, versatility and efficiency.

### References


