CS 312: Algorithms **Spring 2018**

Discussion 4

Your Name: ____________________ Collaborators: ____

You will be randomly assigned groups to work on these problems in discussion section.

Problem 1. Graphs, proofs. The degree of node v in a graph is the number of edges incident to v (equivalently, the number of neighbors of v). Consider the claim and proof below. Identify at least two statements or phrases in the proof that are imprecise or not fully justified.

Claim: Let G be an undirected graph where every node has degree two or more. Then G has a cycle.

Proof: Since every node has two or more neighbors, we can construct a path starting from any node and keep going on that path without ever turning around. The path must eventually loop back on itself, so there is a cycle.

Now discuss with your group how you could make this more precise.

Problem 2. BFS. Recall the definition of BFS. Here is a concrete description from the book pp. 90-91.

- Explore out from node s by distance
- Layer i = nodes at distance i from s
- Returns a tree T
- Runs in $O(n + m)$ time.

Run BFS on the examples drawn on the board. Clearly identify the tree T, the layers $L_0, L_2, \ldots$, and the non-tree edges.

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BFS($s$):
  Set Discovered[$s$] = true
  Set Discovered[$v$] = false for all other $v$
  Set $L[0] = \{s\}$
  Set $i = 0$                                    ▷ layer counter
  Set $T = \emptyset$                            ▷ BFS tree
  while $L[i]$ is not empty do
      Initialize an empty list $L[i+1]$          ▷ next layer
      for each node $u \in L[i]$ do
          for each edge $(u, v)$ incident to $u$ do
              if Discovered[$v$]= false then
                  Set Discovered[$v$] = true
                  Add edge $(u, v)$ to $T$
                  Add $v$ to $L[i+1]$
              end if
          end for
      end for
      Set $i = i + 1$
  end while
```

Problem 3. (Graph Traversal) K&T Chapter 3, Exercise 4.

This is the “butterfly problem” from the homework. Hint: try to modify BFS.