CS 312: Algorithms	**Spring 2018**

Homework 10

Your Name: ____________________________

Collaborators and sources: ____

You may work in groups, but you must write solutions yourself. List collaborators on your submission.

To prove that a problem X is NP-complete, don't forget to:

- Show that $Y \leq_P X$ where Y is a known NP-complete problem
- Prove that the reduction is correct (it outputs YES on a "yes" instance, and NO on a "no" instance).
- Prove that X belongs to NP.

Most of the work is done after you give the reduction and prove it is correct, which you already know how to do. In Problems 2 and 3, the hint tells you which NP-complete problem Y to use. After Monday, you will understand how to argue that X is in NP—this is very short.

Submission instructions. This assignment is due by noon on Thursday, December 6 in Gradescope (as a pdf file). Please review the course policies on the course home page about Gradescope submissions.

1. **(10 points) Interval Scheduling.** For each of the two questions below, decide whether the answer is (i) "Yes", (ii) "No" or (iii) "Unknown, because it would resolve the question of whether P = NP". Explain your answer. (Hint: don't use answer (ii) "No".)

   (a) Let's define the decision version of the Interval Scheduling Problem from Chapter 4 as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

   Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

   (b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling

2. **(10 points) Diverse Subset.** A store trying to analyze the behavior of its customers will often maintain a two-dimensional array A, where the rows correspond to its customers and the columns correspond to the products it sells. The entry $A[i, j]$ specifies the quantity of product j that has been purchased by customer i.

   Heres a tiny example of such an array A.

	detergent	beer	diapers	cat litter
Raj	0	6	0	3
Alanis	2	3	0	0
Chelsea	0	0	0	7

   One thing that a store might want to do with this data is the following. Let us say that a subset S of the customers is *diverse* if no two of the of the customers in S have ever bought the same product (i.e., for each product, at most one of the customers in S has ever bought it). A diverse set of customers can be useful, for example, as a target pool for market research.

   We can now define the DIVERSE-SUBSET Problem as follows: Given an $m \times n$ array A as defined above, and a number $k \leq m$, is there a subset of at least k of customers that is diverse?

   Show that DIVERSE-SUBSET is NP-complete. (Hint: reduce from INDEPENDENT-SET.)

3. **(10 points) Hitting Set.** Consider a set $A = \{a_1, \dots, a_n\}$ and a collection $B_1, \dots, B_m$ of subsets of A (i.e., $B_i \subseteq A$ for all i). We say that $H \subset A$ is a *hitting set* if H contains at least one element from each B_i, that is $H \cap B_i$ is non-empty for all i (so H "hits" all the sets B_i).

   The HITTING-SET problem is the following: Given a set $A = \{a_1, \dots, a_n\}$, subsets $B_1, \dots, B_m \subset A$, and a number k, is there a hitting set $H \subset A$ of size at most k?

   Prove that HITTING-SET is NP-Complete. (Hint: reduce from VERTEX-COVER.)