CMPSCI 311: Introduction to Algorithms **Spring 2017**

Discussion 5

Your Name: _____ Collaborators: _____

You will be randomly assigned groups to work on these problems in discussion section. List your group members on your worksheet and turn it in at the end of class.

Problem 1. Construction. You're running a company contracted to build dorms at UMass Amherst. Each dorm has a basement, a living space, and a roof. These three components must be built in that order. The college has designed each dorm and each dorm component will need a certain predictable amount of time to build.

Dorm	Basement Time	Living Space Time	Roof Time
Anderson	2 days	2 days	3 days
Burleson	3 days	2 days	2 days
Ciesielski	4 days	3 days	2 days

For legal reasons, your company can only be working on one basement at any given time. However, your company can work on any number of living spaces and roofs in parallel with a basement.

You want to build your dorms in an order such that the last dorm to finish being built will be finished as soon as possible.

(a) Say your contract requires you to build Anderson, Burleson, and Ciesielski. What is an ordering where the last dorm to finish finishes as soon as possible?

(b) Consider the general problem where there are n dorms and for dorm i ($i = 1, 2, \cdots n$), time to build basement, living space and roof are respectively b_i, l_i and r_i. Which of the following greedy rules produces an optimal solution?

- Order dorms by increasing basement time b_i
- Order dorms by decreasing basement time b_i
- Order dorms by increasing living space time plus roof time $l_i + r_i$
- Order dorms by decreasing living space time plus roof time $l_i + r_i$

(c) Now let's prove that the rule you selected is optimal by an exchange argument: Let A be the greedy solution. If O is an optimal solution and $O \neq A$, argue that you can modify O to get a new solution O' that is closer to A and no worse than O (so still optimal). Define an inversion as pair of dorms (i, j) that are out of order with respect to greedy solution.

- Show that if O has an inversion it has a consecutive inversion.

Now suppose that O is an optimal solution with a consecutive inversion i and j. If we swap i and j, it is clear that we get a new schedule O' with one less inversion.

- Show that the overall finish time of O' is no later than O.

This means that O' is still optimal, which completes the exchange argument.

(d) What is the run-time of this algorithm?

Problem 2. Trees. A binary tree is one where each node has at most two children. Prove that in any binary tree the number of nodes with two children is exactly one less than the number of leaves.