CMPSCI 311: Introduction to Algorithms **Spring 2017**

Homework 6

Your Name: ____________________________

Collaborators and sources: ____

You make work in groups, but you must write solutions yourself. List collaborators on your submission.

If you are asked to design an algorithm, please provide: (a) the pseudocod for the algorithm or a precise description of the algorithm in words, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submission instructions. This assignment is due by 11:59pm on Tuesday, May 2 in Gradescope (as a pdf file). Please review the course policy about Gradescope submissions on the course website.

1. **(10 points) K&T Chapter 7, Exercise 7.** Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible *base stations*. We'll suppose there are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also k base stations; the position of each of these is specified by (x, y) coordinates as well.

   For each client, we wish to connect it to exactly one of the base stations. Our choice of connections is constrained in the following ways. There is a range parameter r—a client can only be connected to a base station that is within distance r. There is also a load parameter L—no more than L clients can be connected to any single base station.

   Your goal is to design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station, subject to the range and load conditions in the previous paragraph.

2. **(10 points) K&T Chapter 7, Exercise 8**. Statistically, the arrival of spring typically results in increased accidents and increased need for emergency medical treatment, which often requires blood transfusions. Consider the problem faced by a hospital that is trying to evaluate whether its blood supply is sufficient. The basic rule for blood donation is the following. A persons own blood supply has certain *antigens* present (we can think of antigens as a kind of molecular signature); and a person cannot receive blood with a particular antigen if their own blood does not have this antigen present. Concretely, this principle underpins the division of blood into four types: A, B, AB, and O. Blood of type A has the A antigen, blood of type B has the B antigen, blood of type AB has both, and blood of type O has neither. Thus, patients with type A can receive only blood types A or O in a transfusion, patients with type B can receive only B or O, patients with type O can receive only O, and patients with type AB can receive any of the four types.

   (a) Let s_O, s_A, s_B, and s_{AB} denote the supply in whole units of the different blood types on hand. Assume that the hospital knows the projected demand for each blood type d_O, d_A, d_B, and d_{AB} for the coming week. Give a polynomial-time algorithm to evaluate if the blood on hand would suffice for the projected need.

   (b) Consider the following example. Over the next week, they expect to need at most 100 units of blood. The typical distribution of blood types in U.S. patients is roughly 45 percent type O, 42 percent type A, 8 percent type B, and 3 percent type AB. The hospital wants to know if the blood supply it has on hand would be enough if 100 patients arrive with the expected type distribution. There is a total of 105 units of blood on hand. The table below gives these demands, and the supply on hand.

blood type	supply	demand
O	50	45
A	36	42
B	11	10
AB	8	3

Is the 105 units of blood on hand enough to satisfy the 100 units of demand? Find an allocation that satisfies the maximum possible number of patients. Use an argument based on a minimum-capacity cut to show why not all patients can receive blood. Also, provide an explanation for this fact that would be understandable to the clinic administrators, who have not taken a course on algorithms. (So, for example, this explanation should not involve the words *flow*, *cut*, or *graph* in the sense we use them in this class.)

3. **(10 points) K&T Chapter 8, Exercise 2.** A store trying to analyze the behavior of its customers will often maintain a two-dimensional array A, where the rows correspond to its customers and the columns correspond to the products it sells. The entry $A[i, j]$ specifies the quantity of product j that has been purchased by customer i.

   Heres a tiny example of such an array A.

	detergent	beer	diapers	cat litter
Raj	0	6	0	3
Alanis	2	3	0	0
Chelsea	0	0	0	7

   One thing that a store might want to do with this data is the following. Let us say that a subset S of the customers is diverse if no two of the of the customers in S have ever bought the same product (i.e., for each product, at most one of the customers in S has ever bought it). A diverse set of customers can be useful, for example, as a target pool for market research.

   We can now define the Diverse-Subset Problem as follows: Given an $m \times n$ array A as defined above, and a number $k \leq m$, is there a subset of at least k of customers that is diverse?

   Show that Diverse-Subset is NP-complete.

4. **(10 points) K&T Chapter 8, Exercise 5.** Consider a set $A = \{a_1, \ldots, a_n\}$ and a collection $B_1, \ldots, B_m$ of subsets of A (i.e., $B_i \subseteq A$ for all i). We say that $H \subset A$ is a *hitting set* for the collection if H contains at least one element from each B_i, that is $H \cap B_i$ is non-empty for all i (so H "hits" all the sets B_i).

   The Hitting-Set problem is the following: Given a set $A = \{a_1, \ldots, a_n\}$, subsets $B_1, \ldots, B_m \subset A$, and a number k, is there a hitting set $H \subset A$ of size at most k?

   Prove that Hitting-Set is NP-Complete.

5. **(10 points) K&T Chapter 8, Exercise 17.** You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The Zero-Weight-Cycle Problem is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that Zero-Weight-Cycle is NP-complete.

6. **(0 points).** How long did it take you to complete this assignment?