# CS 103: Lecture 8 Matching Markets

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## Announcements

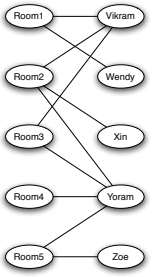
- Happy Mountain Day / Fall Break
- HW 3 assigned next week

## Plan for today

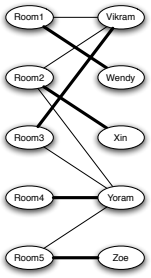
- Finish second-price auctions
- Matching markets
  - Perfect matchings
  - Constricted sets
  - Matching Theorem

## Matching Market

Example: room preferences

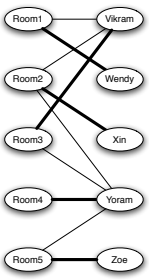



Bipartite graph




Perfect matching

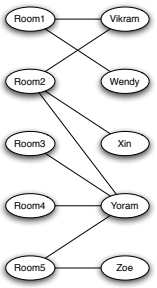
## Matching Market



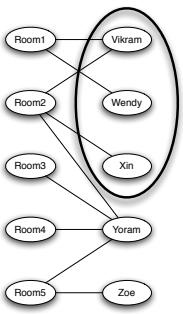

**Bipartite graph**
- two types of nodes
- edges from one type to other

**Perfect matching**: each node L assigned to a single node on the R *to which it is connected by an edge*
- Baristas / shifts
- Classes / rooms
- Planes / gates

## Perfect Matchings

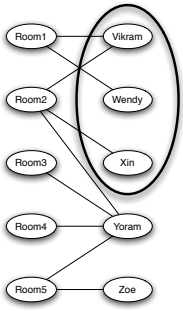



Perfect matching?




No: three people / two rooms

## Constricted Set




$S$ = set of nodes on the right
$N(S)$ = set of left nodes with an edge to a node in $S$

$S$ is **constricted** if $N(S)$ is smaller than $S$

Significance: if bipartite graph has a constricted set, then it has no perfect matching. Can there be other reasons?

## Exercise

**Draw examples on board**

Find a perfect matching or constricted set

## Matching Theorem (König 1931 / Hall 1935)

**Matching Theorem**: if a bipartite graph has no perfect matching, then it must contain a constricted set.

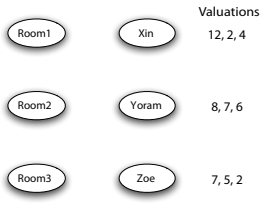
**Discussion**: very clean conceptual picture for resource allocation!

- perfect matching: problem solved!
- constricted set: simple explanation why it is impossible

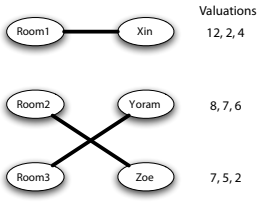
What is missing?

- 0/1 preferences: like a room or not
- want finer-grained notion of preference

## Valuations

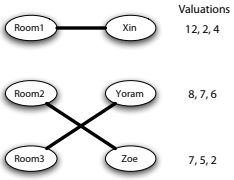



Valuations




Value $12 + 6 + 5 = 23$

## Optimal Assignment



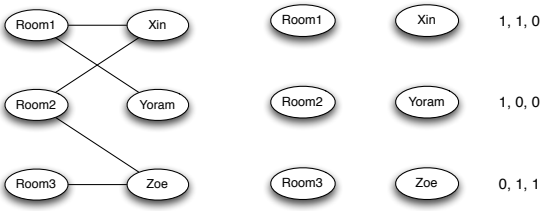

**Total valuation** of an assignment: sum of each individual's valuation

**Optimal assignment**: assignment that maximizes total valuation

Is this assignment optimal? Yes.

Do people get most preferred rooms? No.

## Optimal Assignment / Perfect Matching




Optimal assignment problem generalizes perfect matching