Maintaining Bernoulli Samples Over Evolving Multisets

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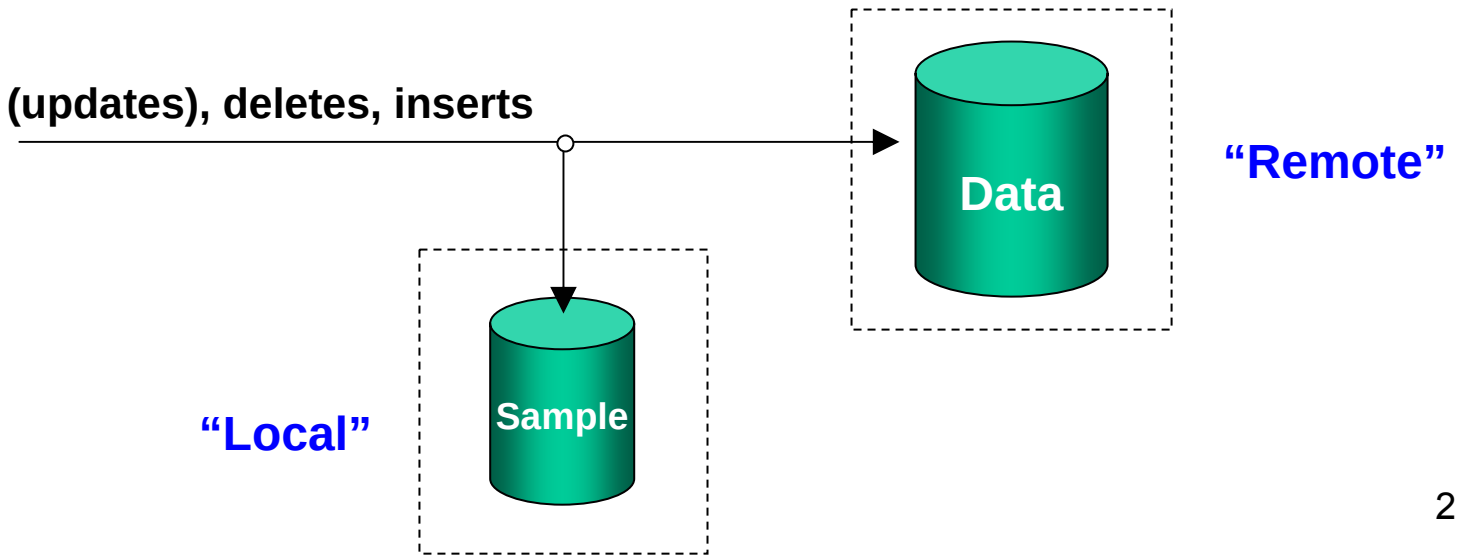
Motivation

- Sampling: crucial for information systems
  - Externally: quick approximate answers to user queries
  - Internally: Speed up design and optimization tasks
- Incremental sample maintenance
  - Key to instant availability
  - Should avoid base-data accesses



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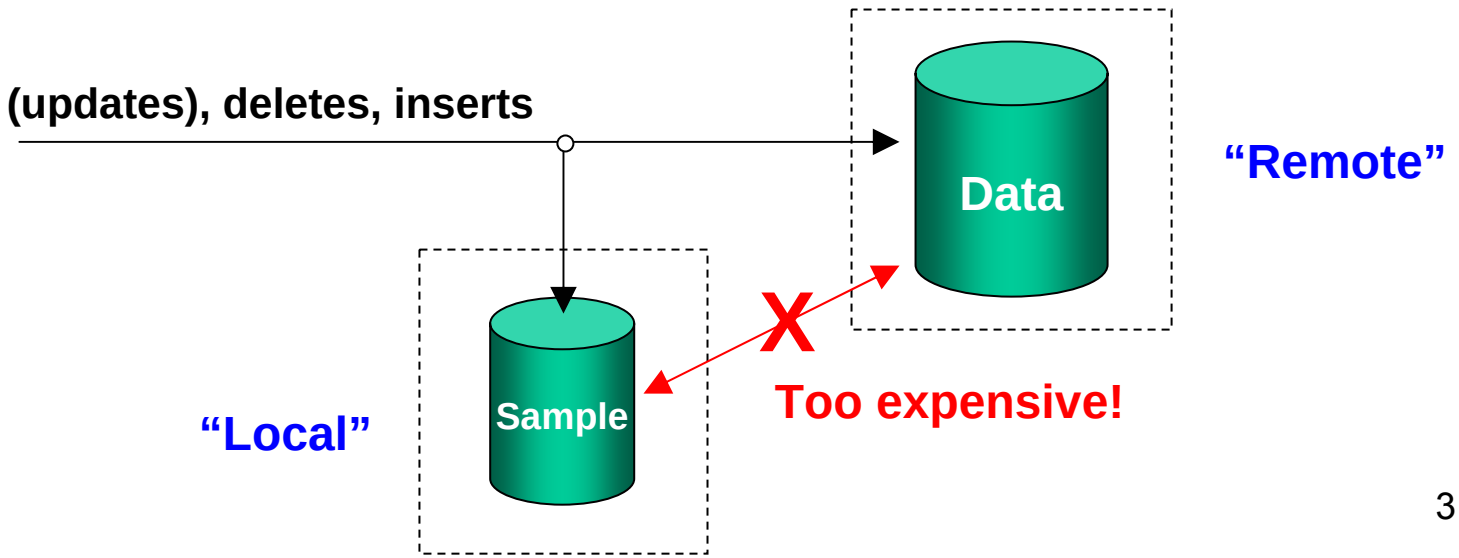
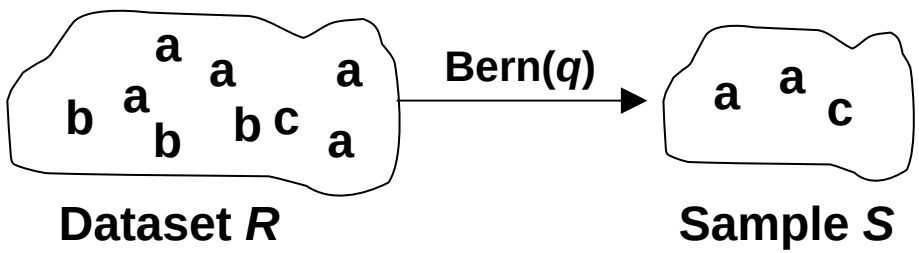


What Kind of Samples?

- Uniform sampling
  - Samples of equal size have equal probability
  - Popular and flexible, used in more complex schemes
- Bernoulli
  - Each item independently included (prob. = *q*)
  - Easy to subsample, parallelize (i.e., merge)
- Multiset sampling (most previous work is on sets)
  - Compact representation
  - Used in network monitoring, schema discovery, etc.



Outline

- Background
  - Classical Bernoulli sampling on sets
  - A naïve multiset Bernoulli sampling algorithm
- New sampling algorithm + proof sketch
  - Idea: augment sample with “tracking counters”
- Exploiting tracking counters for unbiased estimation
  - For dataset frequencies (reduced variance)
  - For # of distinct items in the dataset
- Subsampling algorithm
- Negative result on merging
- Related work

Classical Bernoulli sampling

- Bern(q) sampling of sets
  - Uniform scheme
  - Binomial sample size

$$P\{\,|S| = n\,\} = B(n;|R|,q) = \binom{|R|}{n} q^n (1-q)^{|R|-n}$$

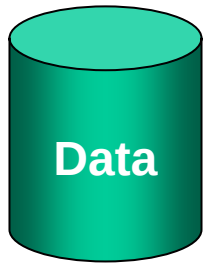
- Originally designed for insertion-only
- But handling deletions from R is easy
  - Remove deleted item from S if present

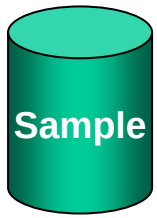
Multiset Bernoulli Sampling

- In a Bern(q) multiset sample:
  - Frequency $X(t)$ of item $t \in T$ is Binomial($N(t)$,q)
  - Item frequencies are mutually independent
- Handling insertions is easy
  - Insert item t into R and (with probability q) into S
  - I.e., increment counters (or create new counters)
- Deletions from a multiset: not obvious
  - Multiple copies of item t in both S and R
  - Only local information is available

A Naïve Algorithm

***N*(*t*) copies of item *t* in dataset**





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A Naïve Algorithm

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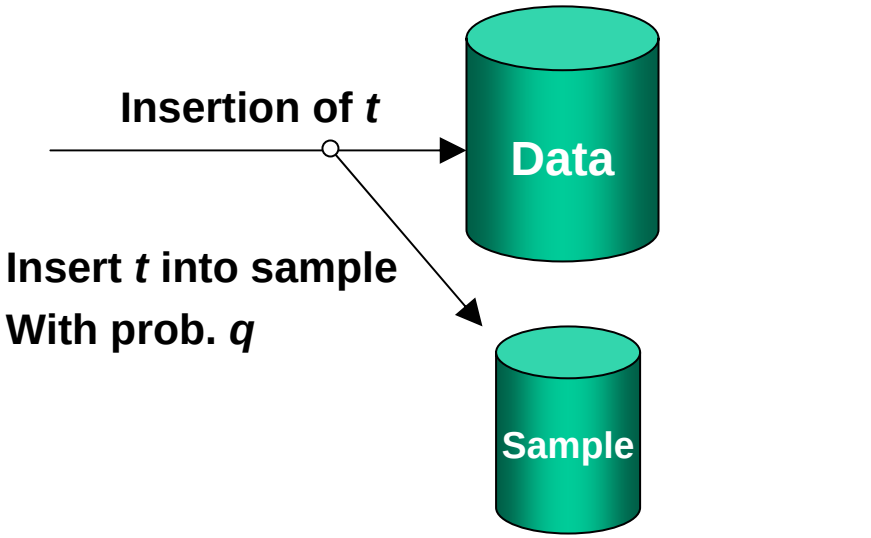

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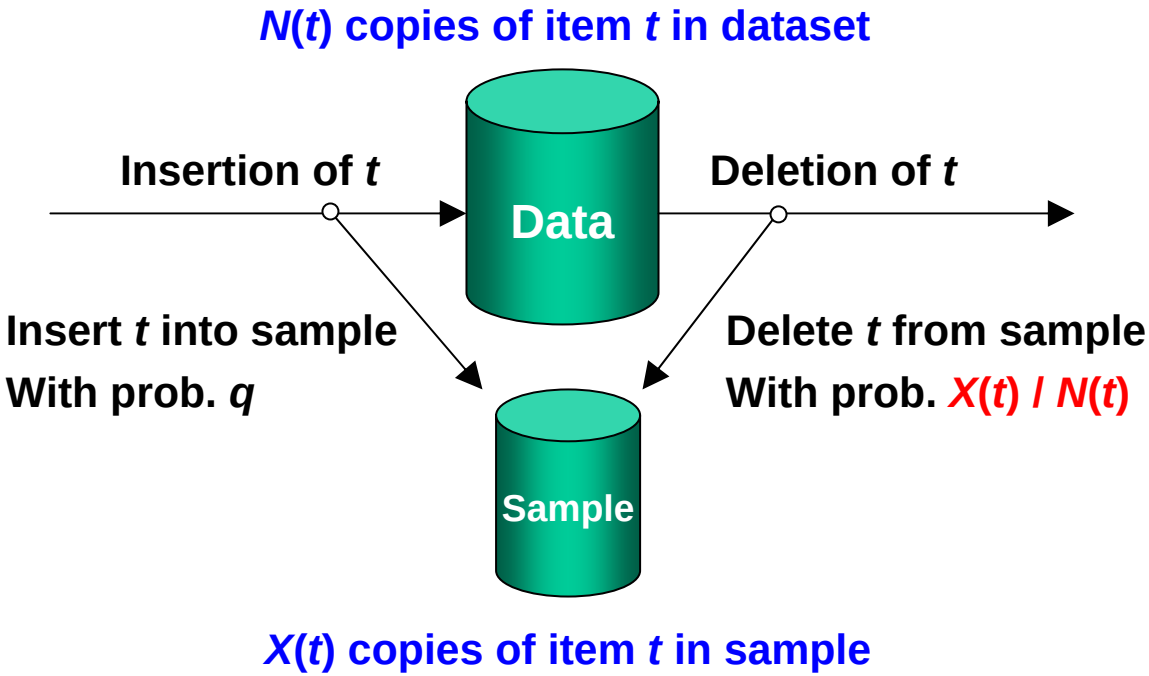

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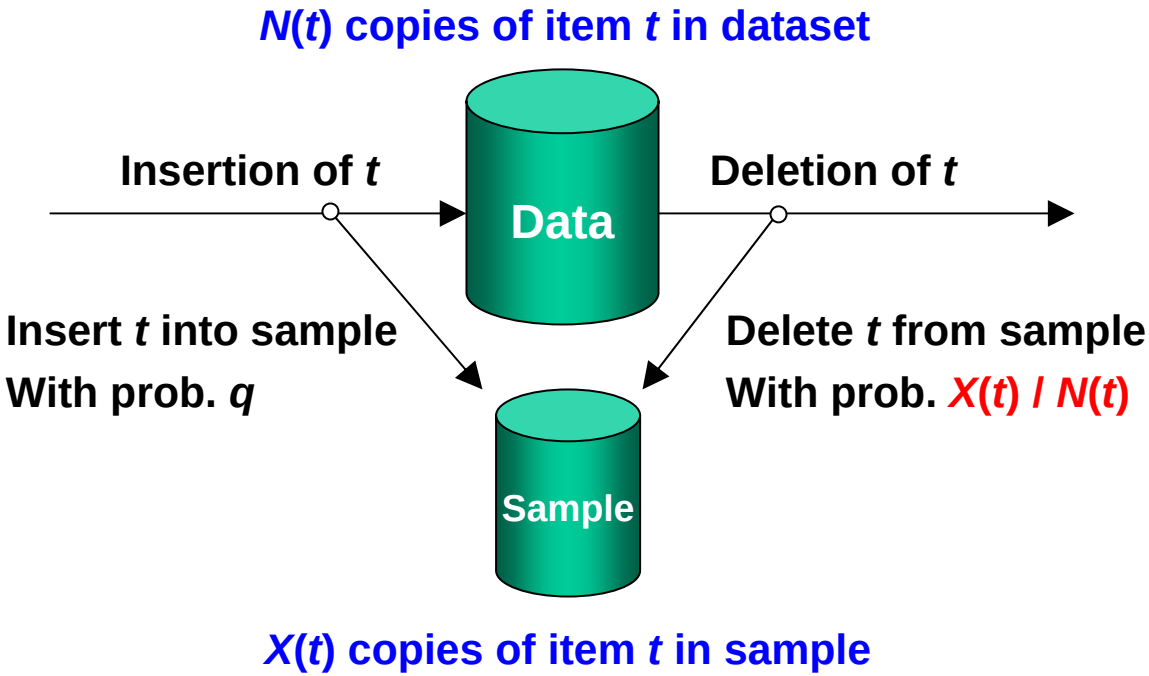

- **Problem:** must know $N(t)$
  - Impractical to track $N(t)$ for every distinct t in dataset
  - Track $N(t)$ only for distinct t in *sample*?
    - **No:** when t first enters, must access dataset to compute $N(t)$

New Algorithm

- Key idea: use **tracking counters** (GM98)
  - After j-th transaction, *augmented* sample S_j is

    $S_j = \{ (X_j(t), Y_j(t)) : t \in T \text{ and } X_j(t) > 0 \}$
    - $X_j(t)$ = frequency of item t in the sample
    - $Y_j(t)$ = net # of insertions of t into R since t joined sample

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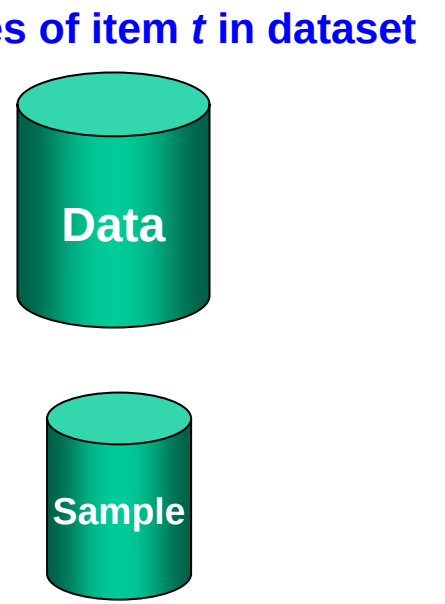

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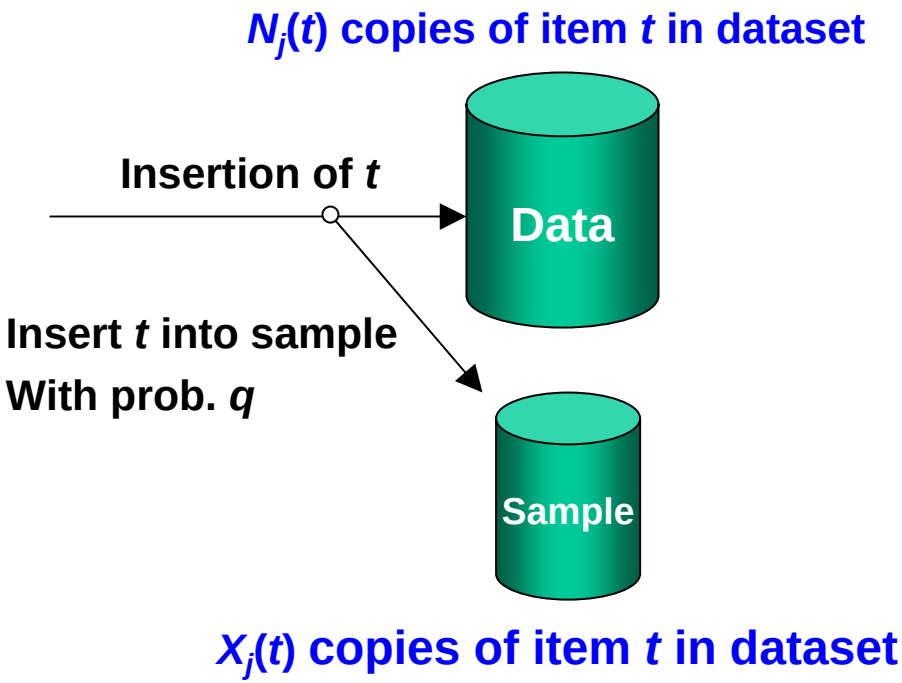

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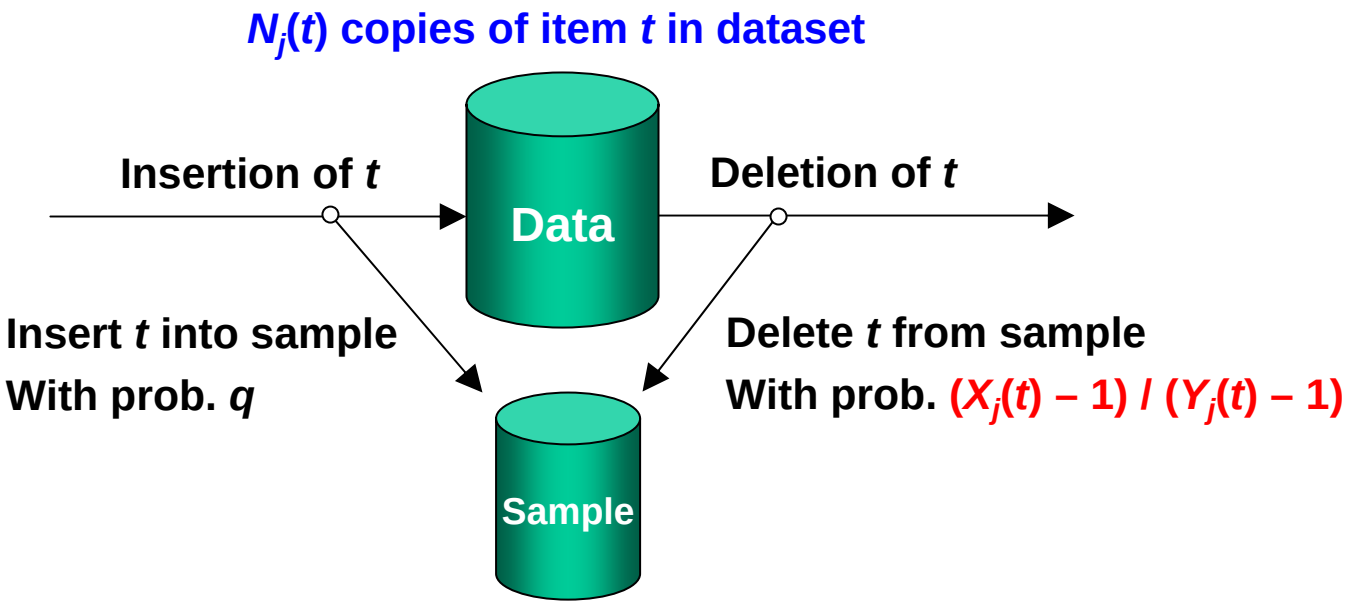

A Correctness Proof

i	R_i	S_i
i^*-1	{t t}	{ }
i^*	{t t t}	{t}
i^*+1	{t t t t}	{t t}
…	…	…
j	{t t t t … t}	{t t … t}
	Y_j - 1 items	X_j - 1 items

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	Y_j - 1 items	X_j - 1 items

Red sample obtained from red dataset via **naïve** algorithm, hence Bern(q)

$$P(X_j = k \mid Y_j = m) = P(X_j - 1 = k - 1 \mid Y_j - 1 = m - 1) = B(k - 1; m - 1, q)$$

Proof (Continued)

- Can show (by induction)

$$P(Y_j = m) = \begin{cases} (1-q)^{N_j} & \text{if } m = 0 \\ q(1-q)^{N_j - m} & \text{otherwise} \end{cases}$$

  - Intuitive when insertions only ($N_j = j$)

- Uncondition on Y_j to finish proof

$$P(X_j = k) = \sum_m P(X_j = k \mid Y_j = m)\, P(Y_j = m)$$

= $B(k\text{-}1; m\text{-}1, q)$ by previous slide

Frequency Estimation

- Naïve (Horvitz-Thompson) unbiased estimator

$$\hat{N}_{X_i} = \frac{X_i}{q} = \frac{X_i - 1}{q} + \frac{1}{q}$$

- Exploit tracking counter:

$$\hat{N}_{Y_i} = \begin{cases} Y_i - 1 + q^{-1} & \text{if } Y_i > 0 \\ 0 & \text{if } Y_i = 0 \end{cases}$$

- Theorem

$$E[\hat{N}_{Y_i}] = N_i \text{ and } V[\hat{N}_{Y_i}] \leq V[\hat{N}_{X_i}]$$

- Can extend to other aggregates (see paper)

Estimating Distinct-Value Counts

- If usual DV estimators unavailable (BH+07)
- Obtain S' from S: insert $t \in D(S)$ with probability

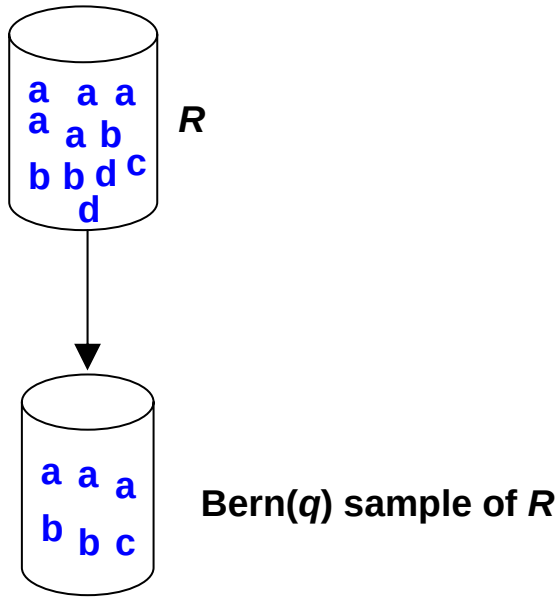
$$p(t) = \begin{cases} 1 & \text{if } Y(t) = 1 \\ q & \text{if } Y(t) > 1 \end{cases}$$

- Can show: $P(t \in S') = q$ for $t \in D(R)$
- HT unbiased estimator: $\hat{D}_{HT} = |S'| / q$
- Improve via conditioning (Var[E[U|V]] ≤ Var[U]):

$$\hat{D}_Y = E[\hat{D}_{HT} \mid S] = \sum_{t \in D(S)} p(t)/q$$

Subsampling

- Why needed?
  - Sample is too large
  - For merging
- Challenge:
  - Generate statistically correct tracking-counter value *Y*'
- New algorithm
  - See paper



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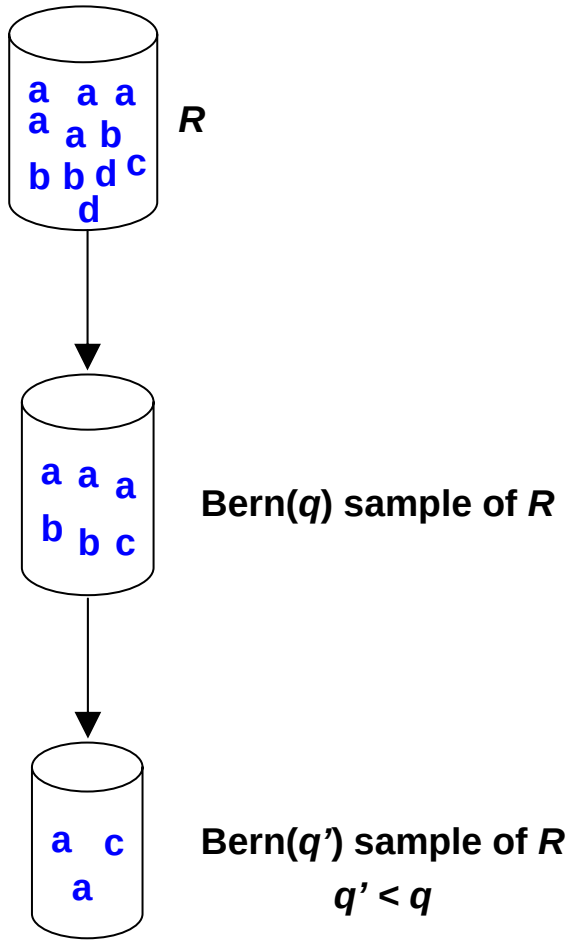
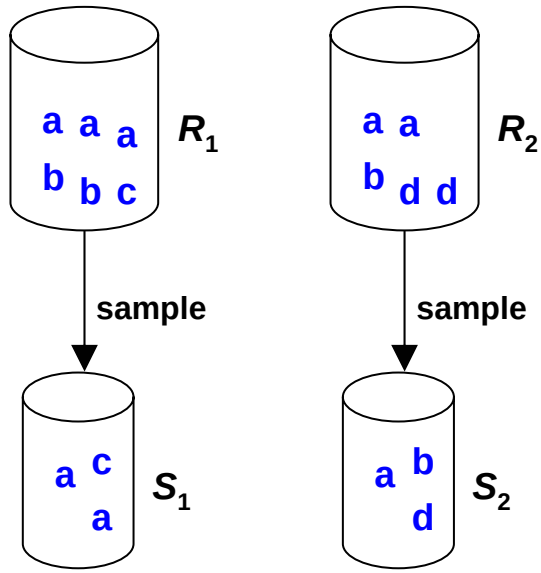


Merging

- Easy case
  - Set sampling or no further maintenance
  - $S = S_1 \cup S_2$
- Otherwise:
  - If $R_1 \cap R_2 \neq \emptyset$ and $0 < q < 1$, then there exists no statistically correct merging algorithm



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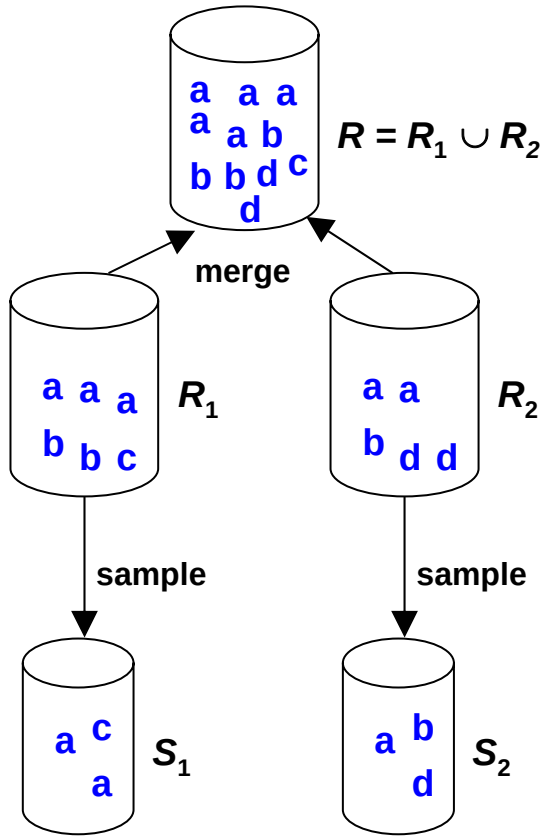

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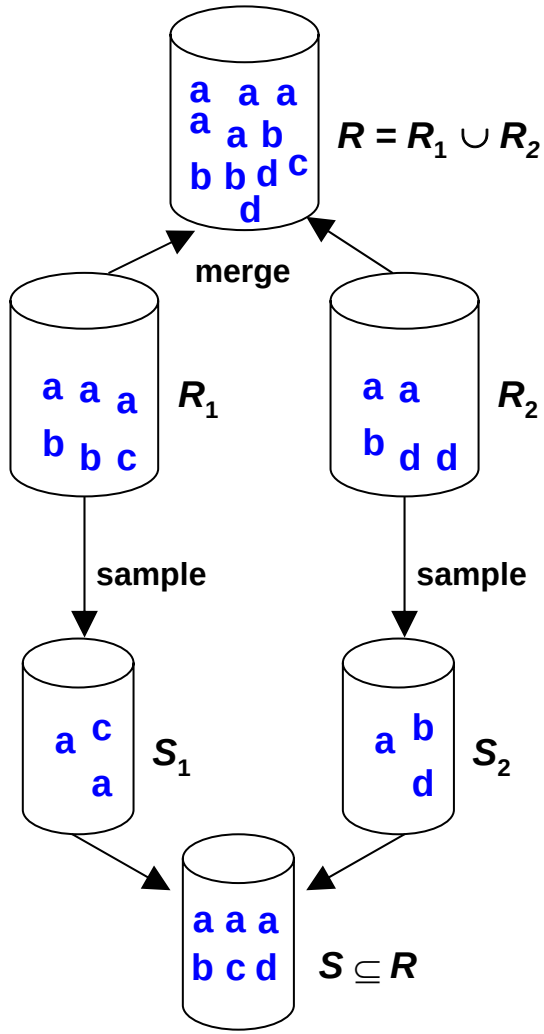

Related Work on Multiset Sampling

- Gibbons and Matias [1998]
  - Concise samples: maintain $X_i(t)$, handles inserts only
  - Counting Samples: maintain $Y_i(t)$, compute $X_i(t)$ on demand
  - Frequency estimator for hot items: $Y_i(t) - 1 + 0.418 / q$
    - Biased, higher mean-squared error than new estimator
- Distinct-Item sampling [CMR05,FIS05,Gi01]
  - Simple random sample of $(t, N(t))$ pairs
  - High space, time overhead

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Backup Slides

Subsampling

- Easy case (no further maintenance)
  - Take Bern(q^*) subsample, where $q^* = q' / q$
  - Actually, just generate X' directly as Binomial(X,q^*)
- Hard case (to continue maintenance)
  - Must also generate new tracking-counter value Y'
- Approach: generate X' then $Y' \mid \{X', Y\}$

Subsampling: The Hard Case

- Generate $X' = \Phi + \Upsilon$
  - $\Phi = 1$ iff item included in S at time i^* is retained
    - $P(\Phi = 1) = 1 - P(\Phi = 0) = q^*$
  - Υ is # of other items in S that are retained in S'
    - Υ is Binomial(X-1,q^*)
- Generate Y' according to "correct" distribution
  - $P(Y' = m \mid X', Y, \Phi)$

Subsampling (Continued)

	i: 1	2	3	4	5	6	
Original sample	t	t	t	t	t	t	$Y_6 = 5, X_6 = 3$
$\Phi = 1$	t	t	t	t	t	t	$Y_6' = 5, X_6' = 2$
$\Phi = 0$	t	t	t	t	t	t	$Y_6' = 3, X_6' = 2$

$$P(Y' = m \mid X', Y, \Phi = 1) = [1 \text{ if } m = Y \text{ and } 0 \text{ otherwise }]$$

$$P(Y' = m \mid X', Y, \Phi = 0) = \frac{X'}{m} \prod_{i=m+1}^{Y-1} \left(1 - \frac{X'}{i}\right) \quad \text{when } X' > 0$$

Generate Y - Y' using acceptance/rejection (Vitter 1984)

Related Work on Set-Based Sampling

- Methods that access dataset
  - CAR, CARWOR [OR86], backing samples [GMP02]
- Methods (for bounded samples) that do not access dataset
  - Reservoir sampling [FMR62, Vi85] (inserts only)
  - Stream sampling [BDM02] (sliding window)
  - Random pairing [GLH06] (resizing also discussed)

More Motivation: A Sample Warehouse