CMPSCI 611: Advanced Algorithms
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Problem Set 5

Out: December 1, 2003
Due: December 9, 2003

1. In this problem we consider an approximation algorithm for the knapsack problem that only has a constant performance ratio (i.e., not as good as the FPTAS considered in class), but runs much more quickly.

   (a) Let the *density* of item i be v_i/w_i (i.e., the value divided by the weight). Consider the following algorithm, where W is the knapsack capacity:

   Let $S = \emptyset$.
   $Current\_weight = 0$
   For each item i in order from largest density to smallest density,
       If $Current\_weight + w_i \leq W$ then
           $S = S + i$.
           $Current\_weight = Current\_weight + w_i$.
   Return S.

   Show that for any positive integer t, there is an input where this algorithm returns a solution that is a factor of t worse than the optimal solution.

   (b) Consider the algorithm that takes the better of two solutions: that returned by the algorithm from part (a), and the solution consisting of the single item of maximum value that fits into the knapsack. Show that this is a 2-approximation.

2. [CLRS] Problem 35-4 (page 1051).

3. An *absolute k-approximation* is an approximation algorithm that returns a solution within k of the optimal solution, regardless of the size of the optimal solution. In other words, the error is additive, instead of multiplicative.

   (a) Show that if, for any constant k, there is an absolute k-approximation for the Metric Traveling Salesman Problem, then $P = NP$. You can assume that the input consists of integer weights.

   (b) The *Chromatic number* of a graph is the minimum number of colors required to assign a color to each vertex in such a way that no edge connects two vertices of the same color. Show that there is an absolute 1-approximation for the Chromatic number of a graph if the input is restricted to planar graphs (i.e., graphs that can be drawn in the plane without edges intersecting). You can use without proof the fact that any planar graph has a Chromatic number of at most 4.

4. [CLRS] Problem 29-1 (page 818).