MapReduce and Distributed Data Analysis

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Dealing With Massive Data

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	Memory	
	Polynomial	Sublinear
	RAM	Sketches External Memory Property Testing

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Single	RAM	Sketches External Memory Property Testing
Multiple	PRAM	

Dealing With Massive Data

Processors	Memory: Polynomial	Memory: Sublinear
Single	RAM	Sketches External Memory Property Testing
Multiple	PRAM	MapReduce Distributed Sketches

What about Data Streams?

	Input	
	Batch	Online
	RAM	Data Streams

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Processors \ Input	Batch	Online
Single	RAM	Data Streams
Multiple	MapReduce	Distributed Sketches Active DHTs

Today's Focus: MapReduce

Multiple Processors:

- 10s to 10,000s processors

Sublinear Memory

- A few Gb of memory/machine, even for Tb+ datasets
- Unlike PRAMs: memory is not shared

Batch Processing

- Analysis of existing data
- Extensions used for incremental updates, online algorithms

Why MapReduce?

Practice:

- Used very widely for large data analysis
- Google, Yahoo!, Amazon, Facebook, Netflix, LinkedIn, New York Times, eHarmony, ...

Is it a Fad?:

- No! (Do Fads last 10 years?)
- Many similar implementations and abstractions on top of MR: Hadoop, Pig, Hive, Flume, Pregel, ...
- Same computational model underneath

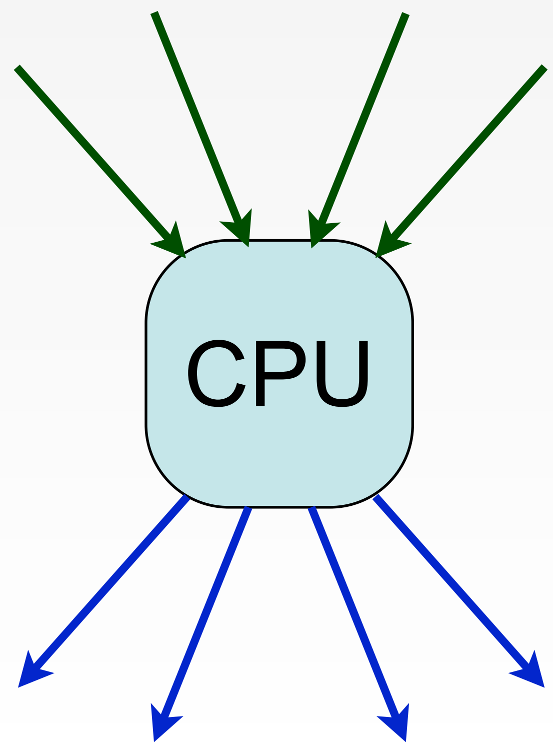
Why?

- Practice that needs theory
- Makes data locality explicit (which sometimes leads to faster sequential algorithms) !

MR Flow

Computation proceeds in rounds.

Each round:



Receive Input

Compute

Produce output

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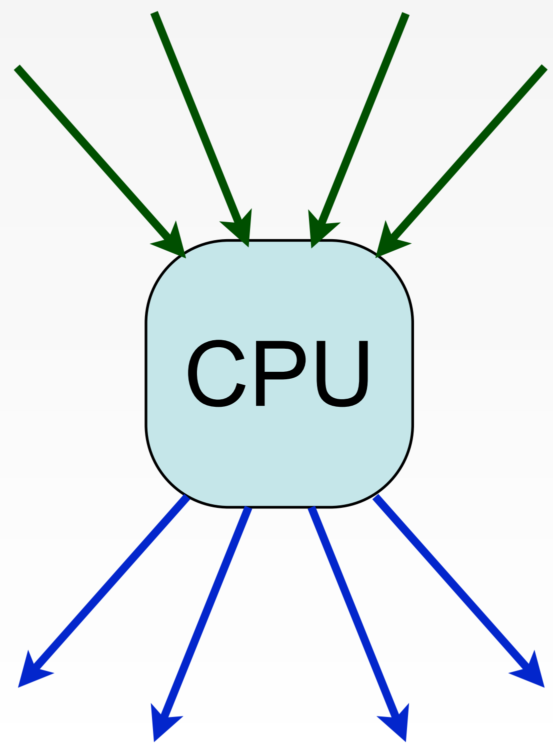


Receive Input: Input must fit into memory

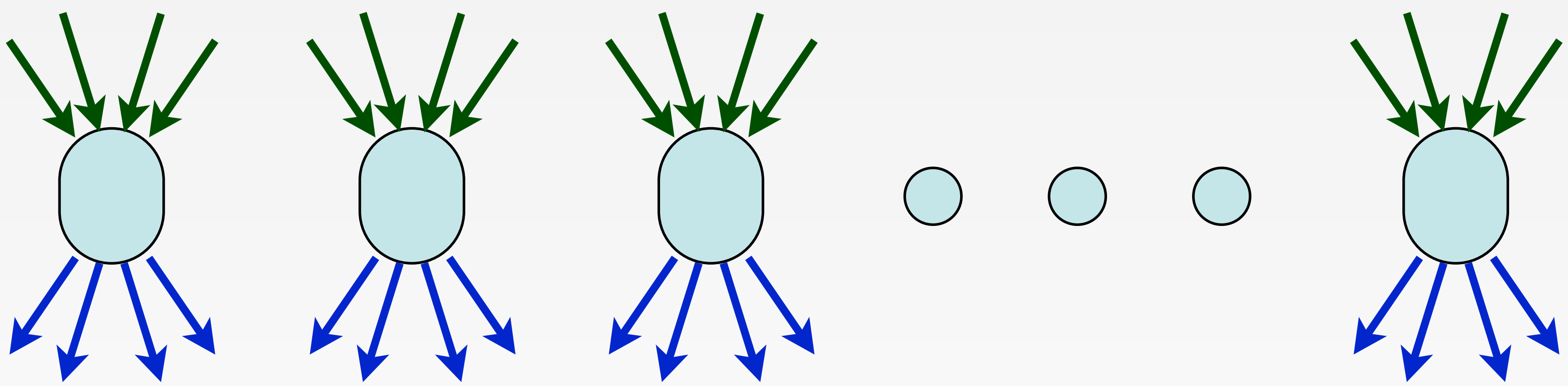
Compute: Full TM

Produce output: Annotated with address of machine in next round

MR Multiple Rounds

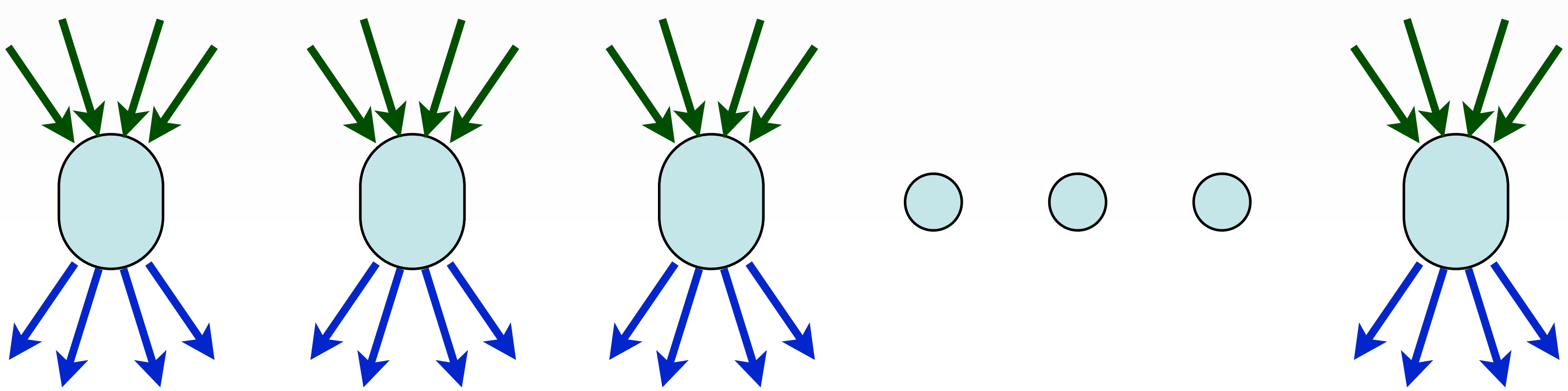
Many Machines...and multiple rounds

Round 1:



Arbitrary communication topology (depends on the output of round 1)

Round 2:



Data Streams vs. MapReduce

Distributed Sum:

- Given a set of n numbers: $a_1, a_2, \ldots, a_n \in \mathbb{R}$, find $S = \sum_i a_i$

Stream:

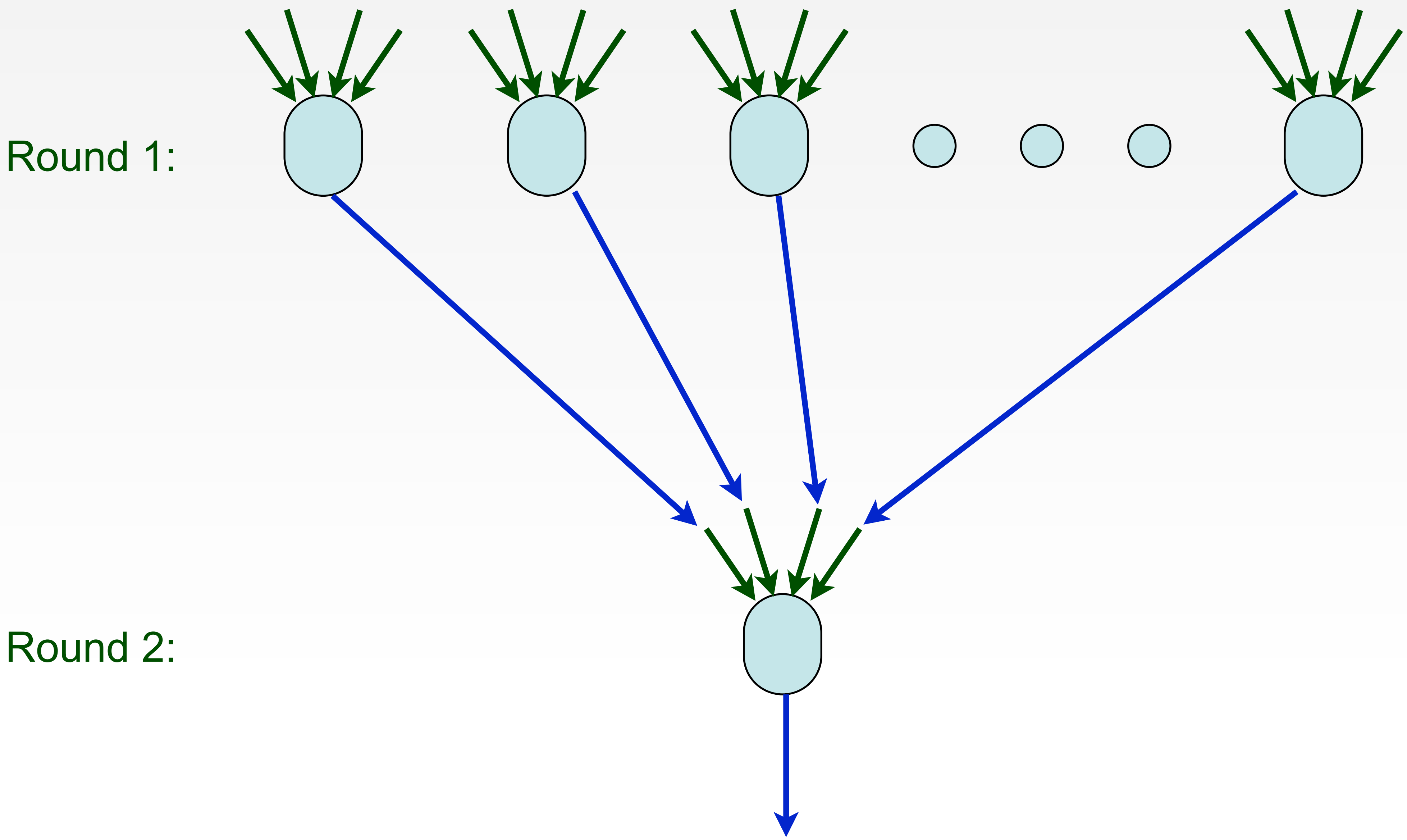
- Maintain a partial sum $S_j = \sum_{i \leq j} a_i$
- update with every element

MapReduce:

- Compute $M_j = a_{jk} + a_{jk+1} + \ldots + a_{j(k+1)-1}$ for $k = \sqrt{n}$ in Round 1
- Round 2: add the $\sqrt{n}$ partial sums.

MR Modeling [Sketch]

Distributed Sum in MR



Modeling

For an input of size n :

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- Insist on sublinear number of machines: $n^{1-\epsilon}$ for some $\epsilon > 0$

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Synchronization

- Computation proceeds in rounds
- Count the number of rounds
- Aim for $O(1)$ rounds

Not Modeling

Lies, Damned Lies, Statistics

- And big-O notation
- And Competitive Analysis
- And...

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Communication:

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- Order of magnitude improvements due to
  - Move code to data (and not data to code)
  - Working with graphs: save graph structure locally between rounds
  - Job scheduling (same rack / different racks, etc)

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- Order of magnitude improvements due to
  - Move code to data (and not data to code)
  - Working with graphs: save graph structure locally between rounds
  - Job scheduling (same rack / different racks, etc)
- Bounded by $n^{2-2\epsilon}$ (total memory of the system) in the model
  - Minimizing communication always a goal

How Powerful is this Model?

Compared to PRAMs:

- Can (formally) simulate PRAM algorithms with MR
- In practice can use same idea without formal simulation
- One round of MR per round of PRAM: $O(\log n)$ rounds total
- Hard to break below $o(\log n)$, need new ideas

How Powerful is this Model?

Different Tradeoffs from PRAM:

- PRAM: LOTS of very simple cores, communication every round
- MR: Many full TM cores, batch communication.

Formally:

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- In practice can use same idea without formal simulation
- One round of MR per round of PRAM: $O(\log n)$ rounds total
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Compared to BSP:

- Closest in spirit
- Do not optimize parameters in algorithm design phase

Outline

Introduction

MapReduce

MR Algorithmics

- Connected Components
- Matchings
- Greedy Algorithms

Open Problems

Algorithmics

Find the core of the problem:

- Reduce the problem size in parallel
- Solve the smaller instance sequentially

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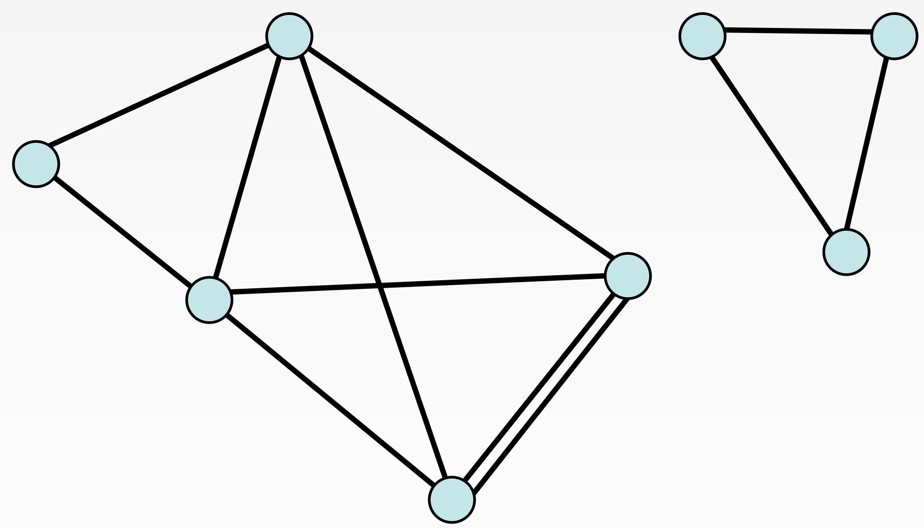
- Reduce the problem size in parallel
- Solve the smaller instance sequentially

Roadmap:

- Identify redundant information
- Filter out redundancy to reduce input size
- Solve the smaller problem
- Use the solution as a seed for the larger problem

Connected Components

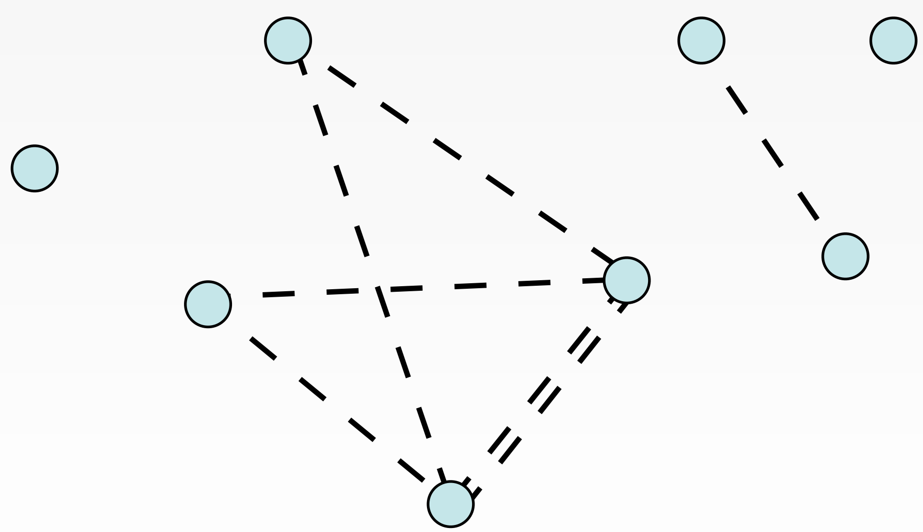
Given a graph:

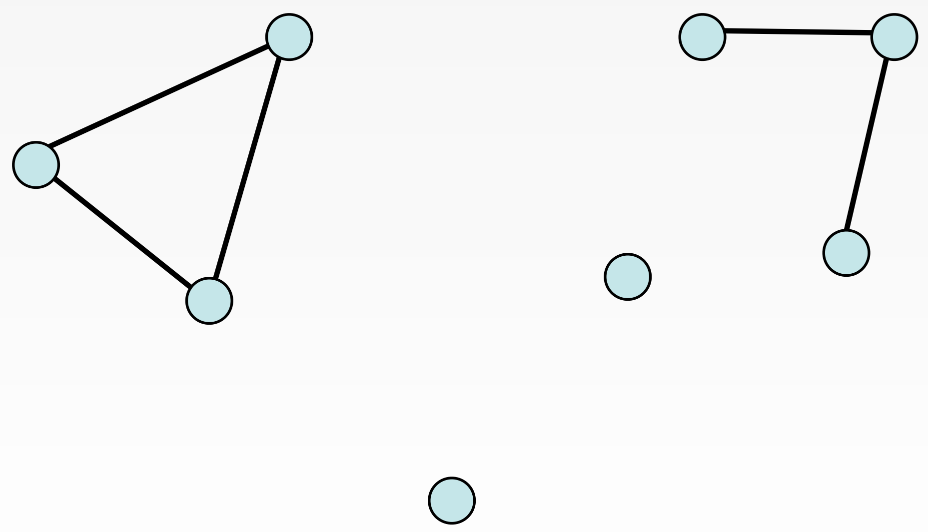


Connected Components

Given a graph:

1. Partition (randomly)





Machine 1

Machine 2

Connected Components

Given a graph:

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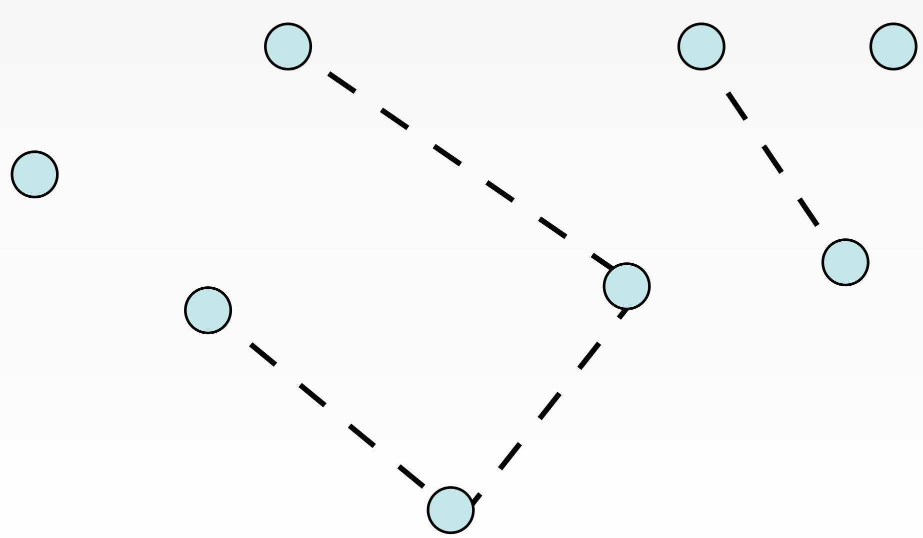
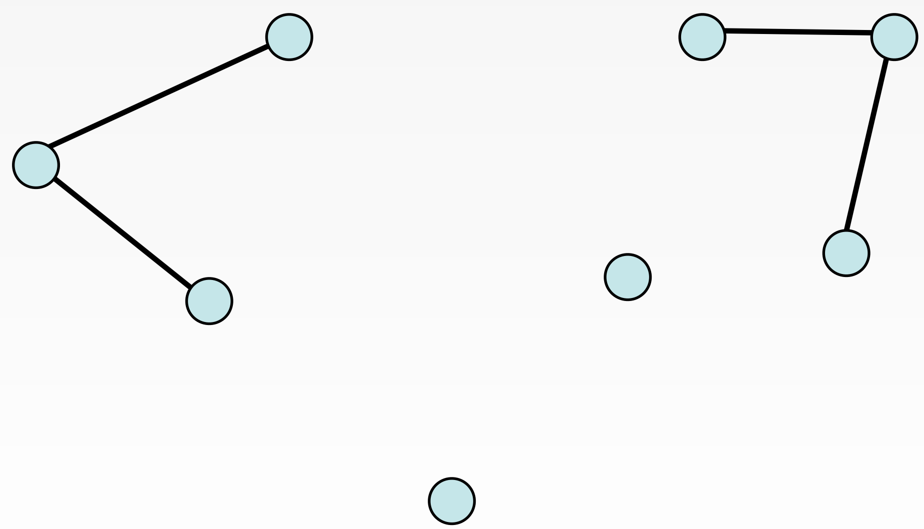
2. Summarize (keep $\leq n-1$ edges per partition)



Machine 1

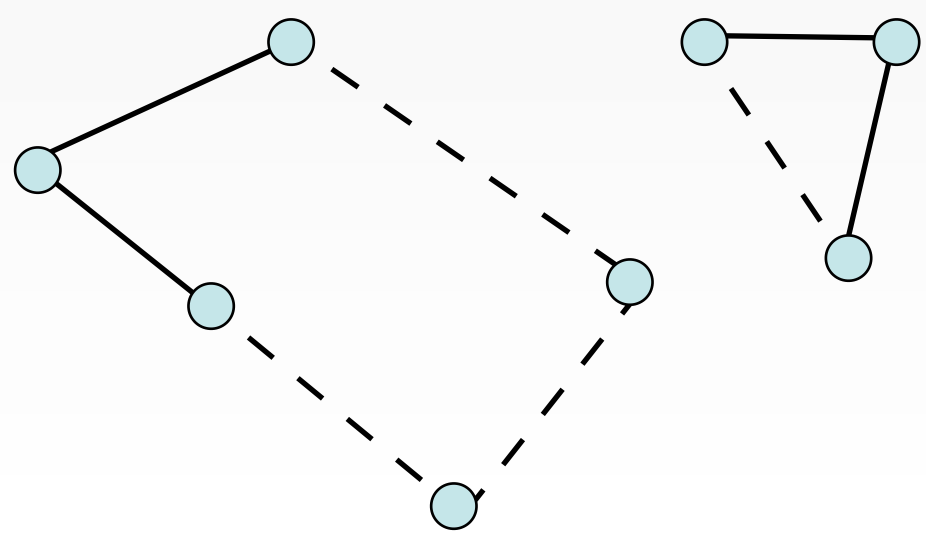


Machine 2

Connected Components

Given a graph:

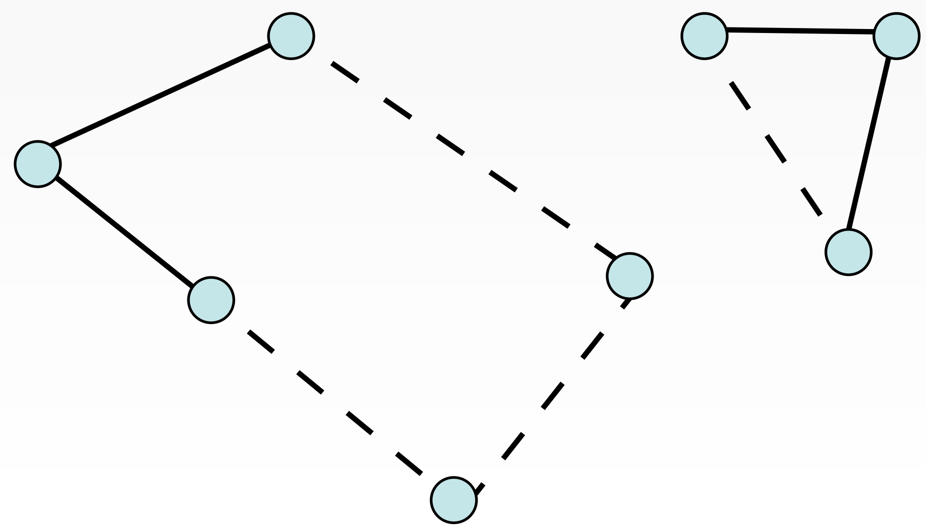
1. Partition (randomly)
2. Summarize (keep $\leq n-1$ edges per partition)
3. Recombine



Connected Components

Given a graph:

1. Partition (randomly)
2. Summarize (keep $\leq n - 1$ edges per partition)
3. Recombine
4. Compute CC's



Analysis

Given: k machines:

- Total Runtime: $O((^m/_k + nk)\alpha(n))$
- Memory per machine: $O(^m/_k + nk)$
  - Actually, can stream through edges so $O(n)$ suffices
- 2 Rounds total

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Finding matchings

- Given an undirected graph $G = (V, E)$
- Find a maximum matching

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- Find a maximal matching

Try random partitions:

- Find a matching on each partition
- Compute a matching on the matchings
- Does not work: may make very limited progress

Looking for redundancy

Matching:

- Could drop the edge if an endpoint already matched

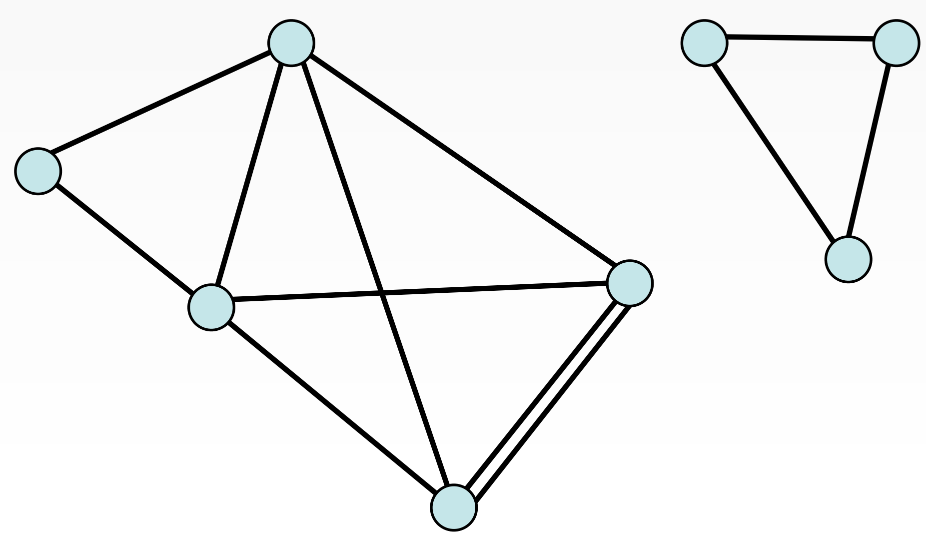
Idea:

- Find a seed matching (on a sample)
- Remove all ‘dead’ edges
- Recurse on remaining edges

Algorithm

Given a graph:

1. Take a random sample



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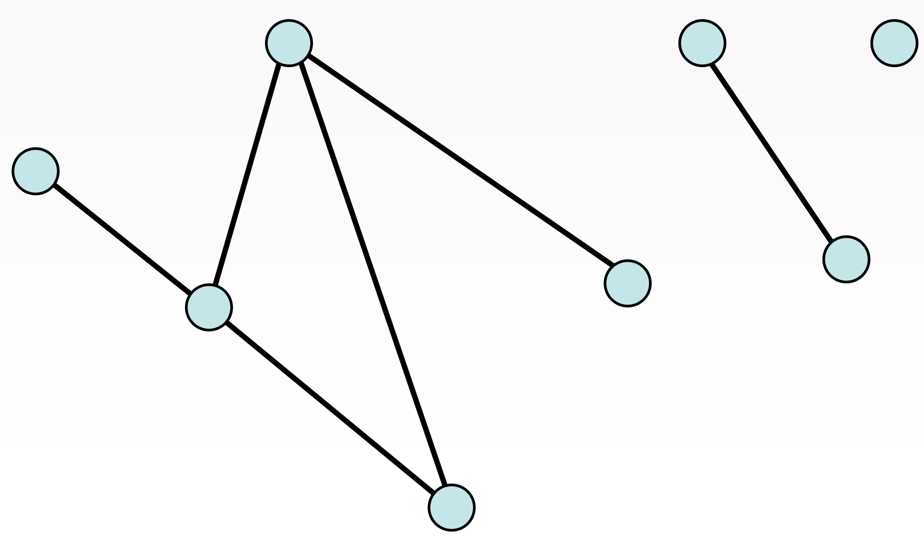
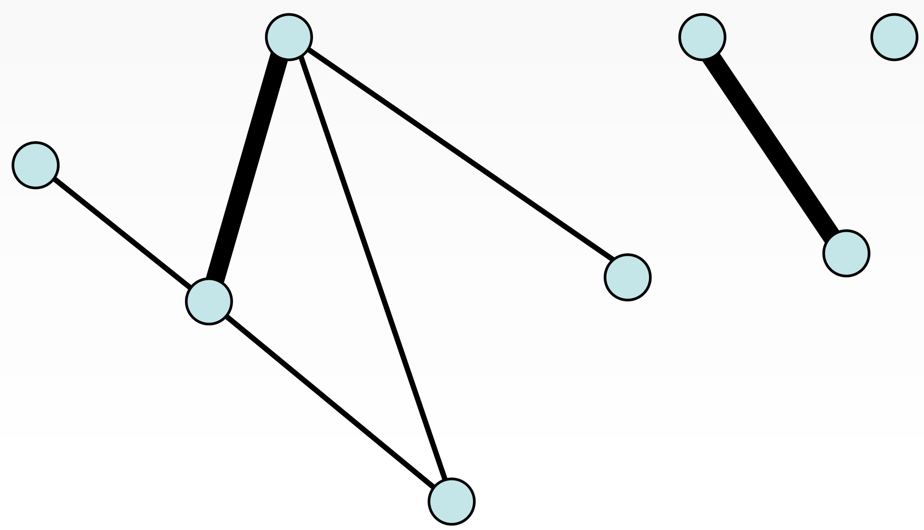


Algorithm

Given a graph:

1. Take a random sample

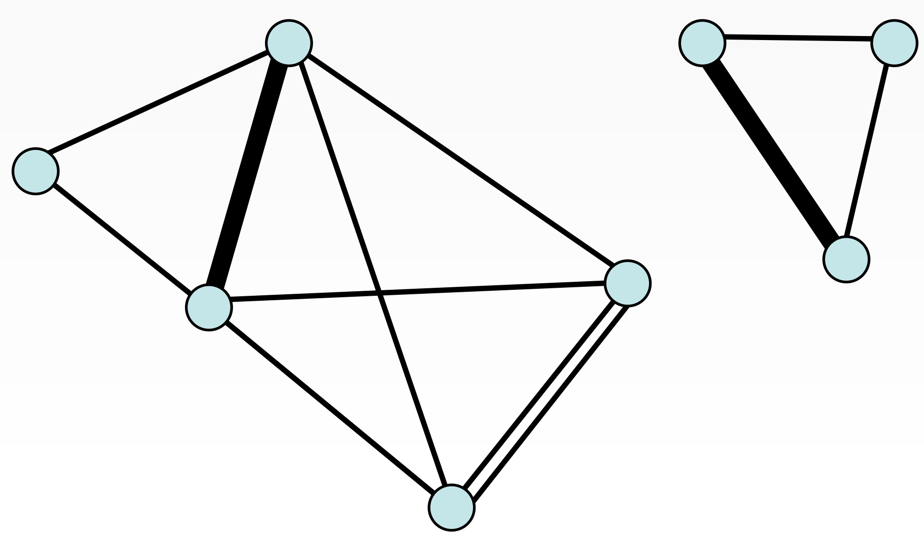
2. Find a maximal matching on sample



Algorithm

Given a graph:

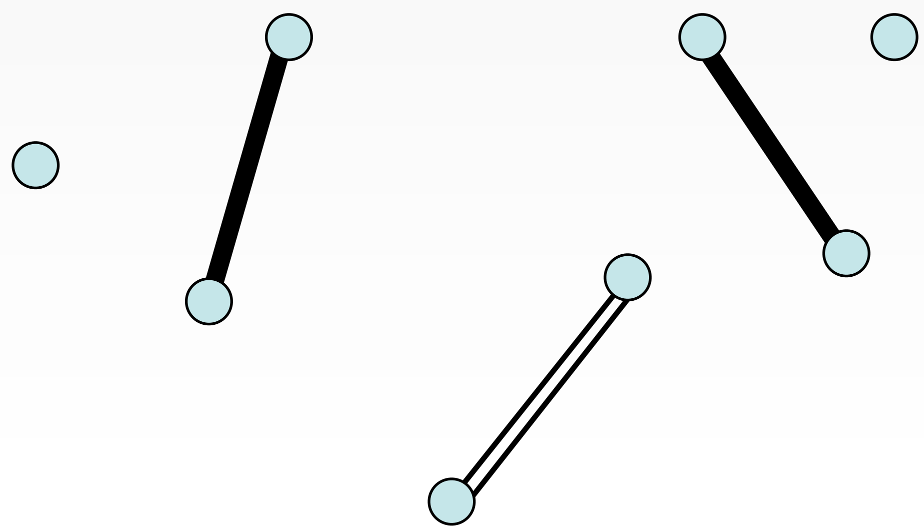
1. Take a random sample
2. Find a maximal matching on sample
3. Look at original graph



Algorithm

Given a graph:

1. Take a random sample
2. Find a maximal matching on sample
3. Look at original graph, drop dead edges



Algorithm

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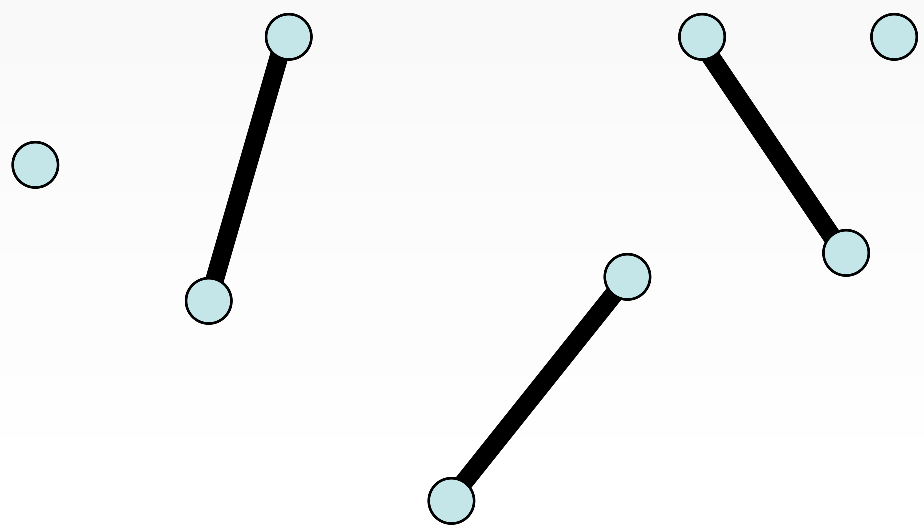
2. Find a maximal matching on sample
3. Look at original graph, drop dead edges
4. Find matching on remaining edges



Analysis

Key Lemma:

- Suppose the sampling rate is $p = \frac{n^{1+c}}{m}$ for some $c > 0$.
- Then with high probability the number of edges remaining after the prune step is at most:

$$\frac{2n}{p} = \frac{2m}{n^c}$$

Analysis

Key Lemma:

- Suppose the sampling rate is $p = \frac{n^{1+c}}{m}$ for some $c > 0$.
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- Proof [Sketch]:
  - Suppose $I \subseteq V$ are unmatched after prune step
  - I must be an independent set in the sampled graph
  - If $|E[I]| > O(n/p)$ then it is an I.S. with probability at most e^{-n}
  - Union bound over all 2^n sets I.

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Corollaries:

- Given n^{1+c} memory, algorithm requires $O(1)$ rounds
- Given $O(n \log n)$ memory, algorithm requires $O(\frac{\log n}{\log \log n})$ rounds.
- PRAM simulations: $\Theta(\log n)$ rounds

Greedy Algorithms

Max k-Cover:

- Given $U = \{u_1, u_2, \ldots, u_n\}$ and sets $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$
- Select $\mathcal{S}^* \subseteq \mathcal{S}$ with $|\mathcal{S}^*| = k$, that maximizes $\bigcup_{S \in \mathcal{S}^*} S$

Classical Algorithm:

- Greedy. Iteratively add largest set
- Remove all of the covered elements
- Guarantees a $1 - 1/e$ approximation to the optimal

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Not so Greedy Algorithms

Not so Greedy Algorithm:

- Iteratively add a set within $(1 + \epsilon)$ of the largest
- Remove all of the covered elements

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- Iteratively add a set within $(1+\epsilon)$ of the largest
- Remove all of the covered elements

Correctness:

- Make almost as much progress as regular greedy algorithm
- Get an $1 - 1/e - O(\epsilon)$ approximation

Streaming Algorithms

Intuition:

- Look for large sets first, then progressively look for smaller sets.
- Natural algorithm:
  - Define a threshold: $v_i = \frac{n}{(1+\epsilon)^i}$
  - Stream through elements, add sets of value at least v_1
  - Stream through elements, add sets of value at least v_2
  - ...until completion

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  - ...until completion

Analysis:

- Every time make a decision within $(1+\epsilon)$ of OPT
- Need $O(\frac{\log \max |S|}{\epsilon})$ passes

Parallel Algorithms

Simulate the Streaming Algorithm

- Given a threshold, v_i, find a sequence of sets each covering at least v_i elements
- Similar to maximal matching:
  - Run the streaming algorithm on a sample
  - Remove the sets whose weight falls below the threshold
  - Repeat until no sets meet the threshold

Parallel Algorithms

Simulate the Streaming Algorithm

- Given a threshold, v_i, find a sequence of sets each covering at least v_i elements
- Similar to maximal matching:
  - Run the streaming algorithm on a sample
  - Remove the sets whose weight falls below the threshold
  - Repeat until no sets meet the threshold

Theorem [MKVV '12]: If each set is sampled with probability $p = km^{-\delta}$ each phase of the streaming algorithm can be simulated in $O(1/\delta)$ rounds

Sample&Prune

High-level primitive:

- Take a sample of the input
- Evaluate the function on the sample
- Prune the input
- Repeat

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- Evaluate the function on the sample
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Much more general:

- General class of greedy algorithms
- (Sub)modular function maximization subject to set of constraints
  - matroid constraints, multiple knapsack constraints

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Overview

MapReduce:

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- Many algorithms “embarrassingly parallel” and don’t use the full power of intermediate steps.

What are the limits of the model:

- Can simulate PRAM algorithms. But are there bigger speedups to be had?
- No known lower bounds!
- Even a simpler model is interesting
  - e.g. assume data is partitioned and no replication as allowed

Classical Questions

Prefix Sum:

- Given an array `A[1...n]` compute `B[1...n]` where `B[i]=` $\sum_{j \leq i} A[j]$
- Relatively simple in MR (2 rounds suffices)

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List Ranking:

- Given a permutation as a set of pointers to the next element: `P[i] = j`
- Find rank of each element (number of hops to reach it from `P[0]`)
- Easy in $O(\log n)$ rounds (simulate PRAM algorithm)
- Can you do it faster?

A Puzzle (essentially list ranking)

I give you an undirected graph on n nodes:

- As a list of edges
- With a promise that every vertex has degree exactly 2

You have:

- $n^{2/3}$ machines, each with $n^{2/3}$ memory

Your Mission:

- Tell me if the graph is connected in $o(\log n)$ rounds...
- ...or prove that it cannot be done

Thank You