Chart Parsing

Lecture #3

Introduction to Natural Language Processing

CMPSCI 585, Fall 2004

University of Massachusetts Amherst

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(agglomeration of slides from Jason Eisner)

Today's Main Points

- Refresher for dynamic programming
  How this applies to parsing.
- Two dynamic programming parsers:
  - CYK
  - Earley's algorithm
- Hand out Programming Assignment #1
  "Implement CYK"

Programming languages

```
printf ("/charset [%s",
        (re_opcode_t) *(p - 1) == charset_not ? "^" : "");

assert (p + *p < pend);

for (c = 0; c < 256; c++)
  if (c / 8 < *p && (p[1 + (c/8)] & (1 << (c % 8)))) {
     /* Are we starting a range?  */
     if (last + 1 == c && ! inrange) {
        putchar ('-');
        inrange = 1;
     }
     /* Have we broken a range?  */
     else if (last + 1 != c && inrange) {
        putchar (last);
        inrange = 0;
     }

     if (! inrange)
        putchar (c);

     last = c;
  }
```

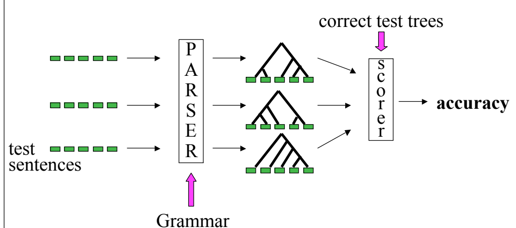
- Easy to parse.
- Designed that way!

Natural languages

```
printf "/charset %s", re_opcode_t *p - 1 == charset_not ? "^" : "";
assert p + *p < pend; for c = 0; c < 256; c++ if c / 8 < *p && p1 +
c/8 & 1 << c % 8 Are we starting a range? if last + 1 == c && !
inrange putchar '-'; inrange = 1; Have we broken a range? else if last
+ 1 != c && inrange putchar last; inrange = 0; if !  inrange putchar
c; last = c;
```

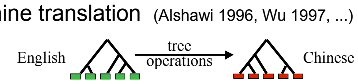
- No {} () [] to indicate scope & precedence
- Lots of overloading (arity varies)
- Grammar isn't known in advance!
- Context-free grammar not best formalism

The parsing problem



Applications of parsing (1/2)

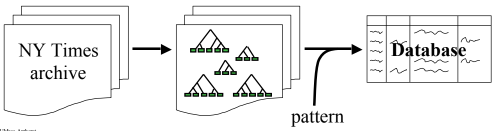
- Machine translation (Alshawi 1996, Wu 1997, ...)



- Speech synthesis from parses (Prevost 1996)
  The government plans to raise income tax.
  The government plans to raise income tax the imagination.
- Speech recognition using parsing (Chelba et al 1998)
  Put the file in the folder.
  Put the file and the folder.

Applications of parsing (2/2)

- Grammar checking (Microsoft)
- Indexing for information retrieval (Woods 1997)
  ... washing a car with a hose ... → vehicle maintenance
- Information extraction (Hobbs 1996) (Miller et al 2000)



Parsing State of the Art

- Recent parsers quite accurate, e.g.,
  - *A Maximum-Entropy-Inspired Parser*
    Eugene Charniak
    Proceedings of NAACL-2000.
  - *Three Generative, Lexicalised Models for Statistical Parsing*
    Michael Collins
    Proceedings of ACL, 1997.
- Most sentences parsed correctly, or with one error

Last class…

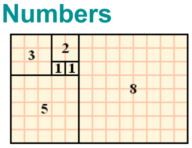
- We defined a CFG,
  where it sits in the Chomsky hierarchy
- Talked about parsing as ***search***…
  …through an exponential number of possible trees
- Gave examples of bottom-up and top-down search.
- Discussed problems:
  - Infinite loop with left-recursive rules
  - Much duplicated work in exponential space… backtracking

Dynamic Programming

- (Not much to do with “programming” in the CS sense.)
- Dynamic programming is efficient in finding optimal solutions for cases with lots of **overlapping subproblems**.
- It solves problems by *recombining solutions* to subproblems, when the subproblems themselves may share sub-subproblems.

DP Example: Calculating Fibonacci Numbers

- F(n) = F(n-1) + F(n-2),
  where F(0)=0, F(1)=1.
  E.g. 0, 1, 1, 2, 3, 5, 8, 13, 21



- Non-DP implementation

```
function fib(n)
 if n is 0 or 1, return n,
 otherwise, return fib(n-1) + fib(n-2)
end function
```


DP Example: Calculating Fibonacci Numbers

- DP implementation

```
 function fib(n)
  array f[n];
  f[0] = 0;
  f[1] = 1;
  for i = 2 to requested number do
   f[i] = f[i-1] + f[i-2];
end function
```

Dynamic Programming for Parsing

- Given CFG in Chomsky Normal Form, and an input string, we want to search for valid parse trees.
- What are the intermediate sub-problems?
- What would the dynamic programming table look like?

CKY algorithm, recognizer version

- Input: string of n words
- Output: yes/no (since it's only a recognizer)
- Data structure: n x n table
  - rows labeled 0 to n-1
  - columns labeled 1 to n
  - cell [i,j] lists possible constituents spanning words between i and j

CKY algorithm, recognizer version

- for i := 1 to n
  - Add to [i-1,i] all (part-of-speech) categories for the i[th] word
- for width := 2 to n
  - for start := 0 to n-width
    - Define end := start + width
    - for mid := start+1 to end-1
      - for every constituent X in [start,mid]
      - for every constituent Y in [mid,end]
      - for all ways of combining X and Y (if any)
      - Add the resulting constituent to [start,end] if it's not already there.

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

NP → time
Vst → time
NP → flies
VP → flies
P → like
V → like
Det → an
N → arrow

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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2 VP → VP PP
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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4	–		
2			P 2 V 5	–	
3				Det 1	
4					N 8

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0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4	–		
2			P 2 V 5	–	
3				Det 1	NP 10
4					N 8

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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4	–		
2			P 2 V 5	–	
3				Det 1	NP 10
4					N 8

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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	–		
1		NP 4 VP 4	–	–	
2			P 2 V 5	–	PP 12
3				Det 1	NP 10
4					N 8

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1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_		
1		NP 4 VP 4	_	_	
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_		
1		NP 4 VP 4	_	_	
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	
1		NP 4 VP 4	_	_	NP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	
1		NP 4 VP 4	_	_	NP 18 S 21
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
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time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
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	time 1	flies 2	like 3	an 4	arrow 5
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Follow backpointers …

S

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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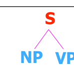

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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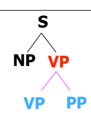

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0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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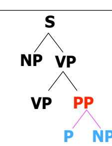

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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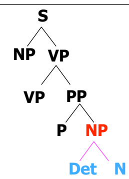

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2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
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Dynamic Programming Parsing 2

- How about a dynamic programming solution for **arbitrary CGF grammars**?
- (Grammars not in Chomsky Normal Form.)

Earley Parser (1970)

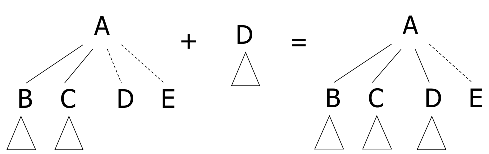
- Nice combination of
  - dynamic programming
  - incremental interpretation
  - avoids infinite loops
  - no restrictions on the form of the context-free grammar.
    A → B C the D of causes no problems
  - $O(n^3)$ worst case, but faster for many grammars
  - Uses left context and optionally right context to constrain search.

Earley Parser

- Input: String of words and grammar
- Output: yes/no
  (i.e. recognizer, but can turn into a parser)
- Data Structure:
  - columns 0 through n,
    corresponding to the gaps between words
  - column j is a list of entries like
    (i, A → X Y . Z W)
    meaning there could be an A starting at i, and we have found the X Y part of it from i to j.

Overview of the Algorithm

- Finds constituents and partial constituents in input
  - A → B C . D E is partial: only the first half of the A



A → B C . D E   A → B C D . E

Overview of the Algorithm

- Proceeds incrementally left-to-right
  - Before it reads word 5, it has already built all hypotheses that are consistent with first 4 words
  - Reads word 5 & attaches it to immediately preceding hypotheses. Might yield new constituents that are then attached to hypotheses immediately preceding *them* …
  - E.g., attaching D to A → B C . D E gives A → B C D . E
  - Attaching E to that gives A → B C D E .
  - Now we have a complete A that we can attach to hypotheses immediately preceding the A, etc.

The Parse Table

- Columns 0 through n corresponding to the gaps between words
- Entries in column 5 look like (3, NP → NP . PP)

  (but we'll omit the → etc. to save space)

  - Built while processing word 5
  - Means that the input substring from 3 to 5 matches the initial NP portion of a **NP → NP PP** rule
  - Dot shows how much we've matched as of column 5
  - Perfectly fine to have entries like **(3, VP → is it . true that S)**

The Parse Table

- Entries in column 5 look like (3, NP → NP . PP)
- What will it mean that we have this entry?
  - *Unknown right context: Doesn't* mean we'll necessarily be able to find a VP starting at column 5 to complete the S.
  - *Known left context: Does* mean that some dotted rule back in column 3 is looking for an S that starts at 3.
    - So if we actually do find a VP starting at column 5, allowing us to complete the S, then we'll be able to attach the S to something.
    - And when that something is complete, it too will have a customer to *its* left …
    - In short, a top-down (i.e., goal-directed) parser: it chooses to start building a constituent not because of the input but because that's what the left context needs. In **the spoon**, won't build **spoon** as a verb because there's no way to use a verb there.
    - So any hypothesis in column 5 *could* get used in the correct parse, if words 1-5 are continued in just the right way by words 6-n.

Earley's Algorithm, recognizer version

- Add **ROOT → . S** to column 0.
- For each j from 0 to n:
  - For each dotted rule in column j, (including those we add as we go!) look at what's after the dot:
    - If it's a word w, SCAN:
      - If w matches the input word between j and j+1, advance the dot and add the resulting rule to column j+1
    - If it's a non-terminal X, PREDICT:
      - Add all rules for X to the bottom of column j, wth the dot at the start: e.g. **X → . Y Z**
    - If there's nothing after the dot, ATTACH:
      - We've finished some constituent, A, that started in column I<j. So for each rule in column j that has A after the dot: Advance the dot and add the result to the bottom of column j.
- Output "yes" just if last column has **ROOT → S .**
- **NOTE: Don't add an entry to a column if it's already there!**

Summary of the Algorithm

- Process all hypotheses one at a time in order. (Current hypothesis is shown in blue.)
- This may add new hypotheses to the end of the to-do list, or try to add old hypotheses again.
- Process a hypothesis according to what follows the dot:
  - If a word, **scan** input and see if it matches
  - If a nonterminal, **predict** ways to match it
    - (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible with this limited *right context*)
  - If nothing, then we have a complete constituent, so **attach** it to all its customers

A Grammar

S → NP VP	NP → Papa
NP → Det N	N → caviar
NP → NP PP	N → spoon
VP → V NP	V → ate
VP → VP PP	P → with
PP → P NP	Det → the
	Det → a

An Input Sentence

Papa ate the caviar with a spoon.

0
0 ROOT . S

initialize

Remember this stands for (0, ROOT → . S)

0
0 ROOT . S
0 S . NP VP

predict the kind of S we are looking for

Remember this stands for (0, S → . NP VP)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa

predict the kind of NP we are looking for
(actually we'll look for 3 kinds: any of the 3 will do)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of Det we are looking for *(2 kinds)*

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of NP we're looking for
but we were already looking for these so don't add duplicate goals! Note that this happened when we were processing a left-recursive rule.

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: the desired word is in the input!

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach the newly created NP (which starts at 0) to its customers (incomplete constituents that *end* at 0 and have NP after the dot)

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	
0 Det . a	

predict

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	

predict

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 P . with

predict

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: success!

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: failure

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

attach

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

predict

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict *(these next few steps should look familiar)*

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

scan *(this time we fail since Papa is not the next word)*

0 Papa 1 ate 2			the 3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

scan: success!

0 Papa 1 ate 2			the 3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa 1 ate 2			the 3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa 1 ate 2			the 3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach
(again!)

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach
(again!)

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

attach
(again!)

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6 spoon	7
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

e	2 the	3 caviar	4 with	5 a	6 spoon	7
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

e	2 the	3 caviar	4 with	5 a	6 spoon	7
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	4 PP P NP .
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	5 NP NP . PP
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

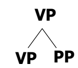
0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

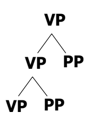
Left Recursion Kills Pure Top-Down Parsing …

VP

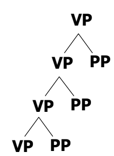
Left Recursion Kills Pure Top-Down Parsing …



Left Recursion Kills Pure Top-Down Parsing …



Left Recursion Kills Pure Top-Down Parsing …



makes new hypotheses ad infinitum before we've seen the PPs at all

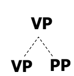
hypotheses try to predict in advance how many PP's will arrive in input

… but Earley's Alg is Okay!

1 VP → . VP PP

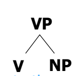
(in column 1)

… but Earley's Alg is Okay!



1 VP → . VP PP

(in column 1)

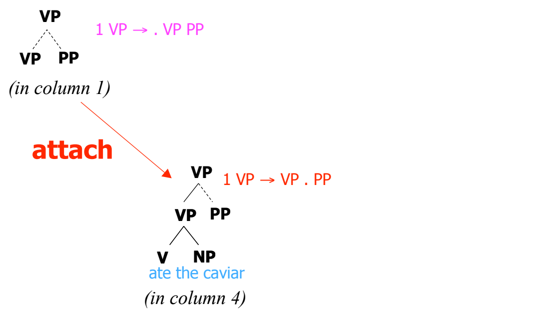


1 VP → V NP .

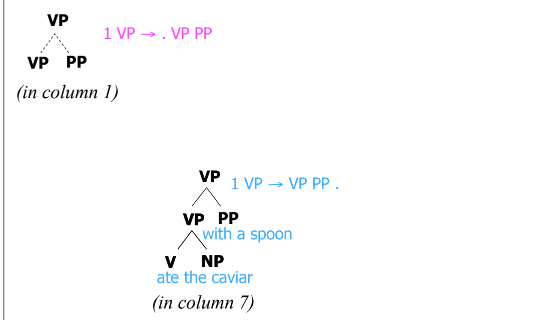
ate the caviar

(in column 4)

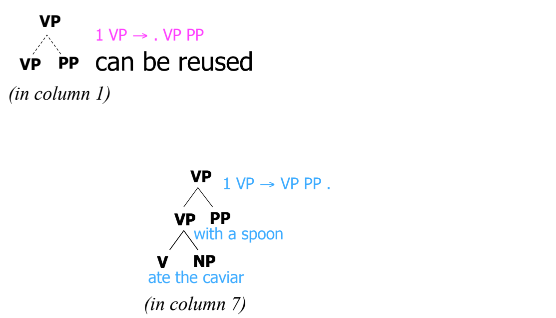
… but Earley's Alg is Okay!



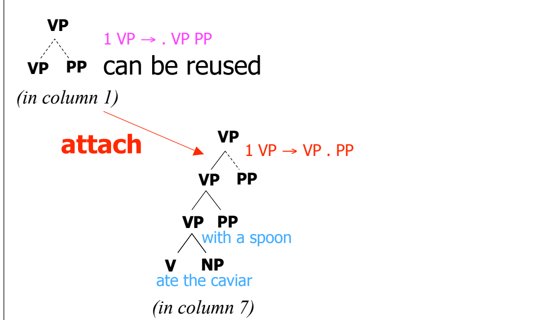
… but Earley's Alg is Okay!



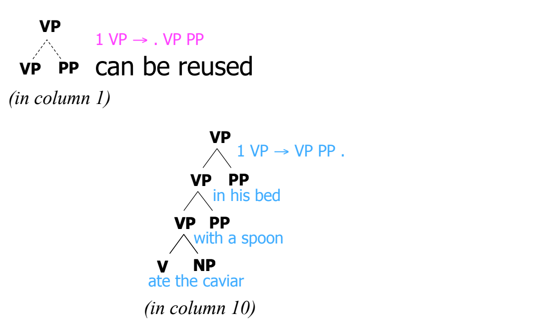
… but Earley's Alg is Okay!



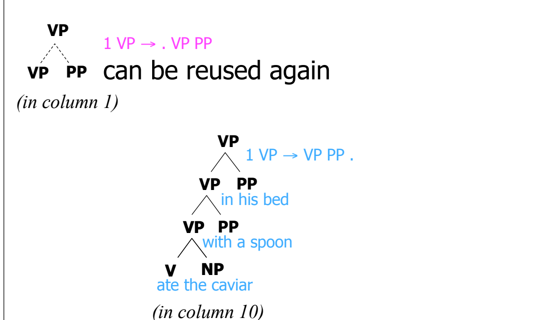
… but Earley's Alg is Okay!



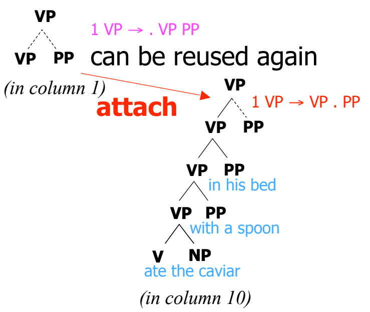
… but Earley's Alg is Okay!



… but Earley's Alg is Okay!



… but Earley's Alg is Okay!



0 Papa	1 ate	2 the	3 caviar	4 with a spoon	…	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed a VP in col 4
col 1 lets us use it in a VP PP structure

0 Papa	1 ate	2 the	3 caviar	4 with a spoon	…	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed that VP = VP PP in col 7
col 1 would let us use *it* in a VP PP structure
can reuse col 1 as often as we need

How to change the parser into a recognizer?

What's the Complexity?

What's the Complexity?

- How many state sets will there be?
  - Length of sentence, n
- How big can the state sets get?
  - Size of grammar, G, times n
- How long does it take to build a state set?
  - Scan
    - Constant time
  - Predict
    - Need to check for duplicates
  - Complete
    - Search previous state set, also check for duplicates, $(Gn)^2$
- Total: $O(n^3)$