Direct Shape Optimization for Strengthening 3D Printable Objects

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Introduction

- Printable 3D objects that need to withstand significant external force.

- Coffee Table
- Wrench
- Coat Hanger
Introduction

- Printable 3D objects that need to withstand significant external force.
Introduction

- Given an input 3D shape,
Given an input 3D shape, the direction and strength of external force,
Given an input 3D shape, the direction and strength of external force, and the boundary,
Introduction

- The object will deform under such external force. The regions under high stress may break.

Stress Analysis
Our goal is to optimize the shape, such that the resulting shape can successfully withstand the external force, and remain as similar as possible to the input shape.
Introduction

- Silhouette image showing the difference between the original (gray) and optimized (purple) shape.

Original vs. Optimized
Related Work

- Strengthening Printable Objects
  - Hollowing [SVB*12, LSZ*14, VGB*14]
  - Internal skin framing [WWY*13]
  - Support struts [SVB*12]
  - Change printing directions [HBA13, US13]
  - Part thickening [SVB*12, US13]
  - Controllable shape design [MDLW15]
Related Work

- Shape Optimization in Computer Graphics
  - Balance [PWLSH13]
  - Spinnability [BWBSH14]
  - Aggregate mass [MAB*15]
  - Inverse elastic shape design [CZXZ14]
  - Microstructures [PZM*15, SBR*15]
Related Work

- Shape Optimization in Mechanical Engineering
  - Parametric surface [HM03, WMC08, BCC*10]
  - B-splines/Beziers, subdivision surfaces [BRC16]
  - Level-set [AJT02, AJ08, DMLK13]
  - Specialized topology modifications [All12, BS13]
  - Procedural models [BR88, RG92, Tor93]
Contributions

- A general, extensible method to optimize 3D shapes under physical and geometric constraints.
- Operates directly on the input mesh.
- Integrated physics simulation with optimizer.
  - Derivations of analytic gradient and Hessian
We use [Trelis] for tetrahedralization
Overview

- $X_0$: Reference State
- $X$: Rest State
- $P$: Deformed State
Overview

- $X_0$: Reference State
- $X$: Rest State
- $P$: Deformed State
Overview

$X_0$
Reference State

$X$
Rest State

$P$
Deformed State
Overview

$X_0$
Reference State

$X$
Rest State

$P$
Deformed State
Formulation – Constrained Optimization

Solve for rest state $X$ that minimize an objective function while satisfying the given constraints.

Objective: $\arg\min_X D(X, X^0)$

Constraints:
- $\forall v \in B: x_v = p_v$, $\forall v \notin B: f_v(X, P) = 0$
- $\forall t: \hat{\sigma}_t^2(X, P) < C$
- $g(X, X^0) = 0$

Simulation
Stress
Geometric
1. Simulation Constraints

\[ X_0 \]
Reference State

\[ \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \]

Boundary Conditions
1. Simulation Constraints

$X_0$
Reference State

$\forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0$

Force Equilibrium
1. Simulation Constraints

\[ \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \]

Reference State

\[ X_0 \]

Force Equilibrium
Elasticity Model

Rest State

Deformed State

Restoration Force

$X$

$P$
Elasticity Model

$X$
Rest State

$P$
Deformed State

Diagram showing the transition from rest state to deformed state.
Elasticity Model

Strain Energy: \( U_t(X, P) = \frac{V_t}{2} \varepsilon_t : E \varepsilon_t \)

Strain tensor \( \varepsilon_t \)

Material tensor \( E \)
1. Simulation Constraints

\[ \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \]

Boundary Conditions  Force Equilibrium

\(X_0\)
Reference State
2. Stress Constraints

∀t: \( \hat{\sigma}_t^2 (X, P) < C \)

von Mises Stress
Material’s Yield Strength
3. Geometric Constraints

- **Symmetry** Constraints:  \( S_m x_i = x_j \)

- **Interior Uniformity** Constraints:  \( x_i = \frac{1}{|N(i)|} \sum_{j \in N(i)} x_j \)

- **User-Defined** Constraints

All in the form of linear, equality constraints:

\[ g(X, X^0) = 0 \]
Symmetry Constraints

(a) original shape

\[ S_m x_i = x_j \]
User-Defined Constraints

(a) original shape  (b) w/o user constraints  (c) with user constraints
Without User-Defined Constraints
With User-Defined Constraints
Objective Function

Constraints: \( \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \)
\( \forall t: \partial_t^2 (X, P) < C \)
\( g(X, X^0) = 0 \)

Simulation
Stress
Geometric
Objective Function

Objective: \( \operatorname{argmin}_X D(X, X^0) \)

\[
D(X, X^0) = w_1 D_{\text{intrinsic}}(X, X^0) + w_2 D_{\text{extrinsic}}(X, X^0)
\]

- **Extrinsic**: L2 distance between vertices
  - Preserves overall shape
- **Intrinsic**: transformed surface Laplacians
  - Preserves surface details and smoothness
  - [Sorkine et al. 2004]
Objective Function

(a) original shape
Solving the Optimization

Objective: \( \argmin_X D(X, X^0) \)

Constraints: \( \forall \nu \in B: x_\nu = p_\nu, \forall \nu \notin B: f_\nu(X, P) = 0 \)

- \( \forall t: \hat{\sigma}_t^2(X, P) < C \)
- \( g(X, X^0) = 0 \)

- **Stress Constraints are Inequality constraints:**
  - Use the penalty method.
Objective:  \( \arg \min_X D(X, X^0) + \delta \cdot \sum_t h(C - \hat{\sigma}_t^2(X, P)) \)

Constraints:  \( \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \)

\[ g(X, X^0) = 0 \]

- \( h \) is a penalty function
- \( \delta \) is the penalty weight
  - We start from a small weight and progressively increase it across iterations.
KKT System and Newton’s Method

Objective:  \( \text{argmin}_X D(X, X^0) + \delta \cdot \sum h(C - \hat{\sigma}_t^2(X, P)) \)

Constraints:  \( \forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0 \)

\[ g(X, X^0) = 0 \]

\[
\begin{bmatrix}
  H & J^T \\
  J & 0
\end{bmatrix}
\begin{bmatrix}
  \Delta x \\
  w
\end{bmatrix}
= -
\begin{bmatrix}
  g \\
  b
\end{bmatrix}
\]

- \( g, H \): gradient and Hessian of the objective
- \( b, J \): function value and Jacobian of constraints
Source code, data, and supplemental materials available for download from our paper website.
Results

- Coat Hanger Example:

  Endure 50% more force
Results

- Coat Hanger Example:

  100% more
Results

- Coat Hanger Example: 200% more
Results

- Coat Hanger Example:

  50% more

  100% more

  200% more
Comparison with Local Thickening

(a) Original shape

(b) Our method

- 20% less material
- Better at preserving surface features
Gallery of Results – Force and Boundary
Gallery of Results – 50% More Force
Gallery of Results – 100% More Force
Gallery of Results – 200% More Force
Physical Validation
Physical Validation

- Optimized shapes withstand 100% more force.
- PLA material, 100% infill, 100 micron resolution
- Equalize volume for fair comparisons.
Physical Validation

Table (twisted leg)

- Force (kN)
- Head Extension (mm)
- Optimized
- Original

- 282N
- 425N
Physical Validation

Stool (four-leg)

Force (kN)

Head Extension (mm)

- 523 N
- 1605 N

Optimized
Original
Conclusion

- An algorithm to directly optimize a 3D mesh to make it withstand specified external force.
- Integrates optimization and physics simulation in a unified framework.
- Derivations of analytic gradient and Hessian of the objective function.
- Applications to printable object design.
Limitations and Future Work

- Performance, convergence speed
- Tetrahedralization quality
- Incorporating higher-order Laplacians [BS08]
- Applications to other design goals, such as improving aerodynamic properties of shapes.
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