[It may help you to read sections 7.1 through 7.3 before completing this homework. Prof. McGregor will give lecture L23 which will entirely cover any of the relevant material.] For each of the following worlds $W_i$ and PredCalc formulas $\varphi_i$ do the following steps:

a. Convert $\varphi$ to an equivalent formula $\psi$ in NNF. If $\varphi$ is already in NNF then just let $\psi := \varphi$. If $\varphi$ uses any abbreviations (from the crib sheet for the second test), first change the abbreviation to its meaning and then convert to NNF.

b. Say what $\psi$ means in normal clear English, i.e., avoid words like "forall" and "there exists" and especially avoid variable names.

c. Play the Truth Game as in L22 and L23 to determine whether or not $W \models \psi$. Please play according to the "Moodle" rules, so that the player chooses the first winning move if there is one, play the first possible move, i.e. 0 if $\psi$ starts with a quantifier and $\alpha$ if $\psi = \alpha \vee \beta$ or $\psi = \alpha \wedge \beta$.

$$\varphi_1 \stackrel{\text{def}}{=} \forall x \forall y \ (E(x,y) \ \rightarrow \ A(x) \wedge R(y))$$

$$\varphi_2 \stackrel{\text{def}}{=} \forall x (A(x) \rightarrow \exists y \, E(x,y))$$

$$\varphi_3 \stackrel{\text{def}}{=} \forall x \forall y \ (E(x,y) \wedge E(x,z) \rightarrow y = z)$$

$$\varphi_4 \stackrel{\text{def}}{=} \forall x \ (A(x) \rightarrow \exists! y \, E(x,y))$$

$$\varphi_5 \stackrel{\text{def}}{=} \forall x \forall y \ (E(x,z) \wedge E(y,z) \ \rightarrow \ x = y)$$

$$\varphi_6 \stackrel{\text{def}}{=} \forall x \exists y \ (x \cdot y = 1)$$

$$\varphi_7 \stackrel{\text{def}}{=} \forall y \exists x \ (R(y) \rightarrow E(x,y))$$

$$\varphi_8 \stackrel{\text{def}}{=} \forall x \forall y \ (G(x,y) \rightarrow \exists z \ (E(x,z) \wedge F(z,y)))$$

$$W_5 \stackrel{\text{def}}{=} (\mathbf{N}, A^{W_5} = \mathbf{N}, R^{W_5} = \mathbf{N}, E^{W_5} = \big\{(a, 2a) \ \big| \ a \in \mathbf{N}\big\})$$

$$W_6 \stackrel{\text{def}}{=} \mathbf{Z}_6^*$$

$$W_7 \stackrel{\text{def}}{=} (\mathbf{N}, A^{W_7} = \mathbf{N}, R^{W_7} = \mathbf{N}, E^{W_7} = \big\{(a, 2a) \ \big| \ a \in \mathbf{N}\big\})$$

$$W_8 \stackrel{\text{def}}{=} (\mathbf{N}, E^{W_8} = \big\{(a, \lfloor a/2 \rfloor) \ \big| \ a \in \mathbf{N}\big\}, F^{W_8} = \big\{(a, 2a) \ \big| \ a \in \mathbf{N}\big\}, G^{W_8} = \big\{(a, a - (a\%2)) \ \big| \ a \in \mathbf{N}\big\})$$