In 1, these were longer and more tedious than I meant for them to be. Please skip what had been part \( \chi \), i.e., don’t bother putting the quantifier-free part into CNF! In the following example, \( 2 \rightarrow 1 + 1 \). Note that since it is just a term, we don’t have to expand the abbreviation.

1.0: \( \alpha_0 = \forall x \ (\text{prime}(x) \land 2 | x \rightarrow x = 2) \)
   \[
   \equiv \forall x \ (\neg \text{prime}(x) \lor 2 | x \lor x = 2) \\
   \equiv \forall x \ (\neg \text{prime}(x) \lor 2 | x) \\
   \equiv \forall x \ (\neg \text{prime}(x) \lor 2 | x) \\
   \equiv \forall x \ (x \neq 0 \land x \neq 1 \land \forall y (y | x \rightarrow y = 1 \lor y = x)) \lor \neg \exists z 2 * z = z \lor x = 2) \\
   \equiv \forall x \ (x = 0 \lor x = 1 \lor \neg \forall y (y | x \rightarrow y = 1 \lor y = x) \lor \forall z 2 * z \neq x \lor x = 2) \\
   \equiv \forall x \ (x = 0 \lor x = 1 \lor \exists y \neg (y | x \rightarrow y = 1 \lor y = x) \lor \forall z 2 * z \neq x \lor x = 2) \\
   \equiv \forall x \ (x = 0 \lor x = 1 \lor \exists y (y | x \land y \neq 1 \land y \neq x) \lor \forall z 2 * z \neq x \lor x = 2) \\
   \equiv \forall x \ (x = 0 \lor x = 1 \lor \exists y \exists z_1 (x = 0 \lor x = 1 \lor (y \land z_1 = x \land y \neq 1 \land y \neq x) \lor \forall z 2 * z \neq x \lor x = 2) \\
   \equiv \forall x \exists y \exists z_1 \forall z (x = 0 \lor x = 1 \lor (y \land z_1 = x \land y \neq 1 \land y \neq x) \lor 2 * z \neq x \lor x = 2) \\
   \equiv \forall x \forall z (x = 0 \lor x = 1 \lor (f(x) \land g(x) = x \land f(x) \neq 1 \land f(x) \neq x) \lor 2 * z \neq x \lor x = 2) \\
   \equiv \text{The only prime divisible by 2 is 2.}
   \]

2. Write a natural deduction proof that \( \vdash \forall x \exists y r(x) = y \). \textbf{Hint}: The main operator is \( \forall \), so use use \( \forall - i \). Thus you have the subgoal, \( \exists y r(x_0) = y \) where \( x_0 \) is a new variable. Thus, in the subgoal, since the main operator is \( \exists \), use \( \exists - i \). Thus your sub-subgoal is to proven \( r(x_0) = t_0 \) for some term \( t_0 \) that you have to construct. The main connective in the sub-subgoal is “\( = \)”, so I suggest you use \( = - i \).

3. Note that for problem 3 – if you haven’t already – you should reread Rosen’s section on functions. I show how to do:

\[ E_7 = \text{“r is an onto function”} \]

Thus, \( \varphi_7 \) is neither. Let \( U^A_7 = U^B_7 = \{0, 1\} \). Let \( r^A_7 = \{(0, 0), (1, 1)\} \) and \( r^B_7 = \{(0, 0), (1, 0)\} \)