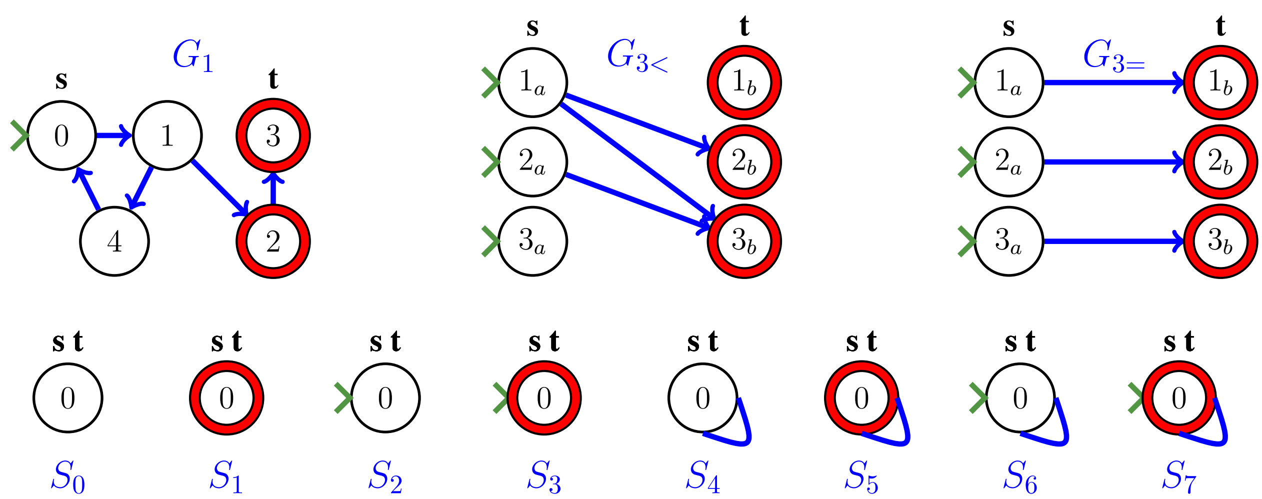
CS250: Disc4: Hw2 Ans and Review for First Test 10/5/16



1-2 This problem and the next concern representing functions as graphs. We have already talked about this via the Arrow Diagrams in [Epp]. See also the L3 slides and the solutions to the R9 Quiz in the L10 slides. Write forumulas in PredCalc(Σ_{garst}) that express the following. (These use the Greek letter rho (ρ) for Relation.)

(a) $\rho_a \equiv$ The Domain is contained in A, i.e., every edge starts at a vertex satisfying A.
$\rho_a \;\equiv\; \forall xy(E(x,y) \to A(x))$ true in $G_{3<}, G_{3=}, S_0, S_1, S_2, S_3, S_6, S_7$; all but G_1, S_4, S_5

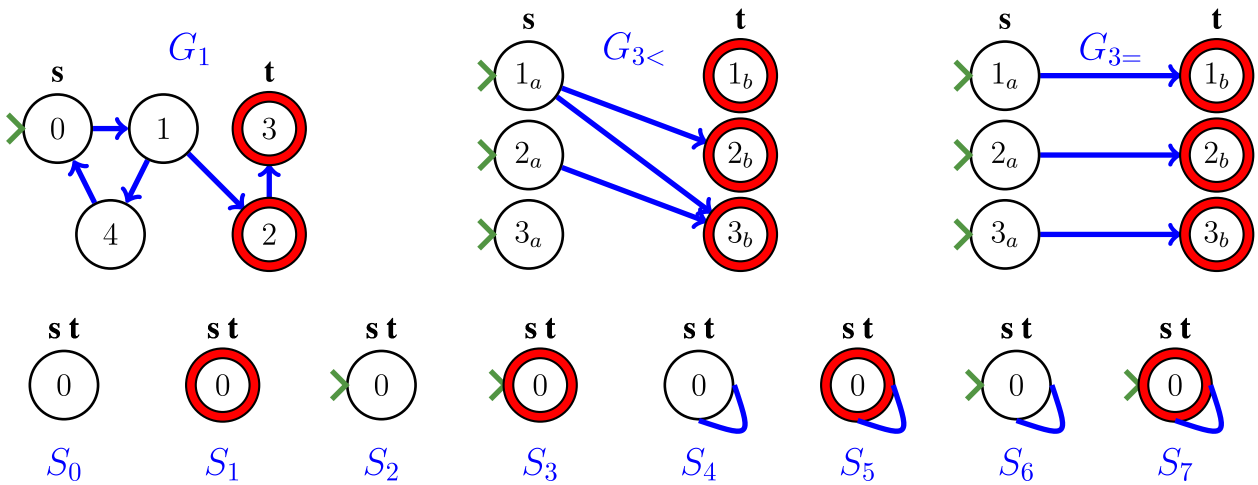
(b) $\rho_b \equiv$ The Domain contains A, i.e., every A vertex has an edge coming out of it.
$\rho_b \;\equiv\; \forall x \exists y(A(x) \to E(x,y))$ true in $G_1, G_{3=}, S_0, S_1, S_4, S_5, S_6, S_7$; all but $G_{3<}, S_2, S_3$

(c) $\rho_c \equiv$ The Range is contained in R, i.e., every edge ends at a vertex satisfying R.
$\rho_c \;\equiv\; \forall x \forall y(E(x,y) \to R(y))$ true in $G_{3<}, G_{3=}, S_0, S_1, S_2, S_3, S_5, S_7$; all but G_1, S_4, S_6

(d) $\rho_d \equiv$ The Range contains R, i.e., every R vertex has an edge going into it.
$\rho_d \;\equiv\; \forall y \exists x(R(y) \to E(x,y))$ true in $G_1, G_{3=}, S_0, S_2, S_4, S_5, S_6, S_7$; all but $G_{3<}, S_1, S_3$

(e) $\rho_e \equiv A$ and R are disjoint.
$\rho_e \;\equiv\; \forall x({\sim}A(x) \vee \;{\sim}R(x))$ true in $G_1, G_{3<}, G_{3=}, S_0, S_1, S_2, S_4, S_5, S_6$; all but S_3, S_7

(f) $\rho_f \equiv$ The relation is single valued, i.e., no vertex has two distinct edges leaving it.
$\rho_f \;\equiv\; \forall xyz(E(x,y) \wedge E(x,z) \to y = z)$ true in $G_{3=}, S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7$; all but $G_1, G_{3<}$



3-4 Consider the vocabulary of sets: $\Sigma_{\text{set}} = (\in^2[\text{infix}]; \emptyset)$ where "$\in$" represents the set membership relation, and the constant symbol "$\emptyset$" represents the emptyset. In this problem, we will represent sets as graphs, where the membership relation is represented by edges, and the emptyset is represented by the constant, s. For example, the statement, "The emptyset is empty" would be written as follows in PredCalc(Σ_{set}), $\sigma_a \stackrel{\text{def}}{=} \forall x\,(\sim x \in \emptyset)$. Note that $\sim x \in \emptyset$ is the same as $\sim (x \in \emptyset)$. The parentheses are not necessary, because the "$\sim$" applies to the atomic formula "$x \in \emptyset$". Note also, that we will typically use the abbreviation "$a \notin b$ to mean $\sim a \in b$. Thus, $\sigma_a \equiv \forall x\,(x \notin \emptyset)$. This would be translated into PredCalc(Σ_{garst}) as $\gamma_a \equiv \forall x\,(\sim E(x, s))$. true in all but G_1, S_4. (Note the hw2 Quiz on moodle did not give this option, so everyone will get full credit on that question.)

Write formulas in PredCalc(Σ_{garst}) that express the following.

b. $\gamma_b \equiv$ the axiom of extension: two elements of the universe are equal if they contain exactly the same elements.

$\gamma_b \;\equiv\; \forall xy((\forall z(E(z,x) \leftrightarrow E(z,y)) \to x = y)$ true in S_0, S_4

c. $\gamma_c \equiv x$ is a $subset$ of y.

$\gamma_c \;\equiv\; \forall z(E(z,x) \to E(z,y)$ true in $G_1, G_{3<}, G_{3=}, S_0, S_4$

d. $\gamma_d \equiv x$ is a $proper subset$ of y.

$\gamma_d \;\equiv\; x \neq y \;\wedge\; \forall z(E(z,x) \to E(z,y)$ true in none of the worlds.

e. $\gamma_e \equiv$ No set is a member of itself.

$\gamma_e \;\equiv\; \forall x\; \sim E(x,x)$ true in all of the worlds except S_4