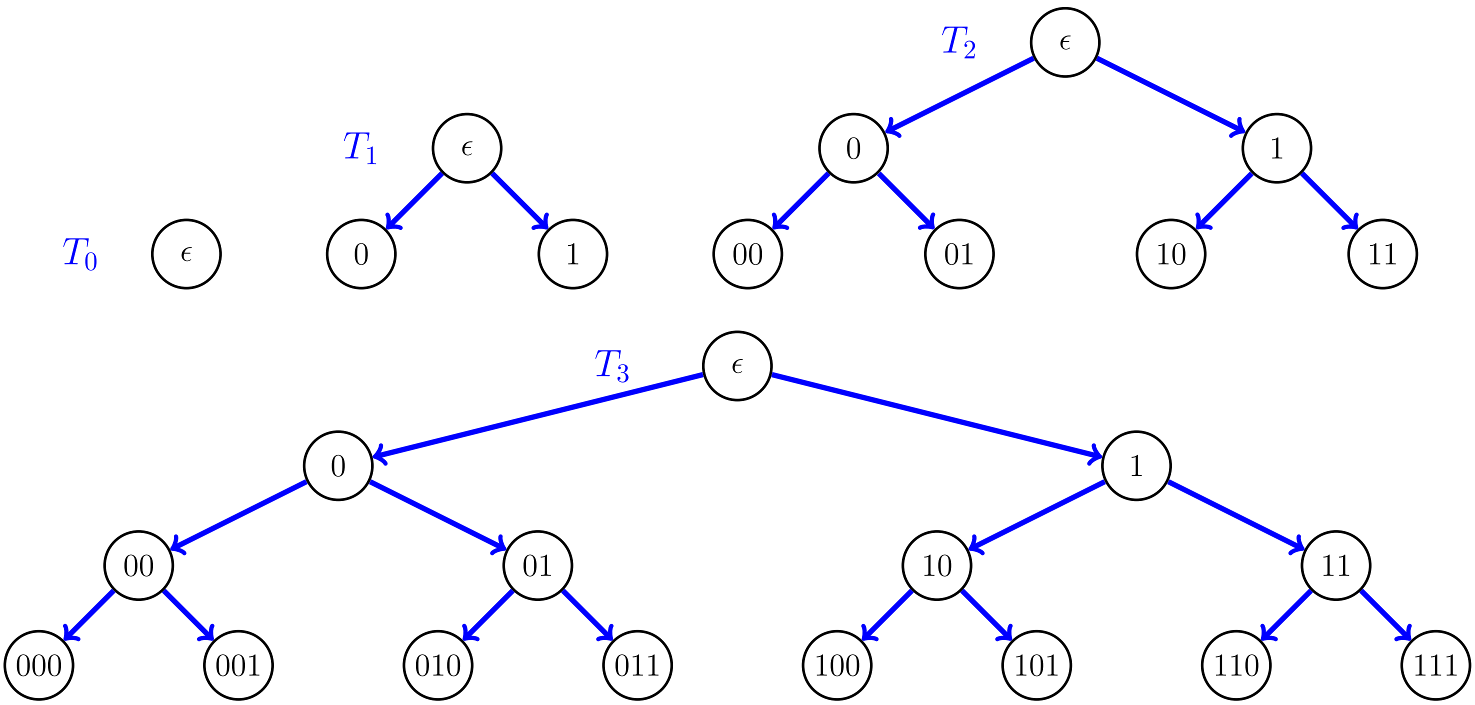
CS250: D6: Mathematical Induction 10/26/16

As usual, you will join random groups of 4 according to the card you are given. Please work together in your groups to understand and solve today's problems. There will be a D6 moodle quiz for you to fill in your answers by Thursday night, 11 p.m.

Def. A **complete binary tree**, T_h, of height h is a directed graph with a single root vertex, ϵ, of height h, and such that each vertex of height $t > 0$ has exactly two children of height $t - 1$. See T_0, T_1, T_2, and T_3 below.



For a world, W, recall that $\|W\|$ denotes the cardinality of the universe, $|W|$. Thus, for a graph, T, $\|T\|$ is the number of vertices of T.

1. Calculate $\|T_0\|$: $\|T_1\|$: $\|T_2\|$: $\|T_3\|$:

2. Make a conjecture of the form, $\forall h \in \mathbf{N}\ \|T_h\| = f(h)$. What is f?

3. Now try to prove your conjucture by induction. The base case should be easy to prove. However, the inductive step may be difficult.

4. Sometimes, as in this case, in order to prove something by induction, it is helpful to have a **stronger**, i.e., more informative hypothesis. What else did you want to know as you tried to prove the inductive case? By the way, the vertices of height 0 in a tree, i.e., those with no outgoing edges are called **leaves**.

5. Make a stronger conjecture of the form $\mathbf{N} \models \forall x\ (\alpha(x))$. What is $\alpha(x)$?

6. In your group, work out and write down a complete and correct inductive proof of your conjecture. Be sure to clearly identify the following, **base case:**

   indHyp: