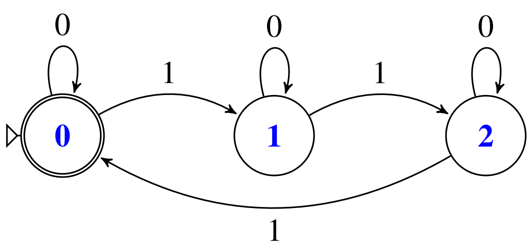
CS250:	D10 Answers: DFA's and Regular Expressions	Fall 2016

1. $L_a = \{w \in \{0,1\}^\star \mid \#_1(w) \equiv 0 \pmod 3\} = \mathcal{L}((0|10^*10^*1)^*)$ L_a is regular and accepted by this DFA:



2. $L_b = \{0^i 1^j 0^{i+j} \mid i,j \in \mathbf{Z}^+\}$ is not regular. We prove this using the Pumping Lemma.

Assume: L_b is accepted by DFA D with n states.

you (G) choose string: $w \in L_b = \mathcal{L}(D)$

Let $w = 0^n 1^n 0^{2n}$

By **pumping lemma**, D chooses $x, y, z \in \{a,b\}^\star$, s.t.,

1. $w = 0^n 1^n 0^{2n} = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbf{N}\ (\ xy^k z \in L_b\ )$

Since $0 < |xy| \leq n$, $y = 0^i,\ 0 < i \leq n$

Thus $xy^0z = 0^{n-i}1^n0^{2n} \in L_b$.

but $0^{n-i}1^n0^{2n} \notin L_b$

$\Rightarrow\Leftarrow$

Therefore L_b is **not DFA acceptable**. $\square$

3. $L_c = \{a^i b^j \mid i,j \in \mathbf{N};\quad i+j \equiv 1 \pmod 2\} = \mathcal{L}(a(aa)^*|((aa)^*b|a(aa)^*bb)(bb)^*)$

L_c is regular and accepted by this DFA: