CS250:	D10: DFA's and Regular Expressions	12/7/16

As usual, break into groups according to the random card you receive as you enter. Work together in your group to solve the following three problems. You will enter your solutions, together with answers to one or two similar problems in the Last Discussion Quiz, due noon, Tuesday, Dec. 13.

Prove whether or not each of the following three languages L is regular. If L is regular, provide **both** a regular expression and a DFA that accepts exactly L. If L is not regular, use the pumping lemma as we did in L31. For your convenience, there is a statement of the Pumping Lemma on the reverse side and the proof that $E = \{a^i b^i \mid i \in \mathbf{N}\}$ is not regular. [Regular means acceptable by a DFA, which is equivalent to being specifiable by a regular expression. We will prove the equivalence in Lecture 33.]

1. $L_a = \{w \in \{0,1\}^\star \mid \#_1(w) \equiv 0 \pmod 3\}$ [binary strings containing a number of 1's divisible by 3]

2. $L_b = \{0^i 1^j 0^{i+j} \mid i, j \in \mathbf{Z}^+\}$

3. $L_c = \{a^i b^j \mid i, j \in \mathbf{N};\quad i + j \equiv 1 \pmod 2\}$

Pumping Lemma for Regular Sets:

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, $\quad n = |Q|$ $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$

Then $\quad \exists x, y, z \in \Sigma^\star$ s.t.

1. $xyz = w$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \geq 0 \quad (xy^k z \in \mathcal{L}(D))$

Prop: $E = \left\{ a^r b^r \;\middle|\; r \in \mathbf{N} \right\}$ is not DFA acceptable.

proof by contradiction:

Assume: E is accepted by DFA D with n states.

you (G) choose string: $\quad w \in E = \mathcal{L}(D)$

Let $w \; = \; a^n b^n$

By **pumping lemma**, D chooses $x, y, z \in \{a, b\}^\star$, s.t.,

1. $w \; = \; a^n b^n \; = \; xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbf{N} \; (\; xy^k z \; \in \; E \;)$

Since $0 < |xy| \leq n, \quad y = a^i, \; 0 < i \leq n$

Thus $xy^0 z = a^{n-i} b^n \in E$.

but $\quad a^{n-i} b^n \notin E$.

F

Therefore E is **not DFA acceptable**. $\square$