In this discussion you will use the Pumping Lemma to prove that two languages are not accepted by DFAs. First, here is a reminder from L30 of the statement and use of the pumping lemma.

**Pumping Lemma for Regular Sets:** Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Let $n = |Q|$. Let $w \in \mathcal{L}(D)$ s.t. $|w| \geq n$. Then $\exists x, y, z \in \Sigma^*$ s.t. the following all hold:

1. $xyz = w$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \geq 0 \ (xy^kz \in \mathcal{L}(D))$

**Prop:** $E = \{a^rb^r \mid r \in \mathbb{N}\}$ is not DFA acceptable.

**proof by contradiction:**

**Assume:** $E$ is accepted by DFA $D$ with $n$ states.

**you (Dumbledore) choose string:** $w \in E = \mathcal{L}(D)$

Let $w = a^nb^n$

By **pumping lemma**, Gandalf chooses $x, y, z \in \{a, b\}^*$, s.t.,

1. $w = a^nb^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^kz \in \mathcal{L}(D))$

Since $0 < |xy| \leq n$, $y = a^i$, $0 < i \leq n$

Thus $xy^0z = a^{n-i}b^n \in E$.

But by the definition of $E$, $a^{n-i}b^n \notin E$.

Therefore $E$ is **not DFA acceptable**.

On the next two sheets you will use the Pumping Lemma. Try to do both problems, but please at least do the first one.
Proposition 1: Let $S_1 = \{ w \in \{a, b, c\}^* \mid \#_a(w) = \#_b(w) + \#_c(w) \}$. Then $S_1$ is not accepted by a DFA.

Proof: Assume that an $n$-state DFA accepts $S_1$.

You choose $w \in S_1$ s.t. $|w| \geq n$:

Gandalf chooses $x, y, z \in \{a, b, c\}^*$ s.t.

1. $w = a^n b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} ( xy^k z \in S_1 )$

You choose $k$ and derive a contradiction from the fact that $xy^k z \in S_1$. □
Proposition 2: Let $P = \{ w \in \{a, b, \}^* \mid w = w^R \}$. Then $P$ is not accepted by a DFA.
[Note that $P$ is the set of palindromes, i.e. words that read the same forward as back, e.g., ablewasiereisawelba.]

Proof: Assume that an $n$-state DFA accepts $P$.

You choose $w \in P$ s.t. $|w| \geq n$:

Gandalf chooses $x, y, z \in \{a, b, c\}^*$ s.t.

1. $w = a^n b^n = xyz$
2. $|xy| \leq n$
3. $|y| > 0$, and
4. $\forall k \in \mathbb{N} \ (xy^k z \in P )$

You choose $k$ and derive a contradiction from the fact that $xy^k z \in P$. 

□