This may seem too easy for a discussion, but I want you all to understand these algorithms very well. Please take the time to make sure that everyone in your group understands all of the steps involved. There will be a question exactly like these discussion problems on t2, where of course you will have to do it without any calculators. I strongly prefer that you do not use calculators in this discussion, because the numbers are not large and I think you will understand this all better if you do it by hand. Dan will spend his time helping any groups that have questions. He will not have time or interest in enforcing a no-calculator rule during this discussion.

For the following natural numbers: \( a_1 = 96, b_1 = 27, a_2 = 105, b_2 = 71 \) and for \( i = 1, 2 \) do the following steps:

1. Run Euclid’s algorithm and show each iteration, e.g., \( a_i = q_i \cdot b_i + r_i \), etc.

2. What is \( \gcd(a_i, b_i) \)?

3. Follow the steps of Euclid’s algorithm backwards to compute \( x_i, y_i \in \mathbb{Z} \) s.t. \( a_i \cdot x_i + b_i \cdot y_i = \gcd(a_i, b_i) \).

4. If \( \gcd(a_i, b_i) = 1 \), then compute \( a_i^{-1} \mod b_i \) and \( b_i^{-1} \mod a_i \). Your answers should be greater than 0 and in the first case less than \( b_i \) in the second case less than \( a_i \).

5. Be careful to check – by multiplying and adding your expressions – that each equation you write is correct!