

Tarski's Definition of Truth

$G \models t_1 = t_2$	iff	$t_1^G = t_2^G$
$G \models P(t_1, \dots, t_n)$	iff	$(t_1^G, \dots, t_n^G) \in P^G$
$G \models \sim \alpha$	iff	$G \not\models \alpha$
$G \models \alpha \wedge \beta$	iff	$G \models \alpha$ and $G \models \beta$
$G \models \alpha \vee \beta$	iff	$G \models \alpha$ or $G \models \beta$
$G \models \forall x(\alpha)$	iff	for all $a \in G $ $G[a/x] \models \alpha$
$G \models \exists x(\alpha)$	iff	exists $a \in G $ $G[a/x] \models \alpha$

Logical Equivalences and Abbreviations

$p \rightarrow q$	\equiv	$\sim p \vee q$
$\sim(p \wedge q)$	\equiv	$\sim p \vee \sim q$
$\sim \forall x \varphi$	\equiv	$\exists x \sim \varphi$
$\sim(p \vee q)$	\equiv	$\sim p \wedge \sim q$
$\sim \exists x \varphi$	\equiv	$\forall x \sim \varphi$
p only if q	\equiv	$p \rightarrow q$
p if q	\equiv	$q \rightarrow p$
p iff q	\equiv	$p \leftrightarrow q$
p is necessary for q	\equiv	$q \rightarrow p$
p is sufficient for q	\equiv	$p \rightarrow q$
p unless q	\equiv	$\sim q \rightarrow p$
$t_1 \neq t_2$	\equiv	$\sim(t_1 = t_2)$
$(\forall x . \alpha)\beta$	\equiv	$\forall x(\alpha \rightarrow \beta)$
$(\exists x . \alpha)\beta$	\equiv	$\exists x(\alpha \wedge \beta)$
$\exists! x(\alpha(x))$	\equiv	$\exists x \forall y(\alpha(x) \wedge (\alpha(y) \rightarrow y = x))$

Natural Deduction Rules

Proviso for \forall -i and \exists -e: x_0 is a "new" variable, i.e., it does not appear in φ , ψ , or Γ .

	introduction	elimination
\wedge	$\frac{\alpha \quad \beta}{\alpha \wedge \beta}$	$\frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$
\vee	$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\beta}{\alpha \vee \beta}$	$\frac{\alpha \vee \beta \quad \alpha \vdash \psi \quad \beta \vdash \psi}{\psi}$
\rightarrow	$\frac{\alpha \vdash \beta}{\alpha \rightarrow \beta}$	$\frac{\alpha \rightarrow \beta \quad \alpha}{\beta} \quad \frac{\alpha \rightarrow \beta \quad \sim \beta}{\sim \alpha}$
F	$\frac{\alpha \quad \sim \alpha}{\mathbf{F}}$	$\frac{\alpha \vdash \mathbf{F}}{\sim \alpha} \quad \frac{\sim \alpha \vdash \mathbf{F}}{\alpha}$
$\sim \sim$	$\frac{\alpha}{\sim \sim \alpha}$	$\frac{\sim \sim \alpha}{\alpha}$
$=$	$\overline{t = t}$	$\frac{t_1 = t_2 \quad \varphi[t_1/x]}{\varphi[t_2/x]}$
\forall	$\frac{\Gamma \vdash \varphi[x_0/x]}{\Gamma \vdash \forall x \varphi}$	$\frac{\forall x \varphi}{\varphi[t/x]}$
\exists	$\frac{\varphi[t/x]}{\exists x \varphi}$	$\frac{\Gamma \vdash \exists x \varphi \quad \Gamma, \varphi[x_0/x] \vdash \psi}{\Gamma \vdash \psi}$