Preamble to the R3 quiz

After you have done reading R3, please print out and understand this sheet. This will be the introduction to the R3 Quiz and you should have this sheet with you when you do the quiz. Please ask on Piazza before doing the R3 quiz if you have any questions about this preamble.

Recall that "R is a **relation** from A to B" means exactly that $R \subseteq A \times B$

and we use the notation $f : A \to B$, pronounced, "f is a **function** from A to B" to mean exactly that $f \subseteq A \times B$, i.e., f is a relation from A to B, and
f is **defined** on domain A: $\forall a \in A\ \exists b \in B\ (a, b) \in f$, and
f is **single valued**: $\forall (a, b), (a', b') \in f\ (a = a' \ \rightarrow\ b = b')$.

From now on, we will use A^2 as an abbreviation for $A \times A$.

The following relations from **Z** to **Z** will be useful. For $k \in \mathbf{Z}^+$, let DkD be the set of ordered pairs of integers, (a, b) such that their difference is divisible by k:

$$\mathrm{D}k\mathrm{D} \stackrel{\text{def}}{=} \left\{(a, b) \in \mathbf{Z}^2 \;\middle|\; \frac{a-b}{k} \in \mathbf{Z}\right\}$$

For example, D2D is the set of pairs of integers whose difference is divisible by 2, i.e.,

$$\mathrm{D2D} = \{(0,0), (0,2), (0,-2), \ldots, (1,1), (1,3), (1,-1), \ldots\}$$

Recall that we are also using $[n]$ as an abbreviation for the set of the first n positive integers: $[n] = \{1, 2, \ldots, n\}$. We often want to restrict relations $R \subseteq \mathbf{Z}^2$ to relations from $[m]$ to $[n]$. Define $R_{n,m}$ to be that restriction:

$$R_{n,m} \stackrel{\text{def}}{=} R \cap ([n] \times [m]) = \left\{(a, b) \in R \;\middle|\; 1 \le a \le n \ \wedge\ 1 \le b \le m\right\}$$

Furthermore, when $n = m$, we will abbreviate $R_{n,n}$ as just R_n. For example, $\mathrm{D2D}_{2,3}$ is exactly the relation A of Example 1.3.1 on page 15 of Epp.