Natural Deduction, which is a complete set of reasoning rules, includes, in addition to the five pairs of PropCalc Rules which we have already studied, three more pairs: $=, \forall, \exists$. The = rules are both easy to understand as are $\forall$-e and $\exists$-i. We will learn these four easy rules in L10. We will save the two remaining hard rules for right after the first test.

The complete list of Natural Deduction Proof Rules is as follows. Don't worry about $\forall$-i or $\exists$-e for now, nor the provisos, which refer only to these two difficult rules.

Natural Deduction Rules for PredCalc

	introduction	elimination	proviso
$\wedge$	$\dfrac{\varphi \quad \psi}{\varphi \wedge \psi}$	$\dfrac{\varphi \wedge \psi}{\varphi} \quad \dfrac{\varphi \wedge \psi}{\psi}$	
$\vee$	$\dfrac{\varphi}{\varphi \vee \psi} \quad \dfrac{\psi}{\varphi \vee \psi}$	$\dfrac{\varphi \vee \psi \quad \varphi \vdash \psi \quad \psi \vdash \psi}{\psi}$	
$\rightarrow$	$\dfrac{\varphi \vdash \psi}{\varphi \rightarrow \psi}$	$\dfrac{\varphi \rightarrow \psi \quad \varphi}{\psi} \quad \dfrac{\varphi \rightarrow \psi \quad \sim\psi}{\sim\varphi}$	
F	$\dfrac{\varphi \quad \sim\varphi}{\mathbf{F}}$	$\dfrac{\varphi \vdash \mathbf{F}}{\sim\varphi} \quad \dfrac{\sim\varphi \vdash \mathbf{F}}{\varphi}$	
$\sim\sim$	$\dfrac{\varphi}{\sim\sim\varphi}$	$\dfrac{\sim\sim\varphi}{\varphi}$	
$=$	$\overline{t = t}$	$\dfrac{t_1 = t_2 \quad \varphi[t_1/x]}{\varphi[t_2/x]}$	
$\forall$	$\dfrac{\Gamma \vdash \varphi[x_0/x]}{\Gamma \vdash \forall x\ \varphi}$	$\dfrac{\forall x\ \varphi}{\varphi[t/x]}$	x_0 does not occur in $\Gamma \cup \{\varphi\}$
$\exists$	$\dfrac{\varphi[t/x]}{\exists x\ \varphi}$	$\dfrac{\Gamma \vdash \exists x\ \varphi \quad \Gamma, \varphi[x_0/x] \vdash \psi}{\Gamma \vdash \psi}$	x_0 does not occur in $\Gamma \cup \{\varphi, \psi\}$

The main rule that Epp discusses in Section 3.4 is $\forall$-e, which she calls "universal instantiation". She uses this together with $\rightarrow$-e. She calls the combined rule, "Universal Modus Ponens". We will instead use the Natural Deduction rule names, $\forall$-e and $\rightarrow$-e.

The following example is famous. Let the unary predicates symbols, M, D refer to being a man, and being mortal, and let the constant symbol, s, refer to Socrates. Thus, $\forall x\,(M(x) \rightarrow D(x))$ means that all men are mortal. Here is the famous proof written as a Natural Deduction.

1	$\forall x\,(M(x) \rightarrow D(x))$	
2	$M(s)$	
3	$M(s) \rightarrow D(s)$	$\forall$-e, 1
4	$D(s)$	$\rightarrow$-e, 2, 3

Note that $\forall$-e says that if we know $\forall x\ \varphi(x)$, then we may conclude $\varphi(t)$ for any term, t. For example, in line 1, we know $\forall x\ \varphi$, where $\varphi \stackrel{\text{def}}{=} (M(x) \rightarrow D(x))$. We can thus conclude $\varphi[s/x]$ in line 3. We substituted the term s for the free variable x in φ. That is $\varphi[s/x] = (M(s) \rightarrow D(s))$.