CS250: Notation

$\mathbf{N}$	$= \mathbf{Z}^{\text{nonneg}} = \{0, 1, 2, \ldots\}$	the **Natural numbers**
$A \times B$	$= \{(a, b) \mid a \in A \wedge b \in B\}$	**cross product** of A, B
A^2	$= A \times A$	
$f : A \to B$	f is a **function** with **domain** A and **co-domain** B	
$\lvert A \rvert$		**cardinality** of set A
$\aleph_0$	$= \lvert\mathbf{N}\rvert$	if $\lvert S\rvert \leq \aleph_0$ then S is **countable**
$W \models \alpha$	W models α	α is true in world W
$\alpha \vdash \beta$	α proves β	we can prove β using assumption α
$\Gamma \vdash \beta$	Γ proves β	we can prove β using assumption set Γ

Easy to identify symbols, from now on

vocabularies	=	$\Sigma, \Sigma_{\mathrm{Tarski}}, \Sigma_{\mathrm{garst}}, \Sigma_{\mathrm{\#thy}}, \ldots$
PredCalc variables	=	$x, y, z, u, v, w, x_1, y_1, z_1, \ldots$
PropCalc variables	=	$p, q, r, p_1, q_1, r_1, \ldots$
constant symbols	=	$s, t, k, k_1, \ldots$
function symbols	=	$f, g, h, f_1, g_1, h_1, \ldots$
predicate symbols	=	$A, R, E, P, P_1, P_2, \ldots$
worlds	=	$W, S, T, G, H, W_1, S_1, T_1, G_1, H_1, \ldots$
elements of $\|W\|$	=	$a, b, c, d, e, a_1, b_1, \ldots$
logical formulas	=	$\alpha, \quad \beta, \quad \gamma, \quad \varphi, \quad \psi, \ \ldots$
	=	**alpha,beta,gamma,phi,psi,** $\ldots$

Abbreviations

$\hookrightarrow$ is an abbreviation for "is an abbreviation for"

$$
\begin{array}{rcl}
t_1 \neq t_2 & \hookrightarrow & \sim(t_1 = t_2) \\
(\forall x \,.\, \alpha)\beta & \hookrightarrow & \forall x(\alpha \to \beta) \\
(\exists x \,.\, \alpha)\beta & \hookrightarrow & \exists x(\alpha \wedge \beta) \\
\exists! x(\alpha(x)) & \hookrightarrow & \exists x \forall y(\alpha(x) \wedge (\alpha(y) \to y = x)) \\
x < y & \hookrightarrow & x \leq y \ \wedge \ x \neq y \\
x|y & \hookrightarrow & \exists z\, (x \cdot z = y) \\
\mathrm{prime}(x) & \hookrightarrow & 1 < x \ \wedge \ \forall y\, (1 < y \wedge y|x \ \to \ y = x)
\end{array}
$$

Precedence of of Logical Operators

1. $\forall, \exists, \sim$ bind the **tightest**
2. $\wedge, \vee, \oplus$
3. $\rightarrow, \leftrightarrow$

$\rightarrow$ binds **from right**: $\alpha \rightarrow \beta \rightarrow \gamma \;\equiv\; \alpha \rightarrow (\beta \rightarrow \gamma)$

$$\alpha_1 \rightarrow \alpha_2 \rightarrow \alpha_3 \rightarrow \alpha_4 \quad \equiv \quad \alpha_1 \rightarrow (\alpha_2 \rightarrow (\alpha_3 \rightarrow \alpha_4))$$

if α_1 then (if α_2 then (if α_3 then α_4))

$$\begin{aligned} \varphi &\stackrel{\text{def}}{=} \forall x\, E(x,y) \rightarrow y = t \\ &\equiv (\forall x\, E(x,y)) \rightarrow y = t \end{aligned}$$

$$\psi \stackrel{\text{def}}{=} \forall x\, (E(x,y) \rightarrow y = t)$$

more about $\varphi \not\equiv \psi$ **in L9 when we define truth**