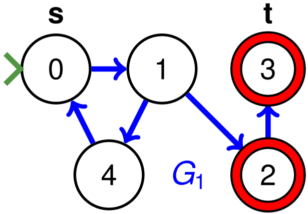
CS250: Discrete Math for Computer Science

L8: More PredCalc

vocabulary of graphs $\Sigma_{\text{garst}} = (A^1, R^1, E^2; s, t)$



$G_1 \in \text{World}[\Sigma_{\text{garst}}]$ = **set of graphs** + pred. A, R; const. s, t

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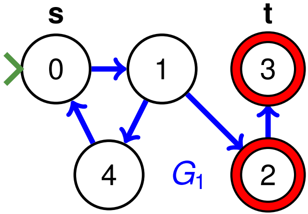

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$$|G_1| \;=\; V^{G_1} \;=\; \{0, 1, \ldots, 4\}$$ the **vertices**

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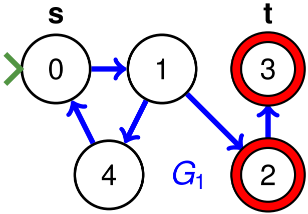

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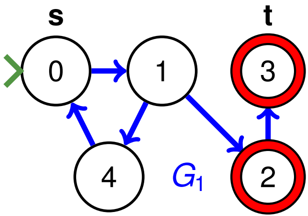

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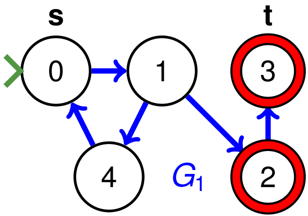

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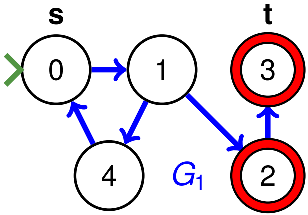
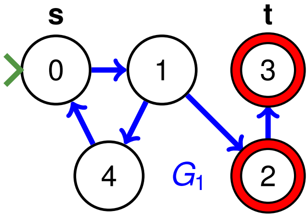
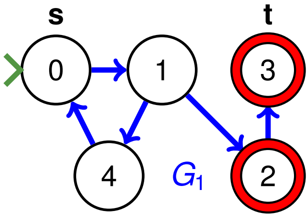

s
t
0
1
3
4
G_1
2



T/F: $G_1 \models \forall x \exists y\, E(x, y)$



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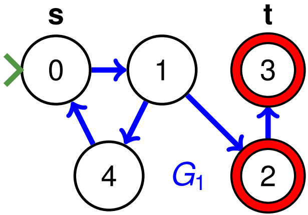

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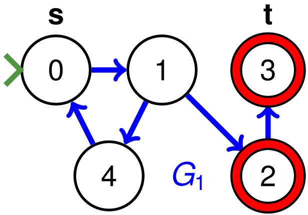

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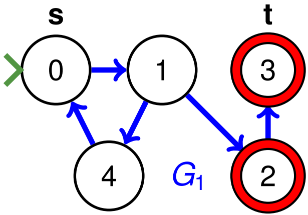

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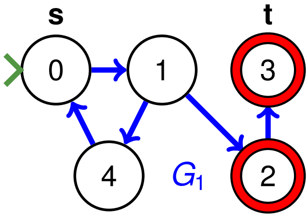

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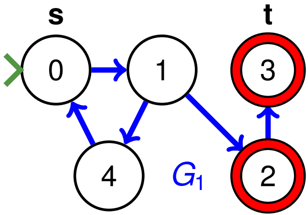

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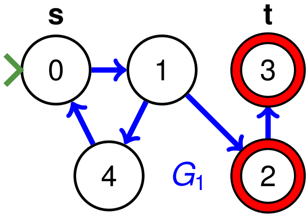

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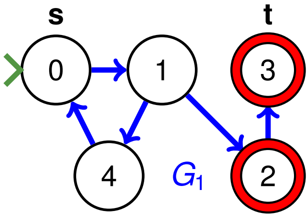

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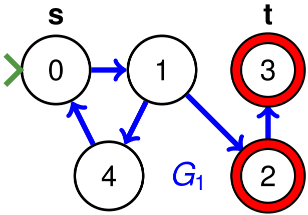
T/F: $G_1 \models A(s) \wedge R(t)$



T/F: $G_1 \models \forall x \exists y\ E(x, y)$		**False**
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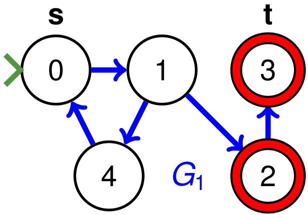

iClicker 8.1 T/F: $G_1 \models \forall x \exists y\ E(y, x)$

A: True **B: False**

T/F: $G_1 \models \forall x \exists y\ E(x, y)$ **False**

T/F: $G_1 \models \exists x \forall y\ \sim E(x, y)$ **True**

T/F: $G_1 \models \exists x\ E(x, x)$ **False**

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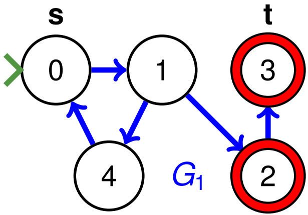
T/F: $G_1 \models A(s) \wedge R(t)$ **True**



iClicker 8.2 T/F: $G_1 \models \forall x \exists y\, (\sim R(x) \to E(x,y))$

A: True **B: False**

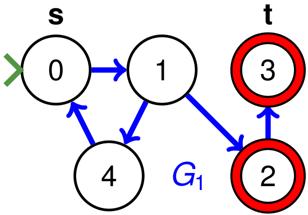
T/F: $G_1 \models \forall x \exists y\; E(x,y)$ **False**

T/F: $G_1 \models \exists x \forall y\; \sim E(x,y)$ **True**

T/F: $G_1 \models \exists x\; E(x,x)$ **False**

T/F: $G_1 \models \forall x \sim E(x,x)$ **True**

T/F: $G_1 \models A(s) \land R(t)$ **True**



iClicker 8.3 T/F: $G_1 \models \exists x\,(x = s \wedge A(x))$

A: True **B: False**

T/F: $G_1 \models \forall x \exists y\, E(x,y)$ **False**

T/F: $G_1 \models \exists x \forall y\, \sim E(x,y)$ **True**

T/F: $G_1 \models \exists x\, E(x,x)$ **False**

T/F: $G_1 \models \forall x \sim E(x,x)$ **True**

T/F: $G_1 \models A(s) \wedge R(t)$ **True**

Easy to identify symbols, from now on

vocabularies = $\Sigma, \Sigma_{\text{Tarski}}, \Sigma_{\text{garst}}, \ldots$

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$$
\begin{aligned}
\textbf{vocabularies} &= \Sigma, \Sigma_{\text{Tarski}}, \Sigma_{\text{garst}}, \ldots \\
\textbf{PredCalc variables} &= x, y, z, u, v, w, x_1, y_1, z_1, \ldots
\end{aligned}
$$

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elements of $\vert\mathbf{W}\vert$	=	$a, b, c, d, e, a_1, b_1, \ldots$

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elements of |W|	=	$a, b, c, d, e, a_1, b_1, \ldots$
logical formulas	=	$\alpha, \quad \beta, \quad \gamma, \quad \varphi, \quad \psi, \ \ldots$

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	=	**alpha,beta,gamma,phi,psi,** ...

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$\exists ! x(\alpha(x))$	$\hookrightarrow$	$\exists x \forall y(\alpha(x) \wedge (\alpha(y) \rightarrow y = x))$

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$$\equiv (\forall x\, E(x, y)) \rightarrow y = t$$

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more about $\varphi \not\equiv \psi$ **in L9 when we define truth**

Negation of PredCalc Formulas

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Negation of PredCalc Formulas

$$\sim\forall x(f) \quad \equiv \quad \exists x(\sim f) \qquad \textbf{deMorgan} \qquad \sim(p \land q) \equiv (\sim p \lor \sim q)$$

$$\sim\exists x(f) \quad \equiv \quad \forall x(\sim f) \qquad \textbf{deMorgan} \qquad \sim(p \lor q) \equiv (\sim p \land \sim q)$$

$$\alpha \quad \stackrel{\text{def}}{=} \quad \forall x \exists y\ E(y, x)$$

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$$\alpha \overset{\text{def}}{=} \forall x \exists y\, E(y, x)$$

$\alpha \equiv$ **"All vertices have an incoming edge."**

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$\alpha \equiv$ **"All vertices have an incoming edge."**

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Usually **best** to write φ in **NNF**

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Answers to R8 Quiz

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Ans = “Some dogs are disloyal”

“There is a dog that is disloyal”

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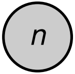
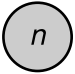
Ans = $\exists$ computer program *P*
(*P* compiles without errors but *P* is not correct)

"There is at least one incorrect computer program
which compiles without errors."

N_1 = 

1. $N_1 \models \forall x(\text{Triangle}(x) \rightarrow \text{Blue}(x))$ **True:** vacuously.
2. $N_1 \models \forall x(\text{Blue}(x) \rightarrow \text{Triangle}(x))$ **True:** vacuously.
3. $N_1 \models \forall x(\text{Square}(x) \rightarrow \exists y \text{LeftOf}(y, x))$ **True:** vacuously.
4. $N_1 \models \forall x(\text{Gray}(x) \rightarrow \text{Circle}(x))$ **True:** All elts. are Circles.
5. $N_1 \models \forall x(\text{Circle}(x) \rightarrow \text{Gray}(x))$ **True:** All Circles are Gray.
6. $N_1 \models \exists x \forall y(\text{LeftOf}(y, x) \vee y = x)$ **True:** there is only 1 elt.

$N_1 \quad = $ 

7. $N_1 \models \exists x \forall y (\text{Above}(y, x) \vee y = x)$
True: there is only 1 elt.

8. $N_1 \models \forall x \exists y (\text{Triangle}(x) \rightarrow \text{LeftOf}(x, y) \wedge \text{Circle}(y))$
True: vacuously.

9. $N_1 \models \forall x \exists y (\text{Triangle}(x) \rightarrow \sim\text{Above}(x, y) \wedge \sim\text{Above}(y, x) \wedge \text{Circle}(y))$
True: vacuously.

10. $N_1 \models \exists x \exists y (\text{Square}(x) \wedge \text{Circle}(y) \wedge \mathit{Above}(y, x) \wedge \text{Leftof}(x, y))$
False: there is no square.

Disc. 2 Answers

$R_1 =$ If I don't drink beer with dinner, then I always have fish.

$R_2 =$ Any time I have both beer and fish for dinner,
I do without ice cream.

$R_3 =$ If I have ice cream or don't have beer, then I never eat fish.

$b \equiv$ "I drink beer" $f \equiv$ "I have fish" $i \equiv$ "I have ice cream"

Translate each R_i into PropCalc:

$R_1 \equiv \sim b \rightarrow f$ $R_2 \equiv b \wedge f \rightarrow \sim i$ $R_3 \equiv i \vee \sim b \rightarrow \sim f$

Next, draw a truth table for these three rules.

W	b	f	i	$\sim b \;\rightarrow\; f$	$b \wedge f \;\rightarrow \sim i$	$i \vee \sim b \;\rightarrow \sim f$	S
W_7	1	1	1	1	0	0	0
W_6	1	1	0	1	1	1	1
W_5	1	0	1	1	1	1	1
W_4	1	0	0	1	1	1	1
W_3	0	1	1	1	1	0	0
W_2	0	1	0	1	1	0	0
W_1	0	0	1	0	1	1	0
W_0	0	0	0	0	1	1	0

Finally, using your truth table, try to come up with a simpler PropCalc formula equvalent to $S \stackrel{\text{def}}{=} R_1 \wedge R_2 \wedge R_3$.

$S \;\equiv\; b \wedge (\sim i \vee \sim f)$ "I always have beer, and I never have both fish and ice cream at the same meal."