Generalization of PropCalc:

rich enough to express all of mathematics!
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Let’s start by reviewing the R7 Quiz.
1. $T_1 \models \forall x (\text{Triangle}(x) \rightarrow \text{Blue}(x))$ **True**: all triangles are blue.
2. $T_1 \models \forall x (\text{Blue}(x) \rightarrow \text{Triangle}(x))$ **False**: $e$ is a Blue Square.
3. $T_1 \models \forall x (\text{Square}(x) \rightarrow \exists y \text{LeftOf}(y, x))$ **True**: $a$ is LeftOf all the Squares.
4. $T_1 \models \forall x (\text{Gray}(x) \rightarrow \text{Circle}(x))$ **True**: All Gray elements are Circles.
5. $T_1 \models \forall x (\text{Circle}(x) \rightarrow \text{Gray}(x))$ **False**: Not all Circles are Gray.
6. $T_1 \models \exists x \forall y (\text{LeftOf}(y, x) \lor y = x)$ **True**: there is a unique rightmost element.
7. $T_1 \models \exists x \forall y (\text{Above}(y, x) \lor y = x)$
   \textbf{False:} there is no unique bottom most element.

8. $T_1 \models \forall x \exists y (\text{Triangle}(x) \rightarrow \text{LeftOf}(x, y) \land \text{Circle}(y))$
   \textbf{True:} every Triangle has a Circle to its right.

9. $T_1 \models \forall x \exists y (\text{Triangle}(x) \rightarrow \sim \text{Above}(x, y) \land \sim \text{Above}(y, x) \land \text{Circle}(y))$
   \textbf{True:} Every Triangle shares its row with a circle.

10. $T_1 \models \exists x \exists y (\text{Square}(x) \land \text{Circle}(y) \land \text{Above}(y, x) \land \text{LeftOf}(x, y))$
    \textbf{True:} witnesses are $x := h$ and $y := d$. 
iClicker 7.1  True or False:

\[ T_1 \models \forall xy (\text{Gray}(x) \land \text{LeftOf}(x, y) \rightarrow \sim \text{Blue}(y)) \]?

A: True  B: False
iClicker 7.1 True or False: 

\[ T_1 = \forall xy (\text{Gray}(x) \land \text{LeftOf}(x, y) \rightarrow \neg \text{Blue}(y)) \]

A: True B: False

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iClicker 7.2 True or False:

\[ T_1 \models \forall x \exists y (\text{Circle}(y) \land \neg \text{Above}(x, y) \land \neg \text{Above}(y, x)) \]?

A: True B: False
\[ T_1 = \]

\[
\begin{array}{cccccc}
a & b \\
& c & \triangle \ \\
& e & f \\
& g & h \\
\end{array}
\]

**Tarski’s World vocabulary:** \[ \Sigma_{\text{Tarski}} = \]

\[
(Circle^1, \ Square^1, \ Triangle^1, \ Blue^1, \ Black^1, \ Gray^1, \ Above^2, \ LeftOf^2; )
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\[ \Sigma_{\text{Tarski}} \text{ has six predicate symbols of arity 1, and two of arity 2.} \]
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A non-empty universe, \( |T_1| = \{a, b, c, d, e, f, g, h, i, j, k\} \)
$T_1 =$

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A **structure** or **world**, $T_1 \in \text{World}[\Sigma_{\text{Tarski}}]$ has:

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and a predicate of the correct arity over its universe for each predicate symbol in $\Sigma_{\text{Tarski}}$. 
$T_1 = \text{Circle}^{T_1} = \{b, d, f, i, k\}$

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\[
\begin{array}{|c|c|}
\hline
\text{Circle} & \text{Square} \\
\hline
\begin{array}{|c|c|c|}
\hline
\text{a} & \text{b} & \text{c} \\
\hline
\text{d} & \text{e} & \text{f} \\
\hline
\text{g} & \text{h} & \text{i} \\
\hline
\text{j} & \text{k} & \text{a} \\
\hline
\end{array}
\end{array}
\]

\[
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Circle$^{T_1} = \{b, d, f, i, k\}$

Square$^{T_1} = \{e, h, j\}$

Triangle$^{T_1} = \{a, c, g\}$

Blue$^{T_1} = \{a, c, e, g\}$

Black$^{T_1} = \{b, h, j\}$
$T_1 =$

Circle$^{T_1}$ = \{b, d, f, i, k\}
Square$^{T_1}$ = \{e, h, j\}
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\[ T_1 = \]

\[
\begin{array}{cccc}
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\text{c} & \text{d} \\
\text{e} & \text{f} \\
\text{g} & \text{h} \\
\text{i} & \text{j} \\
\text{k} & \\
\end{array}
\]

\[
\begin{align*}
\text{Circle}^{T_1} &= \{ b, d, f, i, k \} \\
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\text{Gray}^{T_1} &= \{ d, f, i, k \} \\
\text{Above}^{T_1} &= \{ (a, c), \ldots, (a, k), \ldots, (i, k) \}
\end{align*}
\]

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\[ T_1 = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline
\text{ } & a & b & c & d & e & f & g & h & i & j & k \\ \hline
\text{Circle} & b & d & f & i & k \\ \text{Square} & e & h & j \\ \text{Triangle} & a & c & g \\ \text{Blue} & a & c & e & g \\ \text{Black} & b & h & j \\ \text{Gray} & d & f & i & k \\ \text{Above} & (a, c), \ldots, (a, k), \ldots, (i, k) \\ \text{LeftOf} & (a, b), \ldots, (a, k), \ldots, (j, k) \\ \hline
\end{array} \]

\[ N_1 = \begin{array}{c} n \end{array} \]
\[ T_1 = \begin{align*}
\text{Circle}^{T_1} &= \{a, b, c, d, e, f, g, h, i, j, k\} \\
\text{Square}^{T_1} &= \{b, d, f, i, k\} \\
\text{Triangle}^{T_1} &= \{e, h, j\} \\
\text{Blue}^{T_1} &= \{a, c, g\} \\
\text{Black}^{T_1} &= \{b, h, j\} \\
\text{Gray}^{T_1} &= \{d, f, i, k\} \\
\text{Above}^{T_1} &= \{(a, c), \ldots, (a, k), \ldots, (i, k)\} \\
\text{LeftOf}^{T_1} &= \{(a, b), \ldots, (a, k), \ldots, (j, k)\}
\end{align*}\]

\[ N_1 = \{n\} \]
### Exercise: Which sentences of the R7 quiz does Nishal’s world, $N_1$, satisfy?

$N_1 = \{ n \}$