**Thm:** For every $n$-state NFA, $N = (Q, \Sigma, \Delta, s, F)$, there is an equivalent $2^n$-state DFA.

**proof:** We may assume that $N$ has no $\epsilon$ moves. Let $D = (P(Q), \Sigma, \delta, \{s\}, \{T \subseteq Q | T \cap F \neq \emptyset\})$

$$\delta(T, a) = \bigcup q \in T \Delta(q, a)$$

**Claim:** $\forall w \in \Sigma^\star (\delta^\star (\{s\}, w)) = \Delta^\star (s, w)$

**state D is in after w = the set of states N can be in after w**

**pf:** by induction on $|w|$

**base case:**

$\delta^\star (\{s\}, \epsilon) = \{s\} = \Delta^\star (s, \epsilon)$

**inductive case:** IndHyp: for $|w| = n_0$,

$$\delta^\star (\{s\}, w) = \Delta^\star (s, w).$$

Let $|x| = n_0 + 1$. Then $x = wa$ for some $w \in \Sigma^{n_0}, a \in \Sigma$

$$\delta^\star (\{s\}, xa) = \delta(\delta^\star (\{s\}, w), a) = \bigcup q \in \Delta^\star (s, w) \Delta(q, a) = \Delta^\star (s, xa) \Box$$
**Thm:** For every $n$-state NFA, $N = (Q, \Sigma, \Delta, s, F)$, there is an equivalent $2^n$-state DFA.

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$$\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)$$

**Claim:** $\forall w \in \Sigma^* (\delta^*(\{s\}, w) = \Delta^*(s, w))$

**state $D$ is in after $w$ = the set of states $N$ can be in after $w$**
**Thm:** For every $n$-state NFA, $N = (Q, \Sigma, \Delta, s, F)$, there is an equivalent $2^n$-state DFA.

**proof:** We may assume that $N$ has no $\epsilon$ moves. Let

$$D = (P(Q), \Sigma, \delta, \{s\}, \{T \subseteq Q \mid T \cap F \neq \emptyset\})$$

$$\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)$$

**Claim:** $\forall w \in \Sigma^* (\delta^*({s}, w) = \Delta^*(s, w))$

**state D is in after w = the set of states N can be in after w**

**pf:** by induction on $|w|$

**base case:** $\delta^*({s}, \epsilon) = {s} = \Delta^*(s, \epsilon)$
**Thm:** For every $n$-state NFA, $N = (Q, \Sigma, \Delta, s, F)$, there is an equivalent $2^n$-state DFA.

**proof:** We may assume that $N$ has no $\epsilon$ moves. Let

$$D = (P(Q), \Sigma, \delta, \{s\}, \{ T \subseteq Q \mid T \cap F \neq \emptyset \})$$

$$\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)$$

**Claim:** $\forall w \in \Sigma^* \left( \delta^*(\{s\}, w) = \Delta^*(s, w) \right)$

**state D is in after w = the set of states N can be in after w**

**pf:** by induction on $|w|$

**base case:** $\delta^*(\{s\}, \epsilon) = \{s\} = \Delta^*(s, \epsilon)$

**inductive case:** $\text{indHyp:}$ for $|w| = n_0$, $\delta^*(\{s\}, w) = \Delta^*(s, w)$. 
**Thm:** For every $n$-state NFA, $N = (Q, \Sigma, \Delta, s, F)$, there is an equivalent $2^n$-state DFA.

**proof:** We may assume that $N$ has no $\epsilon$ moves. Let

$$D = (P(Q), \Sigma, \delta, \{s\}, \{T \subseteq Q \mid T \cap F \neq \emptyset\})$$

$$\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)$$

**Claim:** $\forall w \in \Sigma^* (\delta^*(\{s\}, w) = \Delta^*(s, w))$

**state D is in after w = the set of states N can be in after w**

**pf:** by induction on $|w|$  

**base case:** $\delta^*(\{s\}, \epsilon) = \{s\} = \Delta^*(s, \epsilon)$

**inductive case:** **indHyp:** for $|w| = n_0$, $\delta^*(\{s\}, w) = \Delta^*(s, w)$. Let $|x| = n_0 + 1$. Then $x = wa$ for some $w \in \Sigma^{n_0}, a \in \Sigma$
**Thm:** For every \( n \)-state NFA, \( N = (Q, \Sigma, \Delta, s, F) \), there is an equivalent \( 2^n \)-state DFA.

**proof:** We may assume that \( N \) has no \( \epsilon \) moves. Let

\[
D = (P(Q), \Sigma, \delta, \{s\}, \{T \subseteq Q \mid T \cap F \neq \emptyset\})
\]

\[
\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)
\]

**Claim:** \( \forall w \in \Sigma^* \ (\delta^*(\{s\}, w) = \Delta^*(s, w)) \)

**state D is in after w = the set of states N can be in after w**

**pf:** by induction on \(|w|\)  
**base case:** \( \delta^*(\{s\}, \epsilon) = \{s\} = \Delta^*(s, \epsilon) \)

**inductive case:** \( \text{indHyp}: \) for \(|w| = n_0, \delta^*(\{s\}, w) = \Delta^*(s, w). \)

Let \(|x| = n_0 + 1\). Then \( x = wa \) for some \( w \in \Sigma^{n_0}, a \in \Sigma \)

\[
\delta^*(\{s\}, wa) = \delta(\delta^*(\{s\}, w), a) = \delta(\Delta^*(s, w), a)
\]
Thm: For every \( n \)-state NFA, \( N = (Q, \Sigma, \Delta, s, F) \), there is an equivalent \( 2^n \)-state DFA.

proof: We may assume that \( N \) has no \( \epsilon \) moves. Let
\[
D = (P(Q), \Sigma, \delta, \{s\}, \{ T \subseteq Q \mid T \cap F \neq \emptyset \})
\]
\[
\delta(T, a) = \bigcup_{q \in T} \Delta(q, a)
\]

Claim: \( \forall w \in \Sigma^* \ (\delta^*(\{s\}, w) = \Delta^*(s, w)) \)

state \( D \) is in after \( w \) = the set of states \( N \) can be in after \( w \)

pf: by induction on \( |w| \) base case: \( \delta^*(\{s\}, \epsilon) = \{s\} = \Delta^*(s, \epsilon) \)

inductive case: indHyp: for \( |w| = n_0 \), \( \delta^*(\{s\}, w) = \Delta^*(s, w) \).

Let \( |x| = n_0 + 1 \). Then \( x = wa \) for some \( w \in \Sigma^{n_0}, a \in \Sigma \)
\[
\delta^*(\{s\}, wa) = \delta(\delta^*(\{s\}, w), a) = \delta(\Delta^*(s, w), a)
\]
\[
= \bigcup_{q \in \Delta^*(s, w)} \Delta(q, a) = \Delta^*(s, wa) \quad \square
\]
\[ \mathcal{L}(N_2) = \mathcal{L}(D_2) \]
$\mathcal{L}(N_2) = \mathcal{L}(D_2)$